Theory of Quantum Gases and Quantum Coherence",
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DYNAMIC BEHAVIOR OF TRAPPED FERMI GASES

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COLLECTIVE OSCILLATIONS:
TEST OF MANY BODY THEORIES.

- Test of **equation of state** (calculated and/or measured at equilibrium)

- Test of **dynamic theories** (hydrodynamics, TDBdG, Landau theory of Fermi liquids, transport properties)

A few examples:

- Test of equation of state along the **BEC-BCS** crossover, effects of **dimensionality, dipolar forces**

- Collective oscillations at **finite T** (first and second sound)

- **Solitons** (decay and collisions)

HYDRODYNAMIC EQUATIONS AT ZERO TEMPERATURE

$$\frac{\partial}{\partial t} n + \nabla(vn) = 0$$

$$m \frac{\partial}{\partial t} v + \nabla(\frac{1}{2} m v^2 + \mu(n) + V_{ext}) = 0$$

irrotationality

Closed equations for **density** and superfluid **velocity** field

- Have **classical** form (do not depend on Planck constant)
- **Irrotationality** follows from phase of order parameter
- Differ from **rotational** or **viscous hydrodynamics**.
- Hold for both **Bose** and **Fermi** superfluids
- Apply to **macroscopic** (low frequency) motion
- Depend on **equation of state** $\mu(n)$
  (sensitive to quantum correlations, statistics, dimensionality, ...)

After linearization HD equations take the form

$$m\omega^2 \delta n = -\nabla(n\nabla\delta\mu) \quad \Rightarrow \quad m\omega^2\delta\mu = -n\frac{\partial\mu}{\partial n}\nabla^2(\delta\mu) + \nabla V_{ext}\nabla\delta\mu$$

Surface modes ($\nabla^2\delta\mu = 0$)

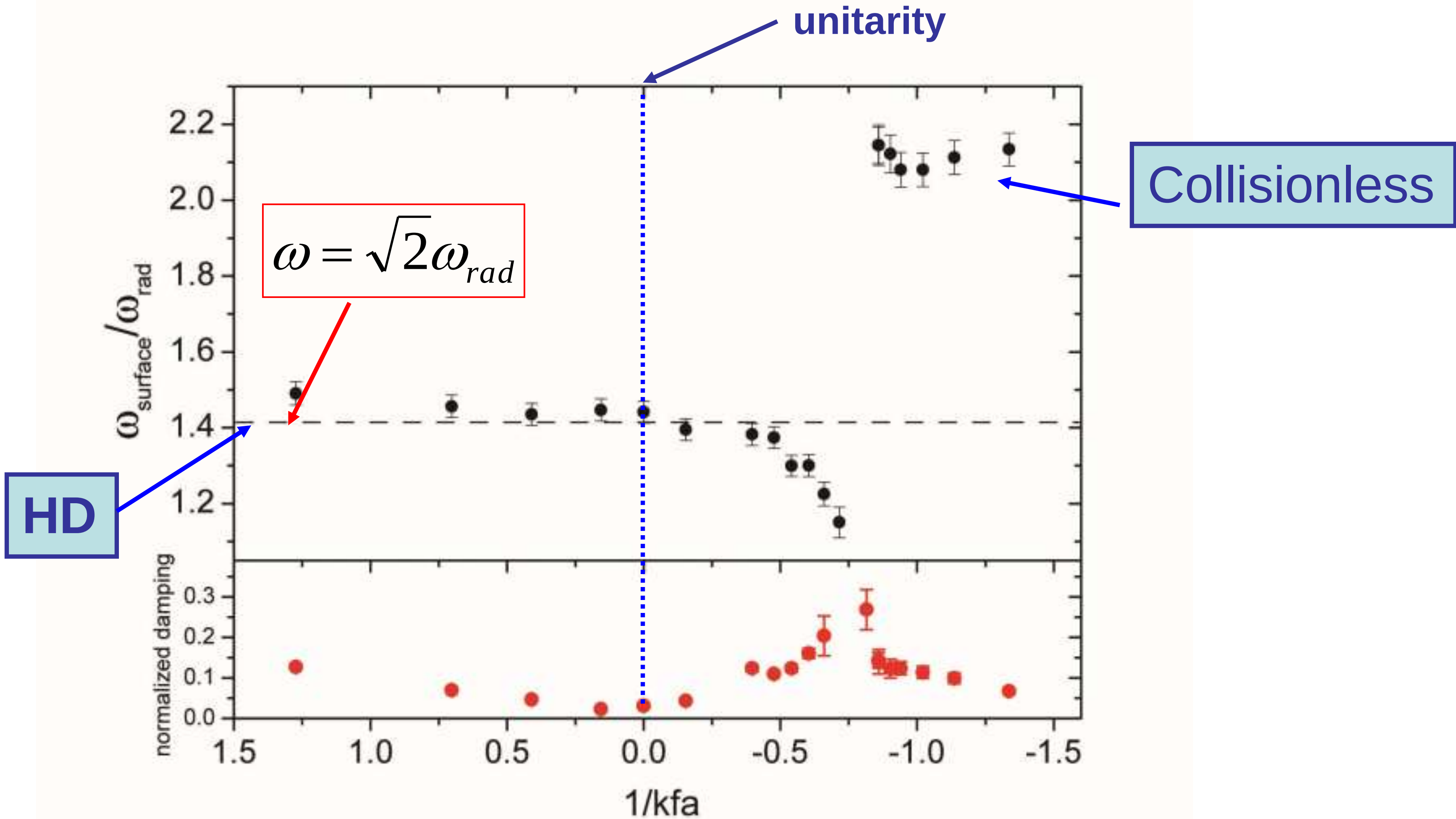
- If $\nabla^2\delta\mu = 0$ HD eqs are insensitive to equation of state

- surface modes are driven by **external potential**, not by surface tension

- For axi-symmetric trapping radial quadrupole mode has frequency $\omega = \sqrt{2}\omega_{rad}$ rather than $\omega = 2\omega_{rad}$ (ideal gas)

- Quadruple mode: useful test of achievement of HD regime

Quadrupole radial mode in Fermi gases
(Innsbruck 2006)



Transition from hydrodynamic to non interacting regime

After linearization HD equations take the form

$$m\omega^2 \delta n = -\nabla n \nabla(\delta\mu) \quad \Rightarrow \quad m\omega^2 \delta\mu = -n\frac{\partial \mu}{\partial n}\nabla^2(\delta\mu) + \nabla V_{ext} \nabla \delta\mu$$

Compression modes ($\nabla^2 \delta\mu \neq 0$)
sensitive to equation of state

- If $\mu \propto n^\gamma$ then $n\partial\mu / \partial n = \gamma(\mu_0 - V_{ext})$ and HD eqs have analytic solutions in the presence of harmonic trapping.

- Radial breathing mode (for $\omega_z << \omega_{rad}$) oscillates with frequency $\omega = \sqrt{2\gamma + 2}\,\omega_{rad}$

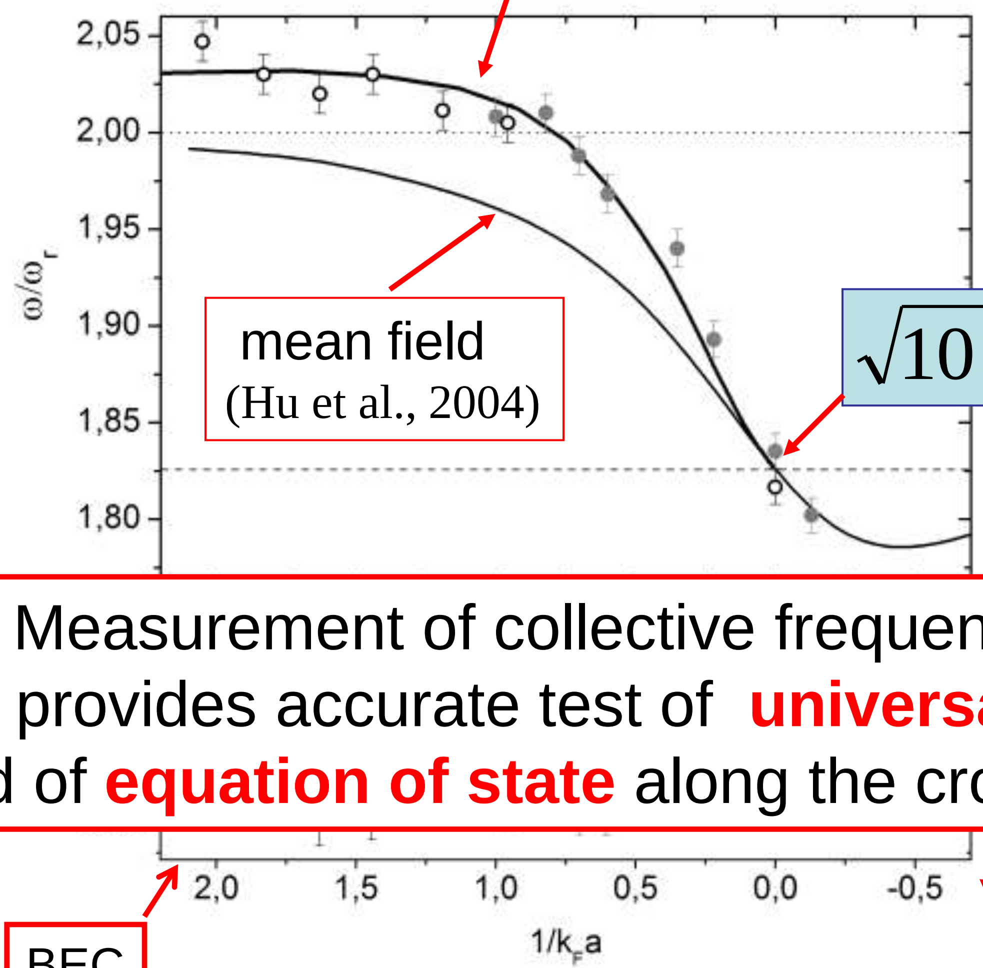
- At unitarity $\gamma = 2/3$ and HD predicts $\omega = \sqrt{10/3}\,\omega_{rad}$

