

Polariton Bose-Einstein condensation: Overview and perspectives

Vincenzo Savona

*Institute of Theoretical Physics
Swiss Federal Institute of Technology Lausanne (EPFL), Switzerland*

Outlook

- **Polariton BEC: The basics**
- **Theoretical considerations**
- **Some recent experimental achievements**
- **Conclusions and outlook**



Polaritons in 3-D semiconductors

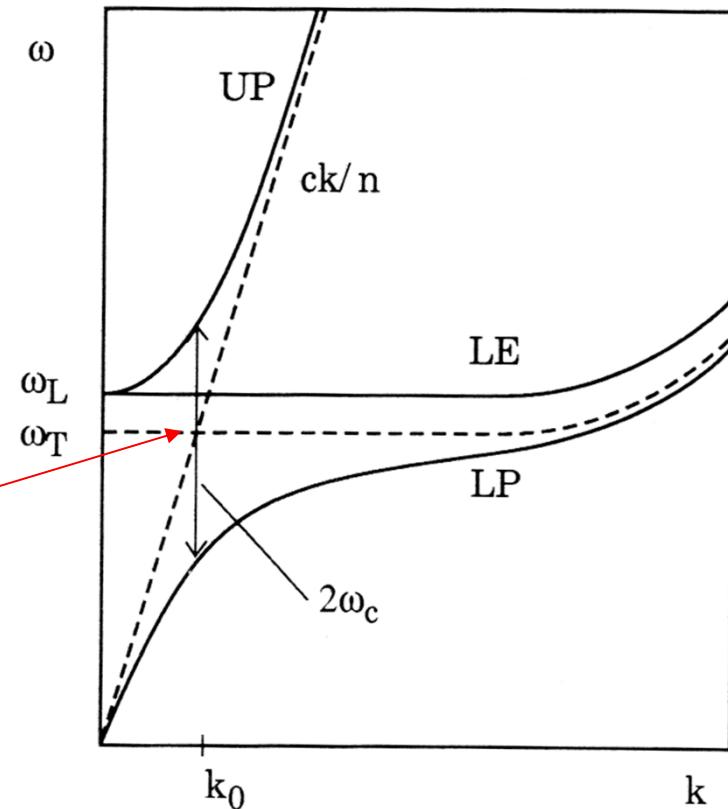
Linear exciton-photon coupling, momentum conserving



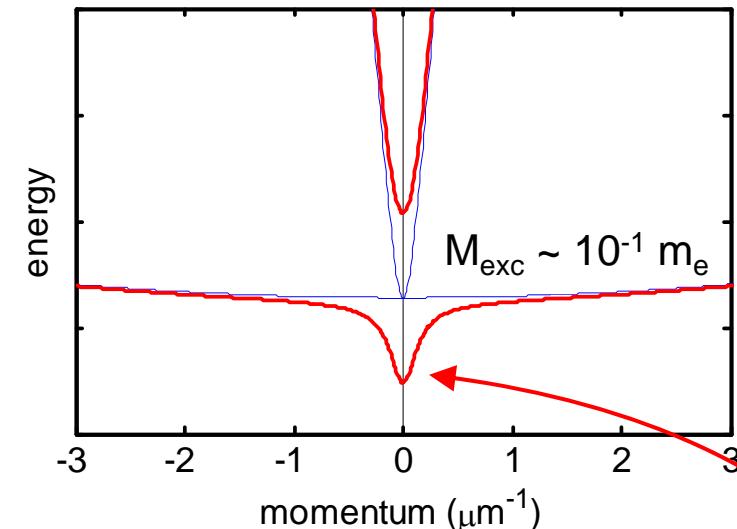
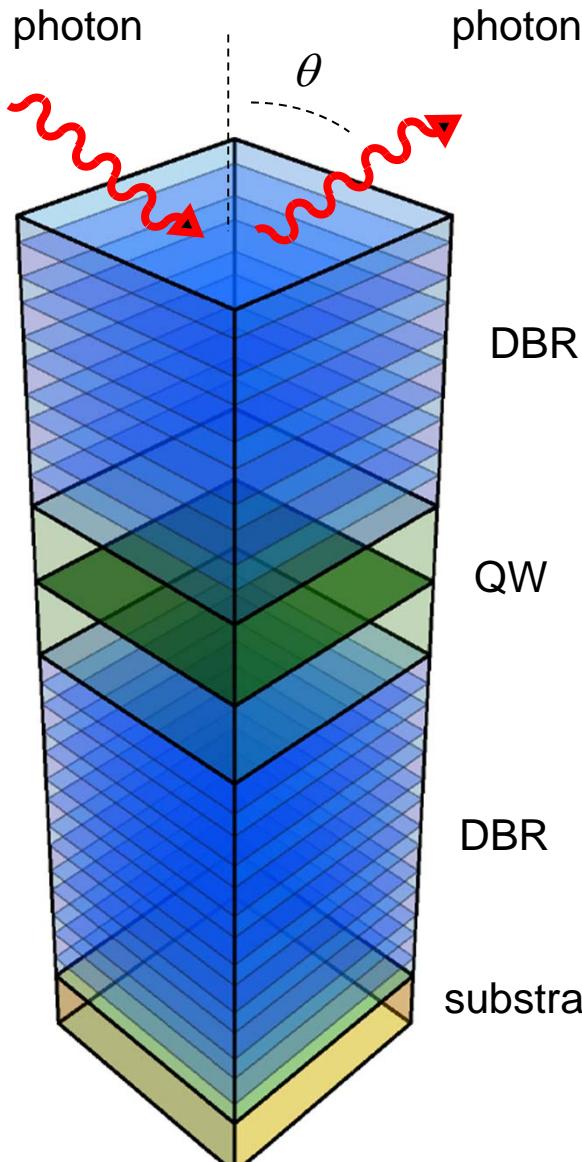
Polaritons as normal modes of the coupled system