Radial breathing mode in strongly interacting Fermi gases

(BCS-BEC crossover)

MC equation of state (Astrakharchick et al., 2005)

Exp: Innsbruck (2007)



mean field (Hu et al., 2004)

$\sqrt{10/3}$

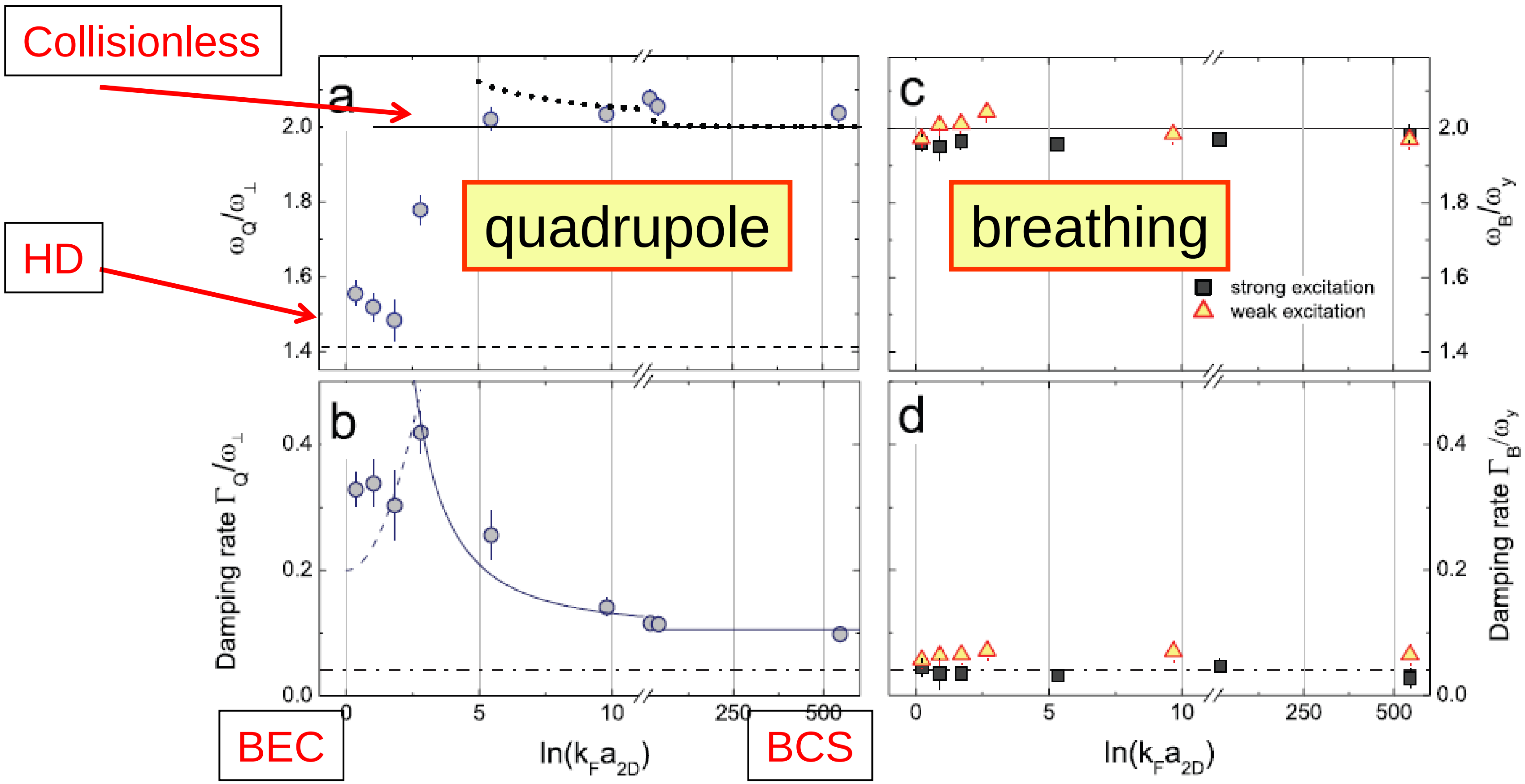
Universality at unitatity

Measurement of collective frequencies provides accurate test of **universality** and of **equation of state** along the crossover!!

BEC

BCS

Collective modes in 2D Fermi gas
(recent exp at Cambridge (Vogt et al PRL 2012)



Questions:

- Is the HD regime due to collisons or to superfluidity ?
- Why is the frequency of the breathing mode constant along the crossover (scale invariance)? log correction to 2D mean field eq. of state $\mu \propto n$

Dipolar forces

$$V_D(\vec{r_1}, \vec{r_2}, \theta) = \frac{d^2(1 - 3\cos^2\theta)}{|\vec{r_1} - \vec{r_2}|^3}$$

Main features of dipolar interaction: anisotropy and long range

First experiments available with magnetic moment (fau , 2005)

Recent advances with electric polar **Fermi molecules** (Boulder 2010)

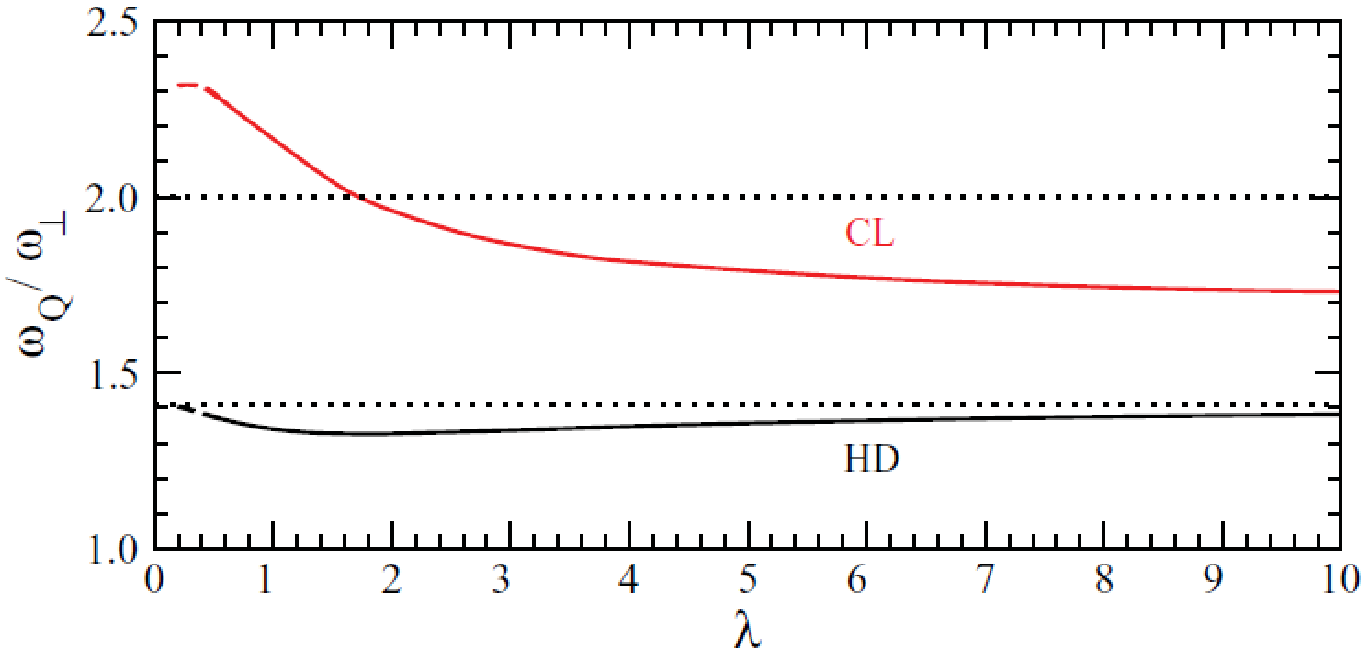
Several theoretical papers on collective oscillations of polar Fermi gases:
Rzazevskii 2004, Pu, 2009, Pelster 2010…

Recent investigation of the transition between HD and collisionless regimes: Marta Abad et al. PRA 2012

Focus on surface **quadrupole** oscillation for axi-sym trap (dipole oriented along z-axis) and effect of **trap deformation**

- Hydrodynamic value $\sqrt{2}\omega_\perp$ only weakly affected by long range nature of dipole force.
- Quadrupole frequency in **collisionless** regime calculated using scaling transformation accounting for Fermi surface deformation (**elastic - zero sound** like effect)
- Shift with respect to ideal gas value $2\omega_\perp$ depends crucially on trap value of **trap deformation** (negative shift for pancakes)

Prediction of **quadrupole frequency** for realistic choice of parameters for trapped KRb polar molecules



$\lambda = \omega_z / \omega_\perp$

- For very pancake configuration system cannot be superfluid (dipole interaction is **repulsive**).
  It will be **collisionless** at low temperatures
- By decreasing deformation the system can become **superfluid** and frequency will jump into **HD** value

A few examples:

- Test of equation of state along the **BEC-BCS** crossover, effects of **dimensionality, dipolar forces**

- Collective oscillations at **finite T** (first and second sound)

- **Solitons** (decay and collisions)

COLLECTIVE OSCILLATIONS AT FINITE TEMPERATURE

Two fluid hydrodynamic theory predicts propagation of first and second sound

First sound is **density** wave
(normal and superfluid components move in phase)

second sound is **temperature** wave

Two-fluid hydrodynamic equations of superfluids (Landau, 1941)

Hold in deep collisional regime $\omega\tau << 1$

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

Ingredients:
- equation of state
- superfluid density

Estimate of collisional time at high temperature (unitary Fermi gas)

(holds also at relatively low T)

Bruun and Smith, 2007

$$\frac{1}{\tau} = \frac{4}{45\pi} \frac{T_F^3}{T^2}$$

⇒

$$\omega_z \tau = \frac{45\pi}{4(6^{1/3})} \frac{T^2}{T_F^2} \frac{\omega_z}{\overline{\omega}_{ho}} \frac{1}{N^{1/3}}$$

HD conditions easily **reached at unitarity** (especially for highly elongated traps). Important advantage of unitary Fermi gas compared to usual BEC's

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

equivalent at T=0

At T=0: $\rho = \rho_S\ ;\ \vec{j} = \rho\vec{v}_S$
eqs. reduce to
T=0 irrotational
superfluid HD equations

*Quite successful to describe the macroscopic dynamic behavior of trapped atomic gases (**expansion**, **collective oscillations**)*

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

Above Tc: $\rho = \rho_N \; ; \vec{j} = \rho\vec{v}_N$ eqs. reduce to standard collisional HD equations (**adiabatic sound**)

Describe the collective oscillations in the normal phase

FIRST AND SECOND SOLUTIONS IN UNIFORM SUPERFLUIDS

H. Hu, E. Taylor, X.-J. Liu, S. Stringari, A. Griffin,
New J.Phys.12. 045040 (2010)

Simple ansatz for variational calculations (uncoupled modes)

1) $\vec{v}_S = \vec{v}_N \equiv \vec{v}$ in phase (**first sound**)

2) $\vec{j} = \rho_S \vec{v}_S + \rho_N \vec{v}_N = 0$ out of phase (**second sound**)

first sound is pure density mode ($\delta T(\vec{r},t) = 0$)

follows from $\rho_N \partial(\vec{v}_N - \vec{v}_S)/\partial t + s\vec{\nabla} T = 0$

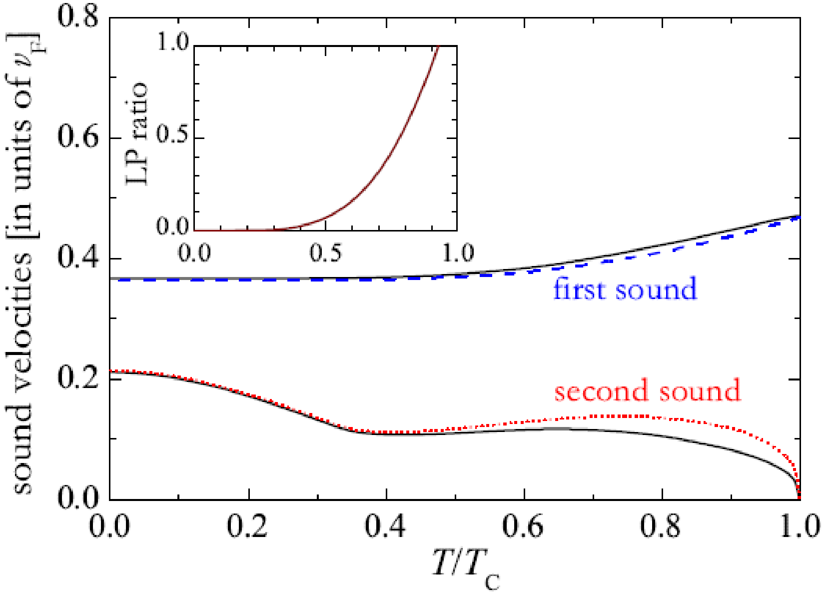
second sound is pure temperature mode ($\delta \rho(\vec{r},t) = 0$)

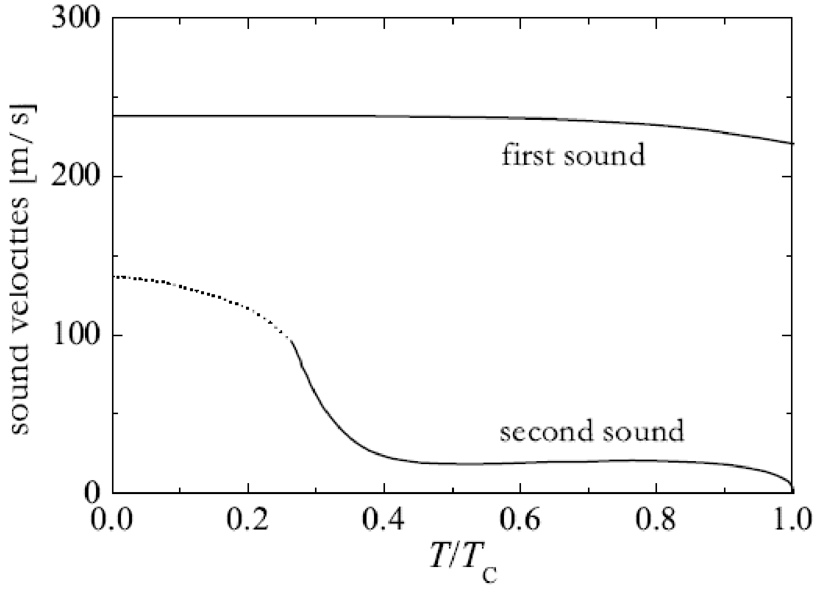
follows from $\partial \rho / \partial t + \vec{\nabla}(\vec{j}) = 0$

Results for first and second sound modes in uniform matter

$$c_1^2 = \left(\frac{\partial P}{\partial \rho}\right)_{\bar{s}} \quad ; \quad c_2^2 = T\,\frac{\bar{s}^2}{c_V}\,\frac{\rho_S}{\rho_N}$$

superfluid density





Unitary Fermi gas
(theory, Taylor, Griffin et al 2009)

Liquid He
(experiment, Peshkov 1946)

FIRST AND SECOND SOLUTIONS IN HARMONICALLY TRAPPED SUPERFLUIDS

with Yan Hua, Lev Pitaevskii

(highly elongated traps)

Previous work in spherical trap

(with Hui Hu, EdTaylor and Allan Griffin)

Predictions of two-fluid HD theory applied to harmonically trapped configurations:

- Frequency of **scaling modes** (quadrupole and breathing) is **not affected by temperature** (result is independent of trap deformation). Scaling modes are first sound modes ($\vec{v}_S = \vec{v}_N$)

- **Higher nodal** modes exhibit **T-dependence**

Why **elongated** configurations ?

- Easier realization of HD condition $\omega_z \tau << 1$

- Easier experimental conditions

- Easier theoretical calculation of normal modes because of 1D nature

- In the presence of tight radial trapping 3D equations can be reduced to 1D form (TF 1D): New equation of state ($P_1(\rho_1) \neq P(\rho)$)

- New role of viscosity and thermal conductivity

From 3D to 1D (Thomas-Fermi)

(Bertaina et al. PRL 2010)

At **T=0** reduction to 1D form is ensured by equation for **superfluid velocity field** and condition $\delta\mu = \delta\mu(z,t)$ applied to low frequency solutions of order ω_z

At finite temperature reduction to 1D form requires not only Usual HD condition $\omega\tau << 1$, but also additional condition $\omega << \omega_\perp^2 \tau$

imposed by viscosity and thermal conductivity in equations for the current and entropy. The new condition yields

$$v_z = v_z(z,t) \quad and \quad \delta T = \delta T(z,t)$$

1D hydrodynamic equations can be derived using **variational** procedure with respect to velocity fields.