$$|\text{polariton}\rangle = \frac{1}{\sqrt{2}} (|\text{photon}\rangle \pm |\text{exciton}\rangle)$$

J. J. Hopfield, Phys. Rev. **112**, 1555 (1958)

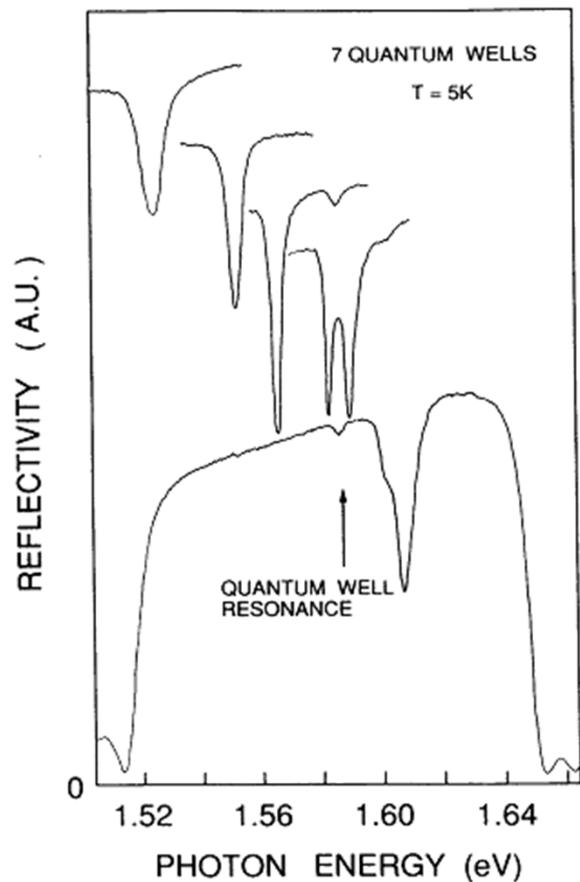


Polaritons in 2-D semiconductor microcavities

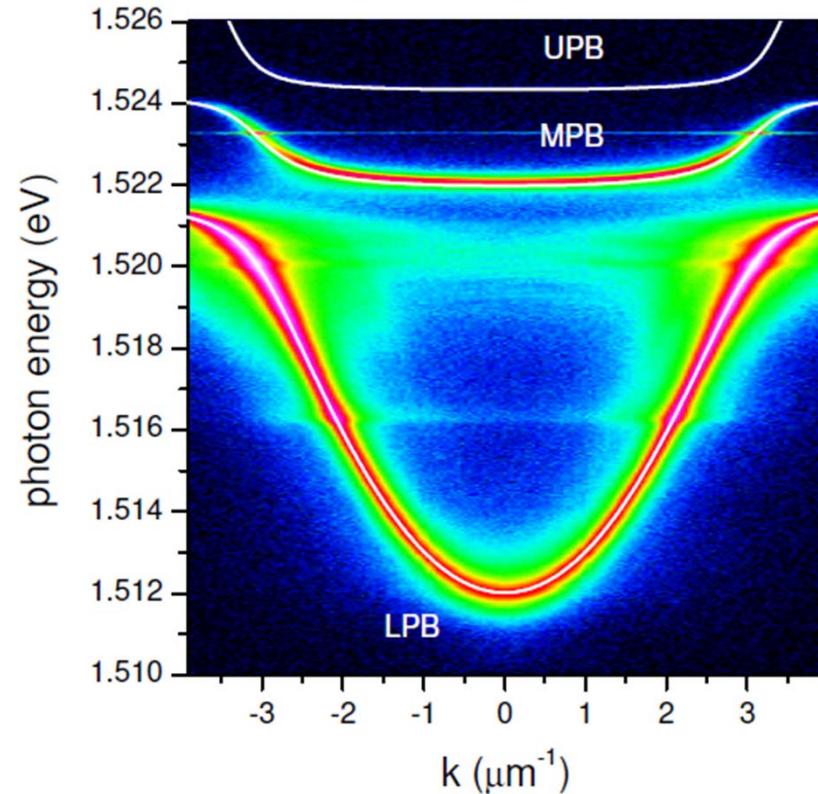


- Optical generation and detection: $k = \frac{\omega}{c} \sin \theta$
- Radiative lifetime: ~ 10 ps
- Very light effective mass $M_{\text{pol}} \sim 10^{-5} m_e$
⇒ Very long coherence length ~ 100 micron
- Polaritons are Bose particles (low density)