For first sound we use the ansatz $v_{zn} = v_{zs} \equiv v$

At unitarity HD equation takes the form

$$m\omega^2 v = -(7/5\rho_1)\partial_z(P_1\partial_z v) + m\omega_z^2 v$$

where $\rho_1 = \int \rho dxdy$

and $P_1 \propto \int dxdyP \propto \rho_1^{7/5} f(T/\rho_1^{2/5})$

are 1D density and pressure,

Solutions of 1D HD equation **at unitarity:**

At **T=0** ($P_1 \propto \rho_1^{7/5}$) solutions are polynomials:
$v \propto z^k + az^{k-2} + ...$ with dispersion:

$$\omega^2 = \frac{1}{5}(k+1)(k+5)\omega_z^2$$

At **high T** ($P_1 = T\rho_1 / m$) one instead finds:

$$\omega^2 = \frac{1}{5}(7k+5)\omega_z^2$$

k=0 (center of mass, $\omega = \omega_z$) and
k=1 (axial breathing, $\omega = \sqrt{12/5}\omega_z$) frequencies are **temperature independent**

Higher k-modes have richer nodal structure and **exhibit T-dependence**.

To calculate T-dependence we use a polynomial variational ansatz for the velocity field. Result for the frequency of the k=2 mode:

$$\omega^2(k=2) = \frac{129t_2 - 25}{5(9t_2 - 5)}\omega_z^2$$

with

$$t_2(T) = M_0 M_4 / M_2^2$$

and moments

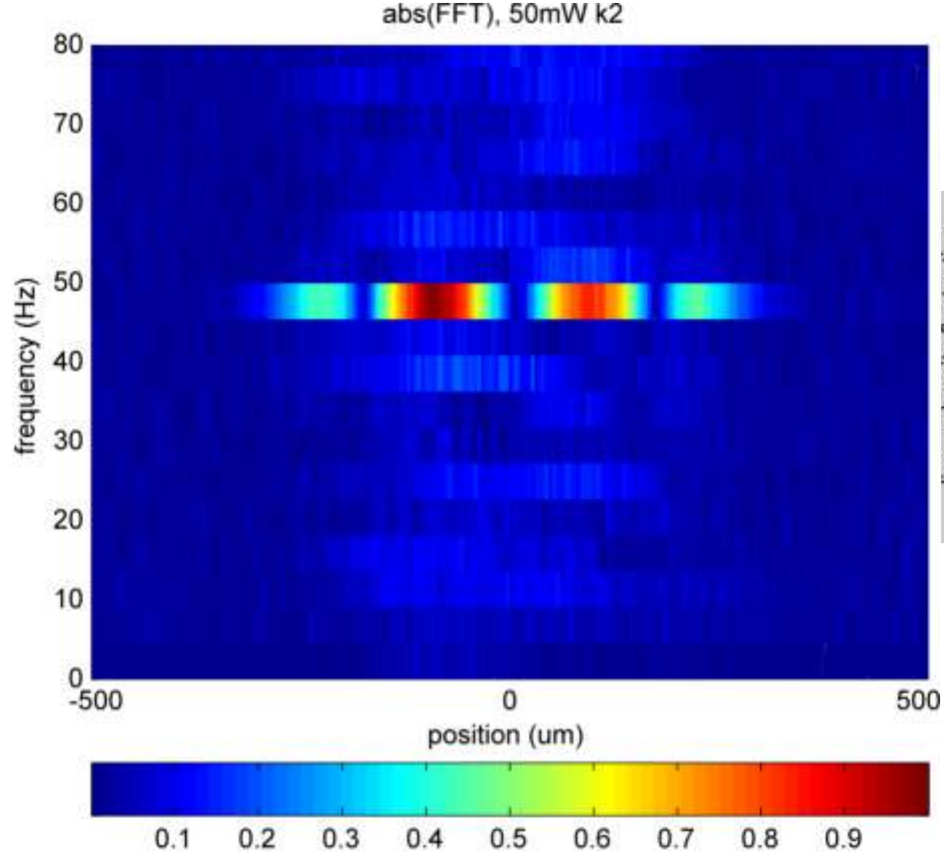
$$M_\ell = \int_{-\infty}^{\beta\mu_0} dx(\beta\mu_0 - x)^{(\ell+1)/2}\rho(x)$$

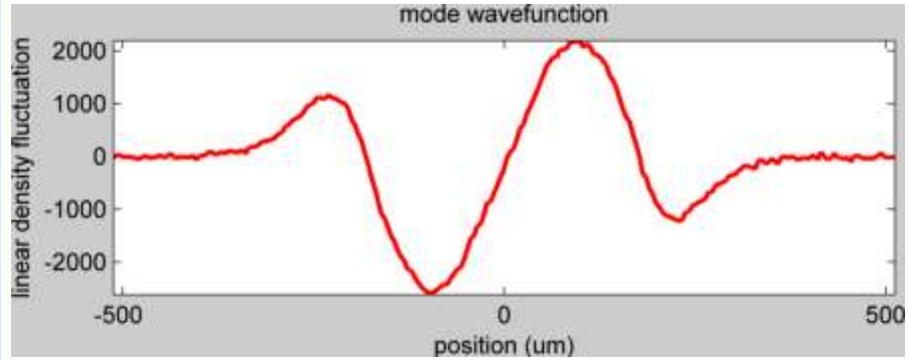
evaluated from **measured MIT equation of state** at unitarity (M. Zwierlein and M. Ku, Mit)

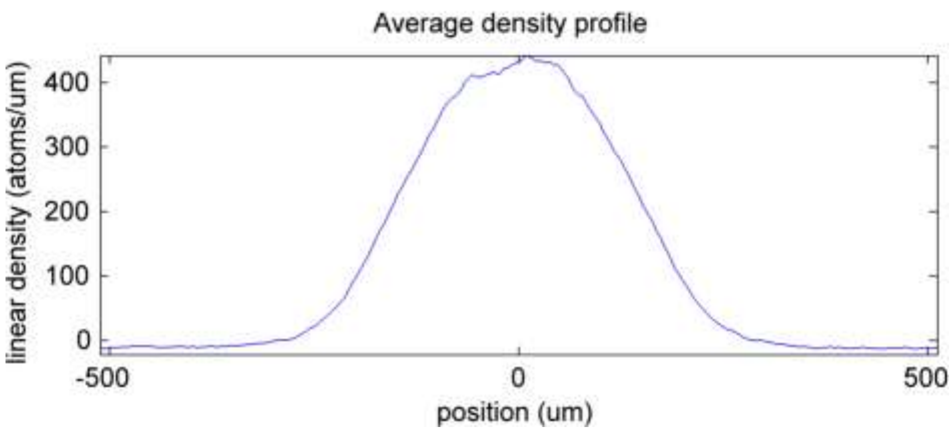
Experimental excitation of axial collective modes in the unitary Fermi gas at $T < T_C$

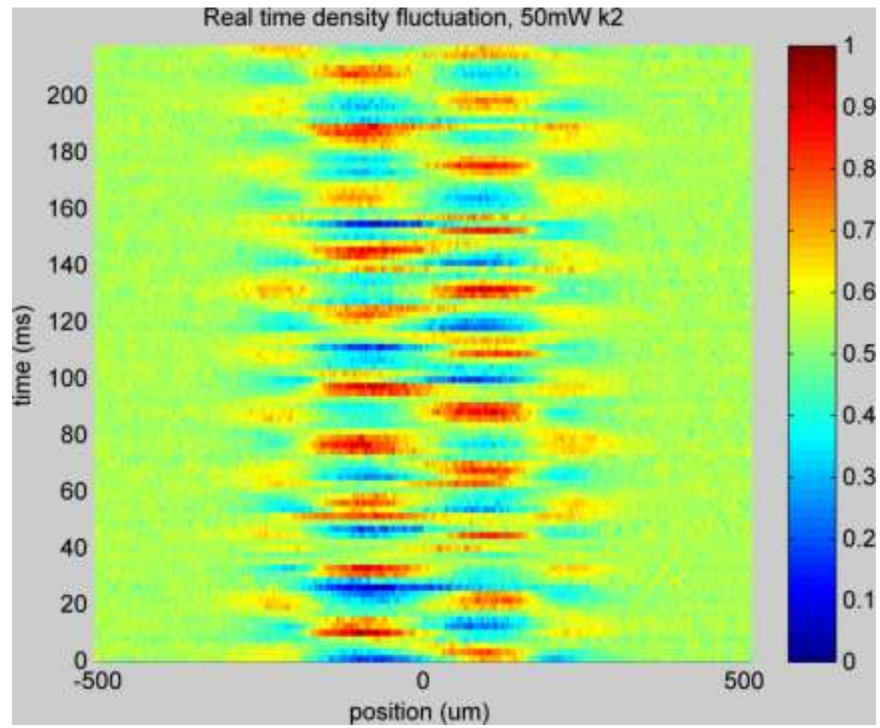
(Innsbruck)