Polaritons in semiconductor microcavities

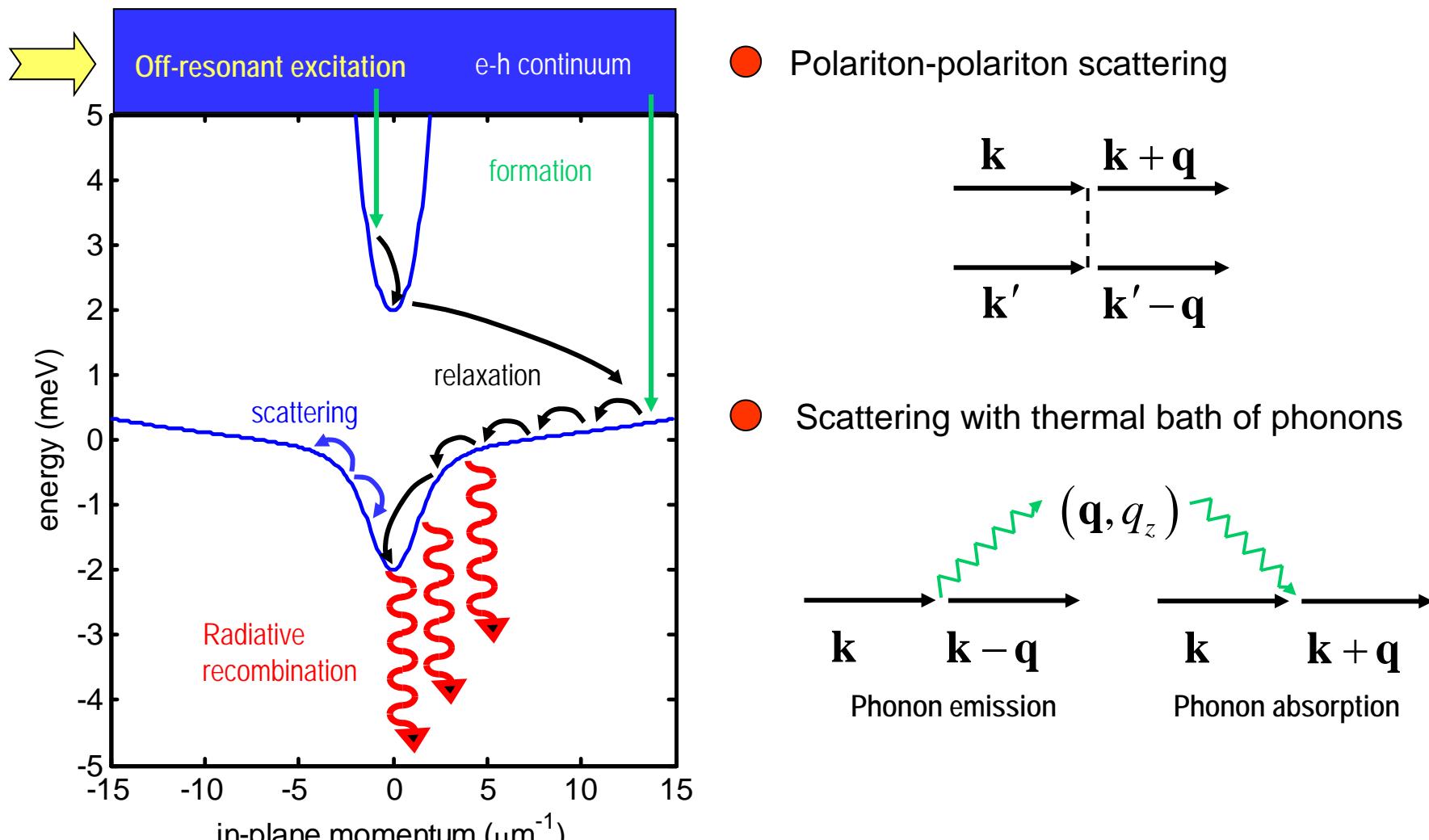


C. Weisbuch et al., PRL **69**, 3314 (1992)



W. Langbein, Proc. ICPS (2002)

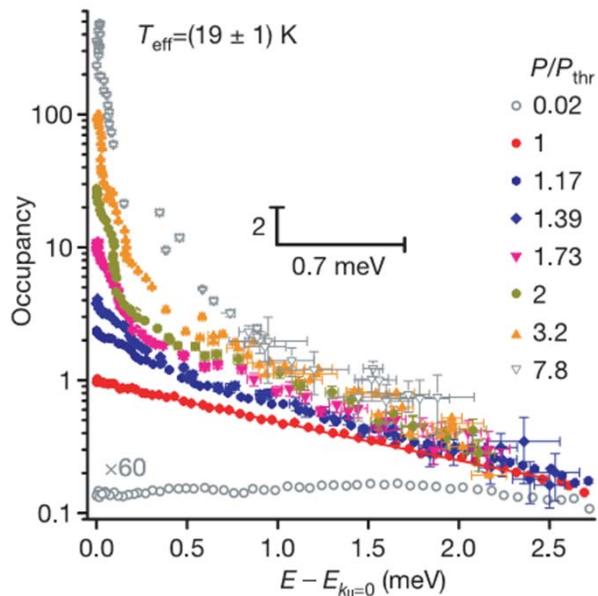
Polariton off-resonant excitation and kinetics



Driven-dissipative regime: always (a bit) out of thermal equilibrium

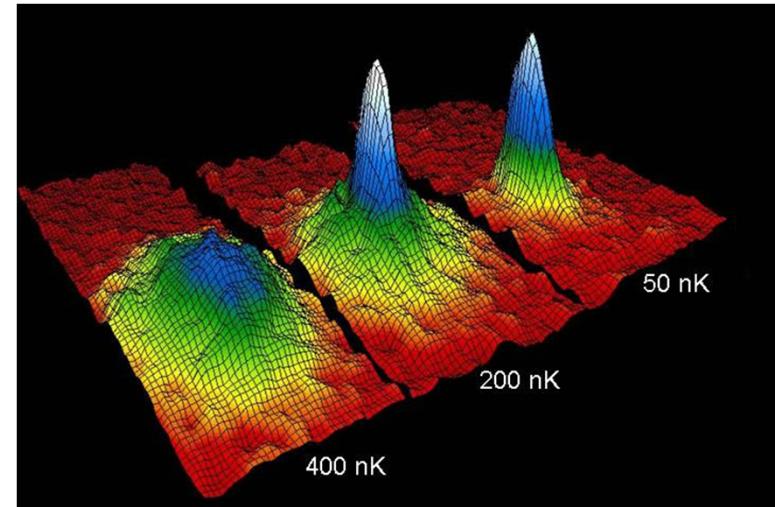
Polariton Bose-Einstein condensation

Polaritons



J. Kasprzak, et al., Nature **443**, 409 (2006)

Cold atoms



M. H. Anderson, et al., Science **269**, 198 (1995)

Also...

Le Si Dang et al., PRL **81**, 3920 (1998)