K2 mode

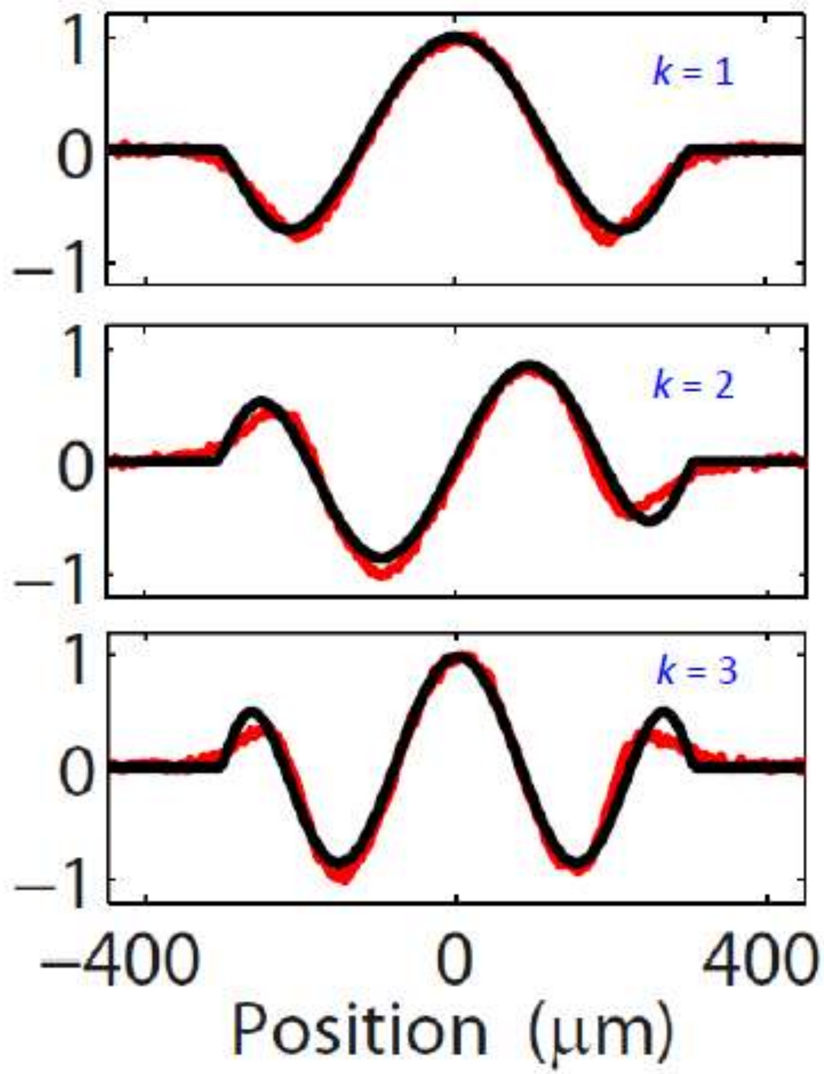

abs(FFT), 50mW k2
frequency (Hz)
80
70
60
50
40
30
20
10
0
-500
0
500
position (um)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

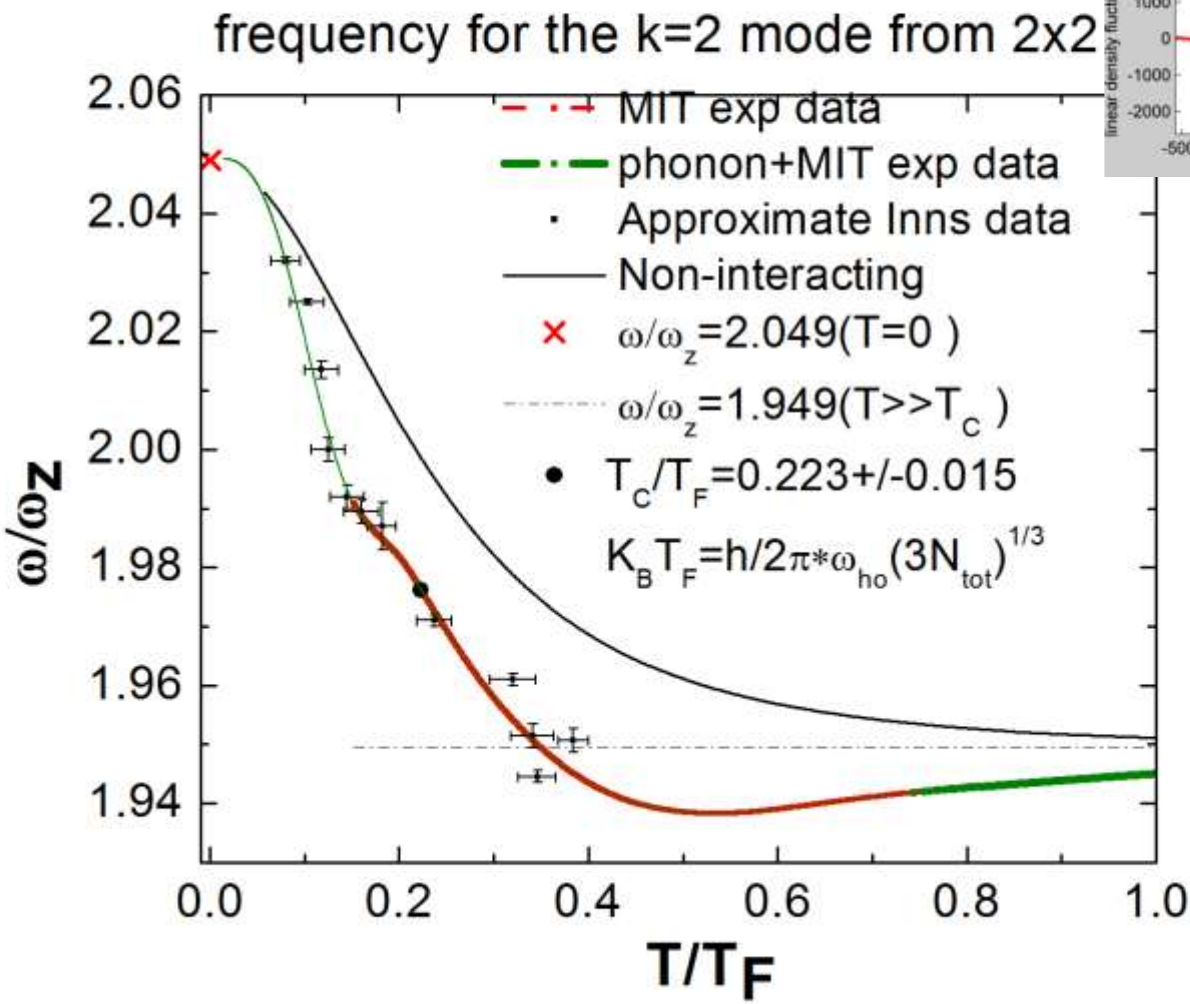

mode wavefunction
linear density fluctuation
2000
1000
0
-1000
-2000
-500
0
500
position (um)

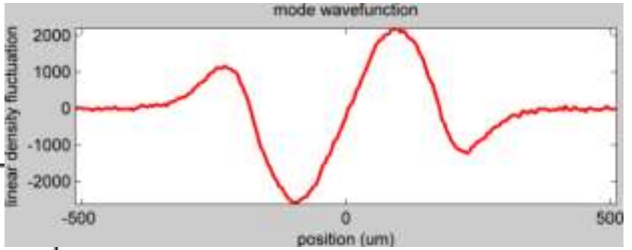

Average density profile
linear density (atoms/um)
400
300
200
100
0
-500
0
500
position (um)


Real time density fluctuation, 50mW k2
time (ms)
200
180
160
140
120
100
80
60
40
20
0
-500
0
500
position (um)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Mode profiles: experiment vs. theoretical prediction







- **Theoretical** curve calculated using 1D HD plus MIT Eq. of state and phonon contribution at very low T
- **Experimental** data: Innsbruck
- First exp verification of transition between superfluid and collisional HD

A few examples:

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Soliton oscillation and collisions in an interacting Fermi gas

Soliton solution of Gross-Pitaevskii eq. P (Tsuzuki 1971)

$$\Psi(z-vt)=\sqrt{n}\left(i\frac{v}{c}+\sqrt{1-\frac{v^2}{c^2}}\tanh\left[\frac{z-vt}{\sqrt{2}\xi}\sqrt{1-\frac{v^2}{c^2}}\right]\right)e^{-i\mu t}$$

- Energy $\varepsilon=\frac{4}{3}\hbar cn\left(1-\frac{v^2}{c^2}\right)^{3/2}$ decreases by increasing velocity

- **Maximum** velocity of soliton given by **sound velocity**

- In harmonic trap soliton oscillates with frequency $\omega_{HO}/\sqrt{2}$

- **Collisions** between solitons **are elastic** (integrability of 1D GP equation)

What happens in interacting Fermi gases ?

- Limiting velocity fixed by **pair breaking** mechanism (smaller than sound velocity in BCS side of resonance)

- In harmonic trap **frequency** of oscillation in interacting Fermi gase is **smaller than in BEC's**

- Collisions are **inelastic** (after the collision soliton can reach the limiting velocity and disappear !)

Results obtained by solving numerically

Time dependent Bogoliubov de Gennes equations

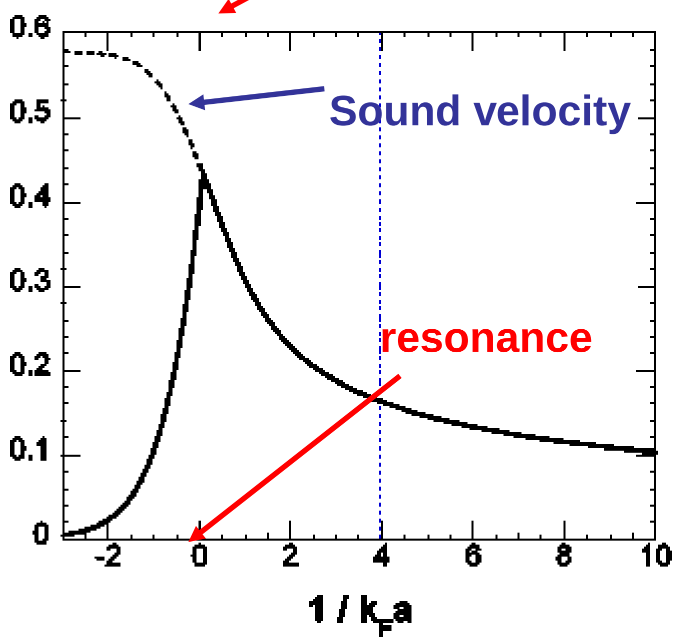
Scott et al. (PRL 2011, NJP 2012)

Landau's critical velocity along the BEC-BCS crossover

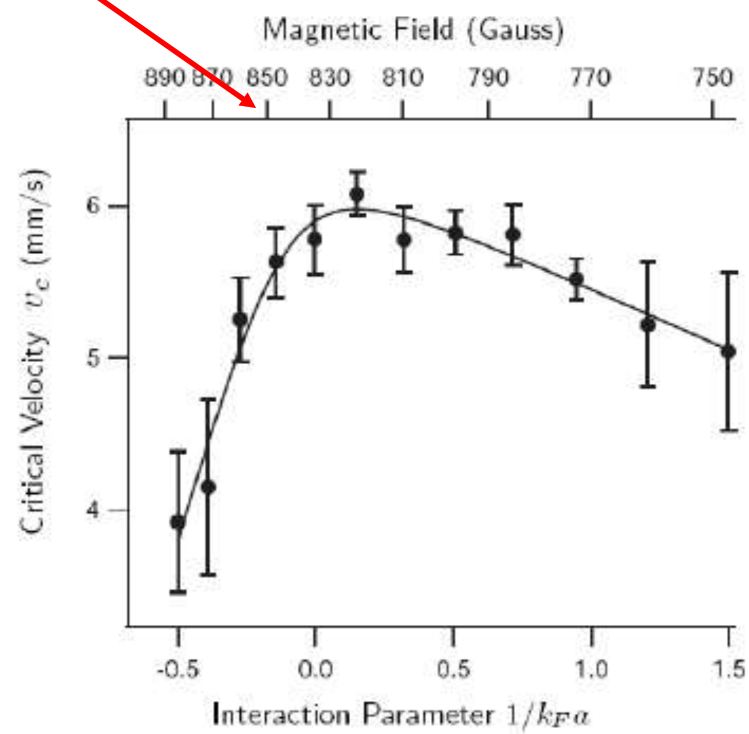
theory

experiment

$$\frac{v_{cr}}{v_F}$$



(Combescot et al, 2006)

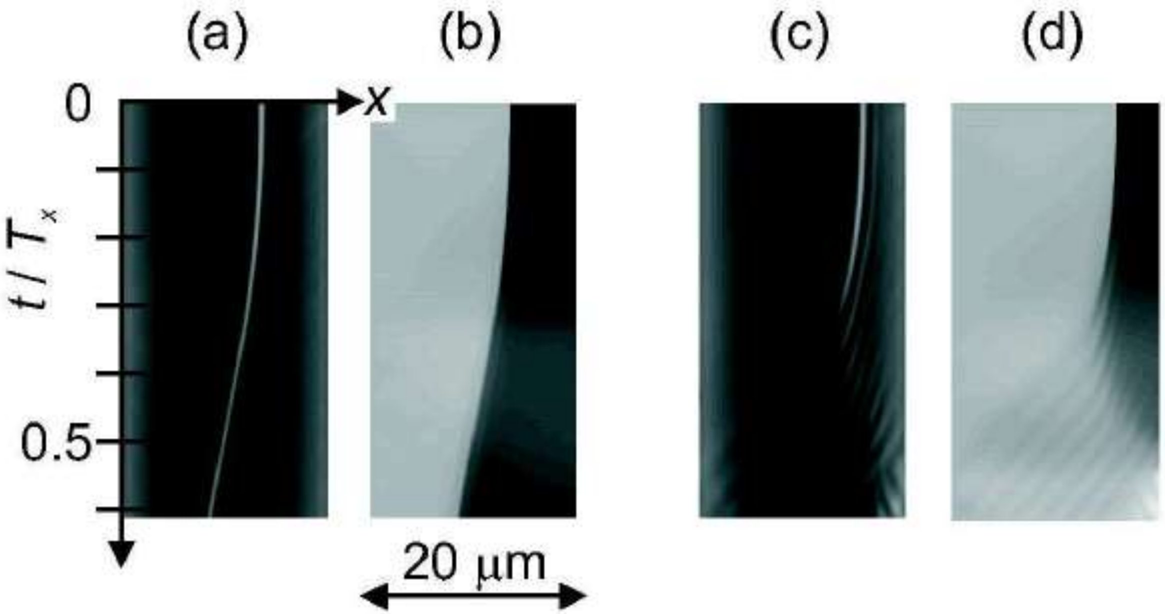


(Miller et al, 2007)

Landau's critical velocity
is
highest near unitarity !!

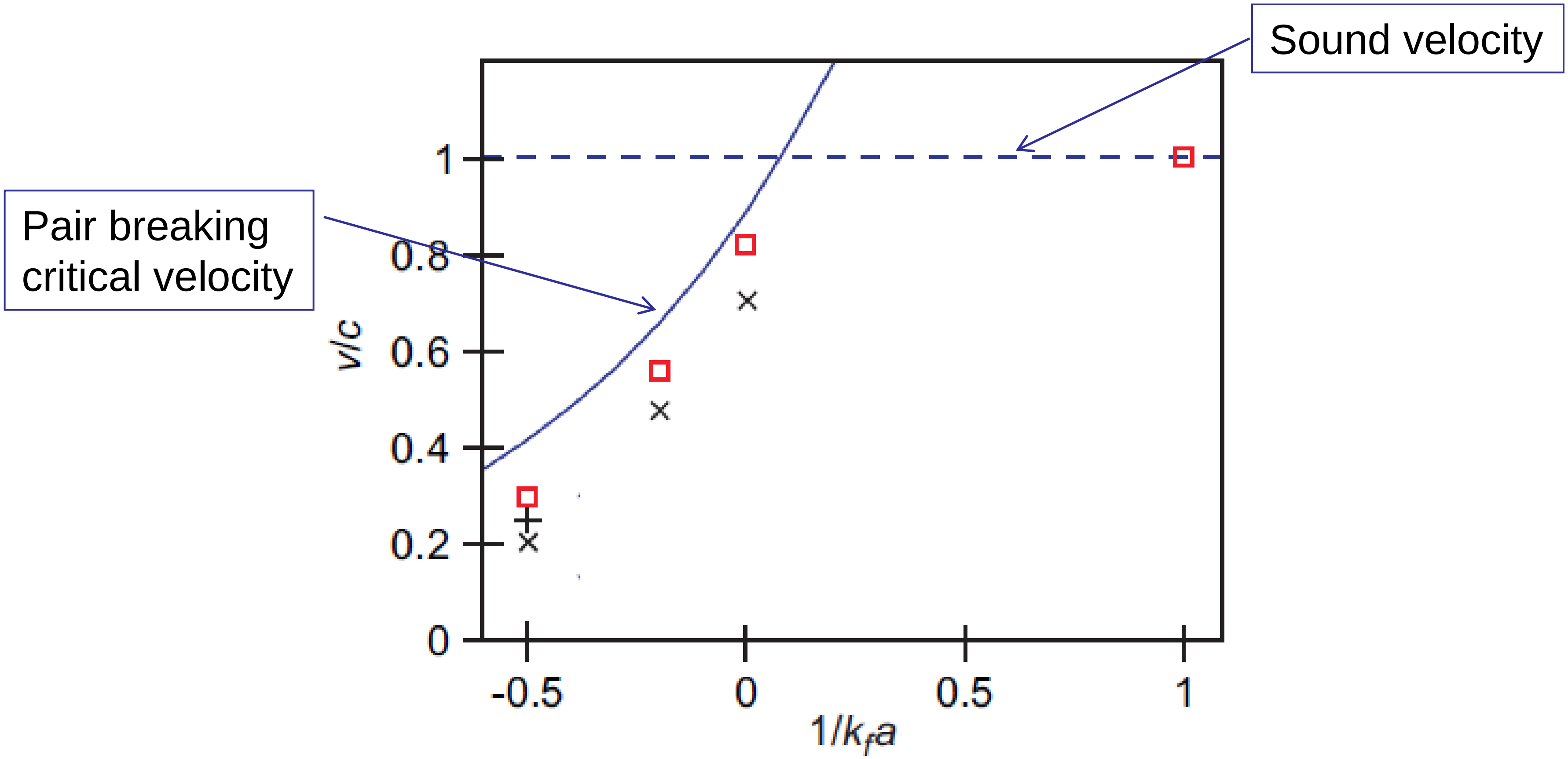
Decay of solitons

- A soliton is cretated at distance x from the center of harmonic trap.
- The harmonic potential accelerates the soliton
- If x is too large the soliton decays because its velocity reaches pair breaking critical velocity



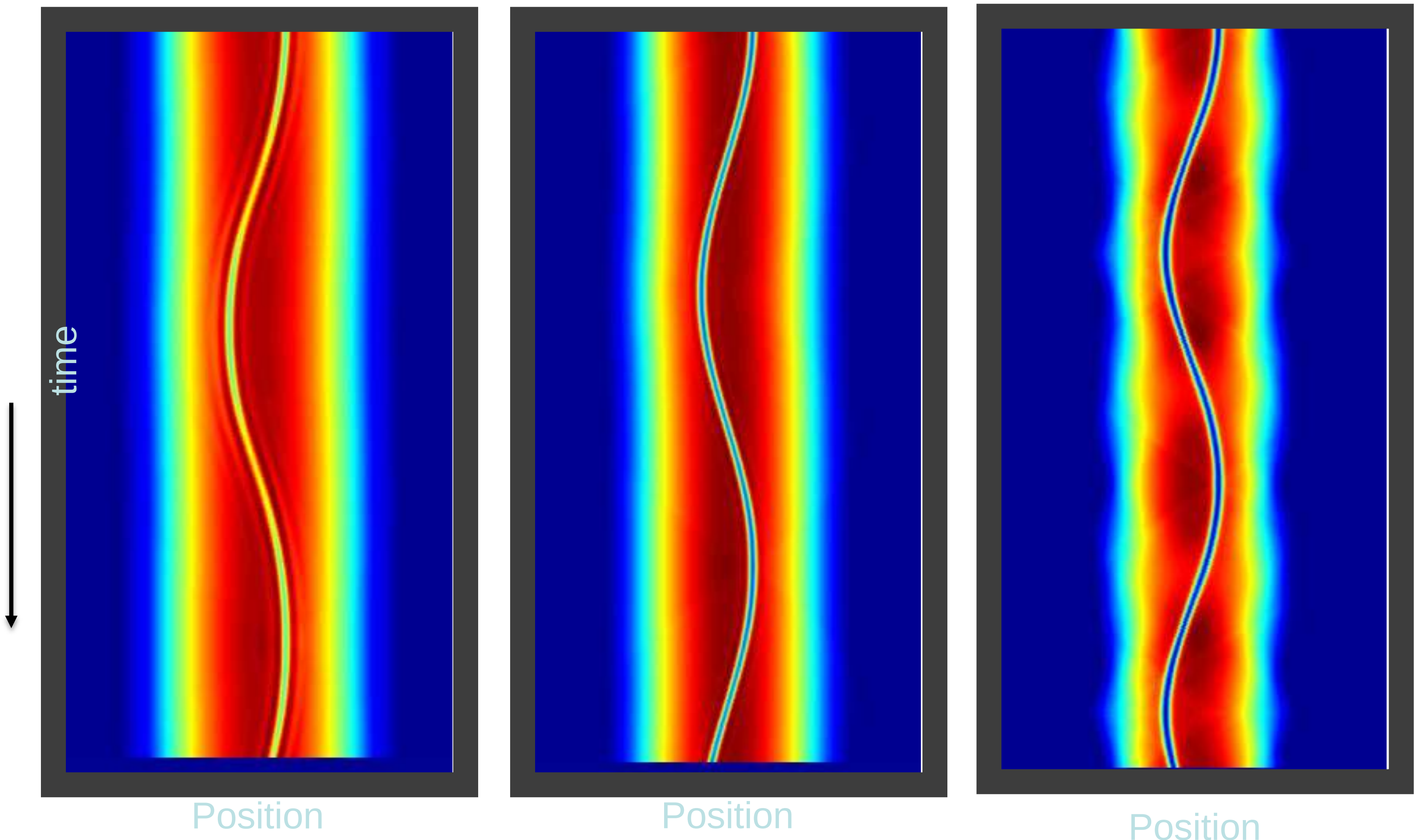
$k_F a = -0.5$

Critical velocity of solitons
calculated solving BdG equations



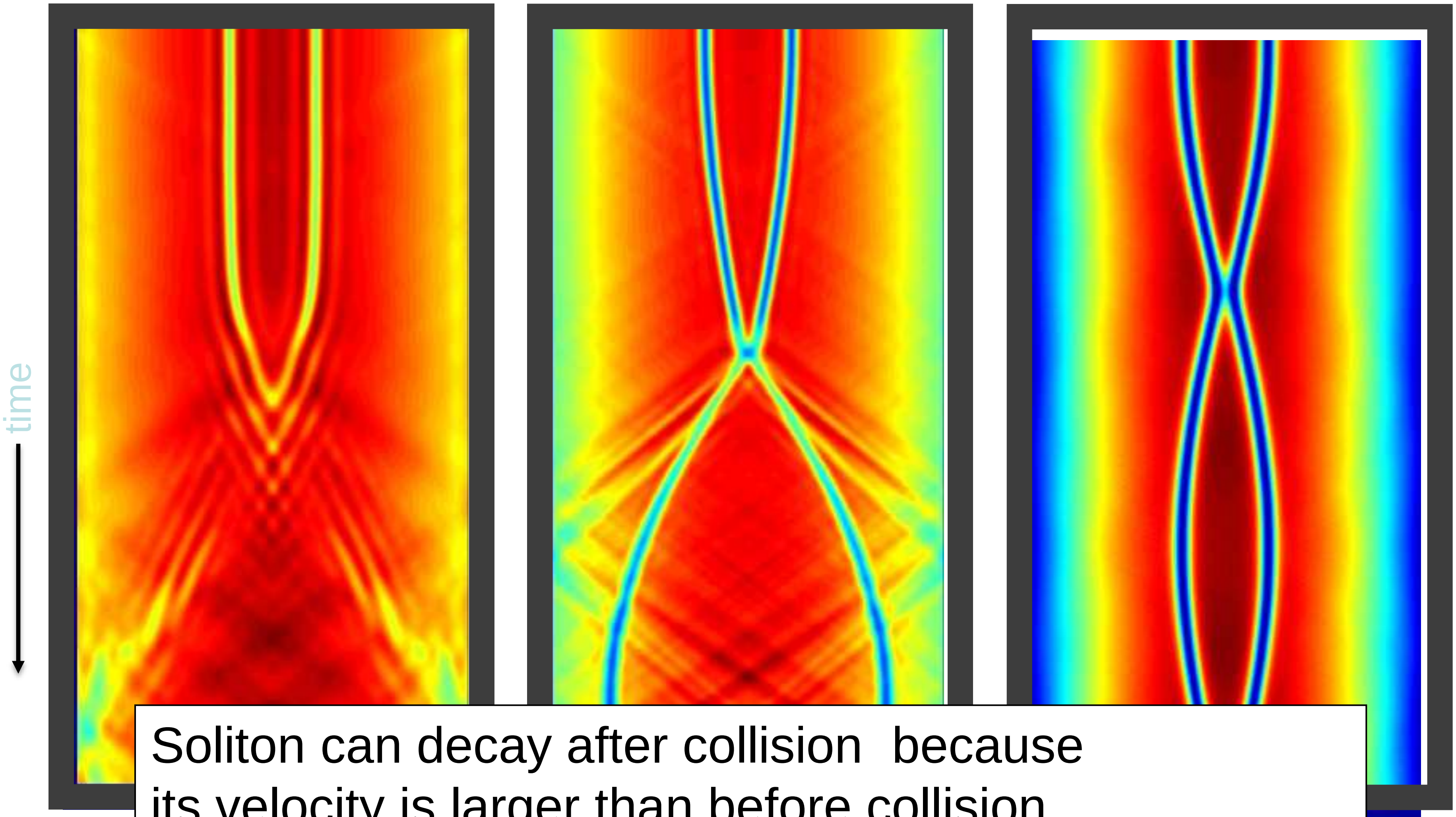
Soliton oscillation in a trap across the crossover
(in BCS frequency is smaller than in BEC)

$1/k_Fa = -0.5$ (BCS) $1/k_Fa = 0$ (Unitarity) $1/k_Fa = 0.5$ (BEC)



Soliton collisions in a trap across the crossover

$1/k_F a = -0.5$ (BCS) $1/k_F a = 0$ (Unitarity) $1/k_F a = 0.5$ (BEC)



Soliton can decay after collision because its velocity is larger than before collision (energy of soliton decreases with increasing velocity)

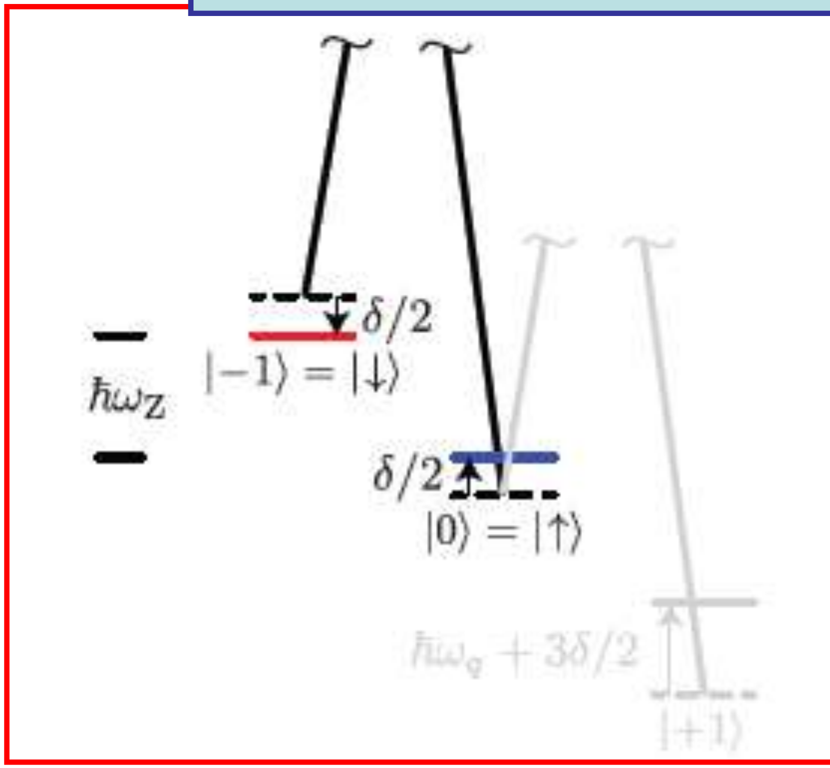
Main conclusions:

The adventure of **collective oscillations** in quantum gases started immediately after the first experimental **realization** of **Bose-Einstein condensation** in **1995**.

It found a new exciting season after the realization of quantum degenarate **Fermi gases**

It is **still** an **active** direction of research as a dynamic test of the **new phases** available with quantum gases.

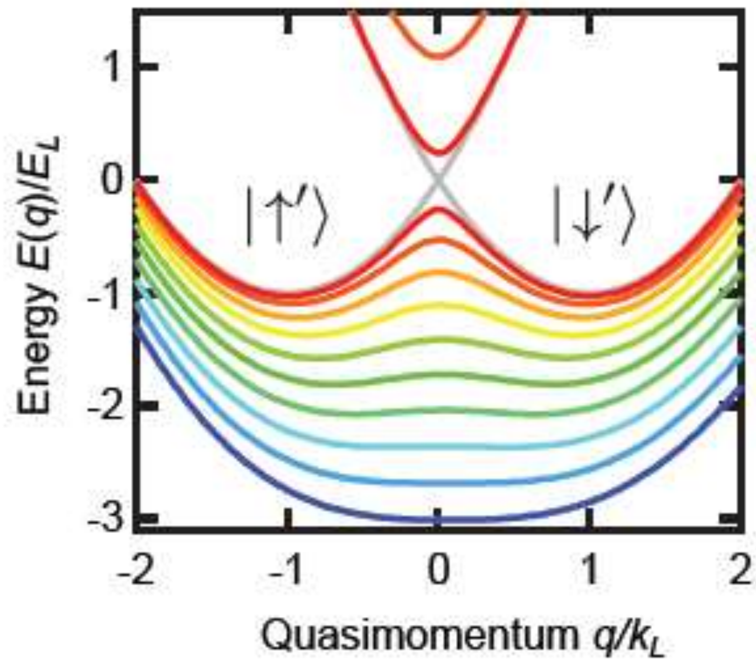
Spin-orbit Hamiltonian with equal Rashba and Dresselhaus couplings. Recent exp implementation with Fermi gases



$$H = \frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z + V_{ext} + V_{2-body}$$

Recent exp implementation in Fermi gases

- Wang et al. arXiv:1204.1887
- Cheuk et al. arXive:1205.3483



Spin-orbit Hamiltonian violates Galilean invariance.
Equation of continuity affected by spin-orbit term

⇩

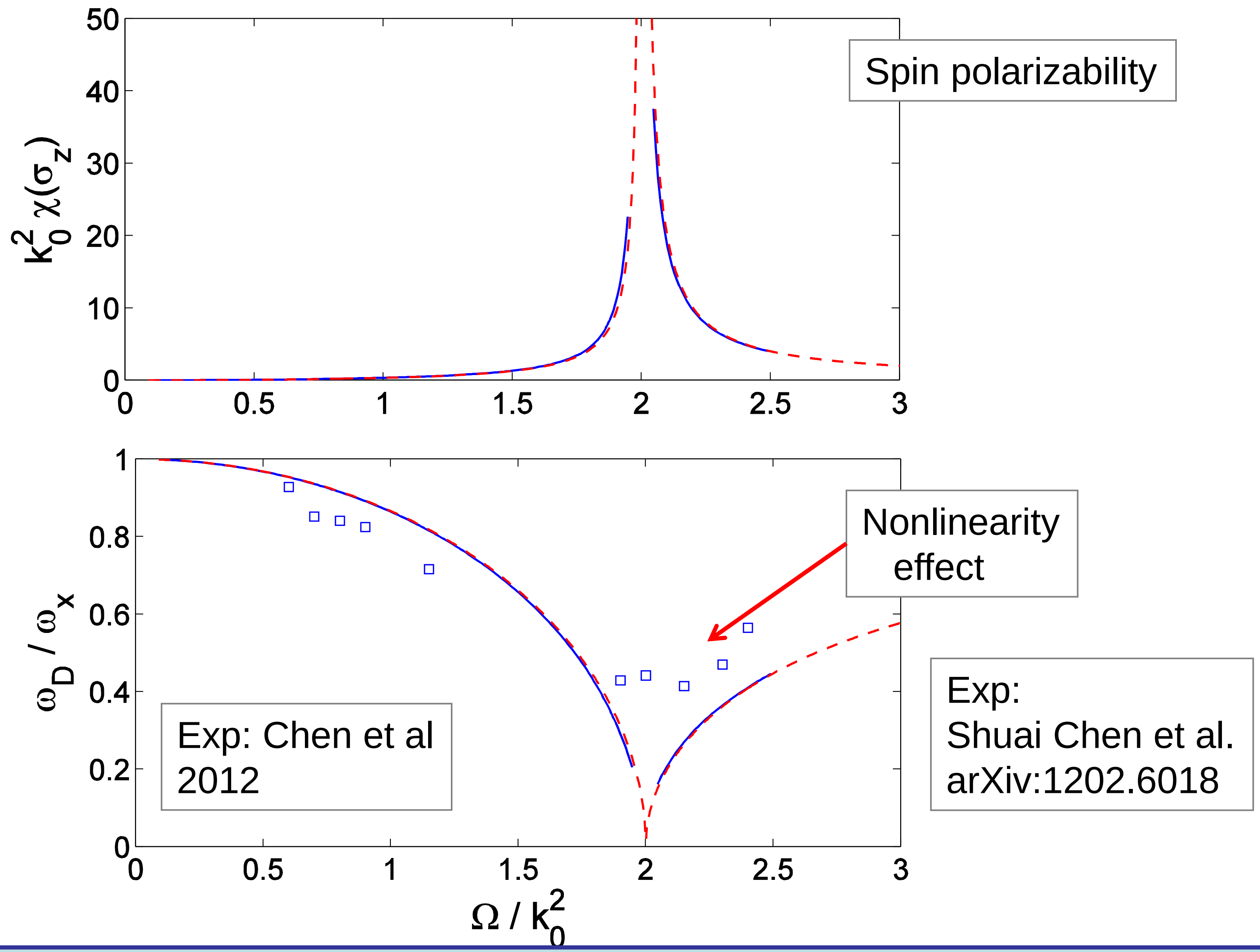
New commutation relation for dipole operator

$$[H, X] = -i\left(P_x - k_0\sigma_z\right)$$

yields new sum rule estimate for dipole frequency:

$$\omega_D^2 = \omega_{ho}^2 \frac{1}{1 + k_0^2 \chi(\sigma_z)}$$

Center of mass frequency quenched with respect to oscillator frequency ω_x due to coupling with spin degree of freedom. Key role played by **Spin polarizability** $\chi(\sigma_z)$



Dipole frequency of spin-orbit coupled BEC gas quenched with respect to oscillator value ω_x (Yun Li et al arXiv:1205.6398)