J. Bleuse et al., J. Crystal Growth **184/185**, 750 (1998)

H. Deng, et al., PNAS **100**, 15318 (2003)

R. Balili, et al., Science **316**, 1007 (2007)

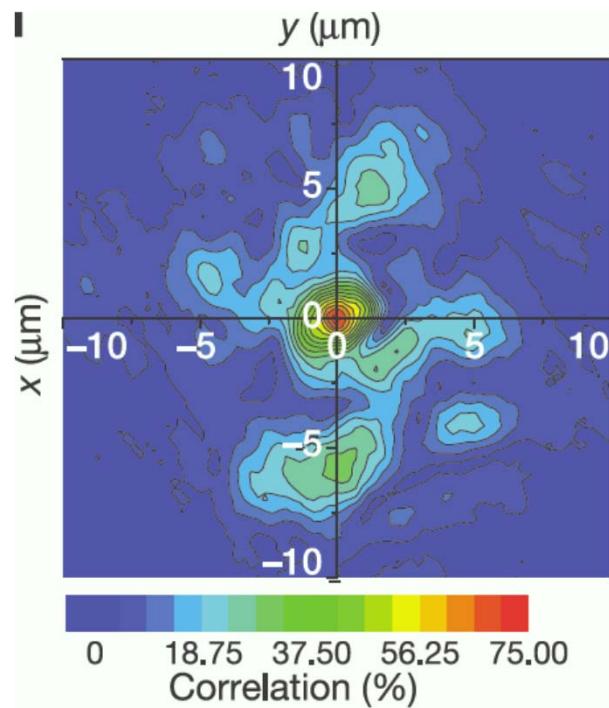
S. Christopoulos, et al., PRL **98**, 126405 (2007) (**GaN system at room temperature**)

S. Utsunomiya et al., Nature Physics **4**, 700 (2008)

E. Wertz et al., APL **95**, 051108 (2009)

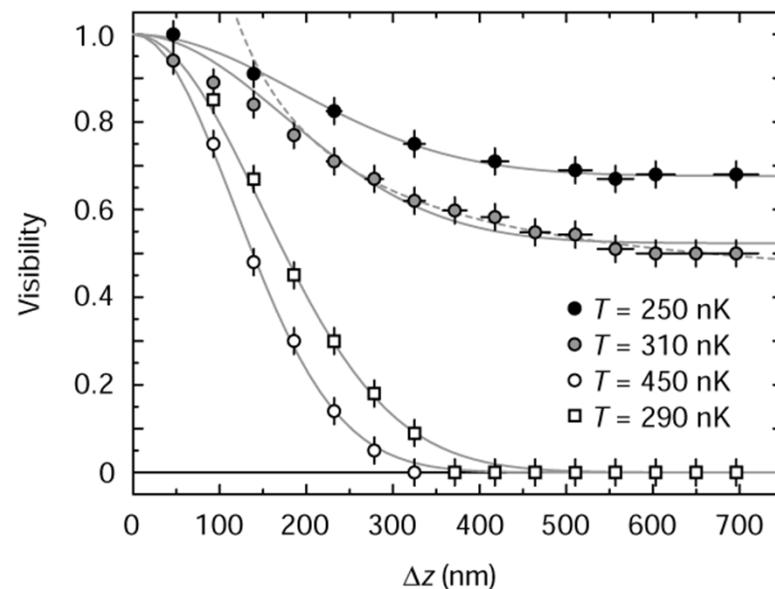
Off-diagonal long-range order

Polaritons



J. Kasprzak, et al., *Nature* **443**, 409 (2006)

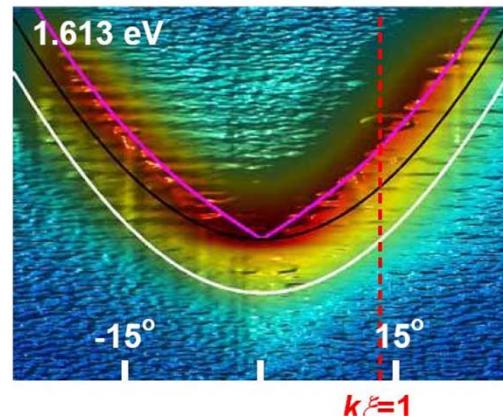
Cold atoms



I. Bloch, T. W. Hänsch and T. Esslinger,
Nature **403**, 166 (2000)

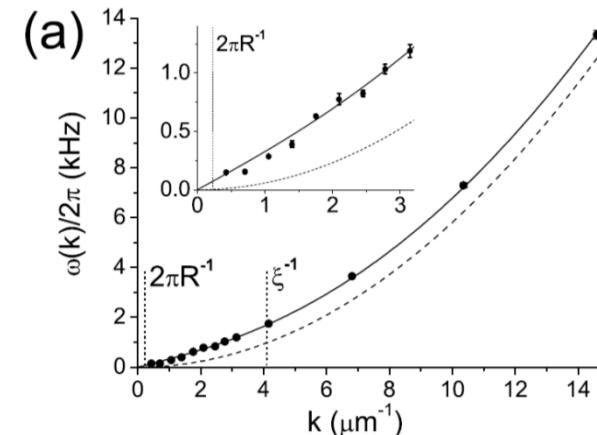
Collective excitation spectrum

Polaritons



S. Utsunomiya et al., Nature Physics **4**, 700 (2008)

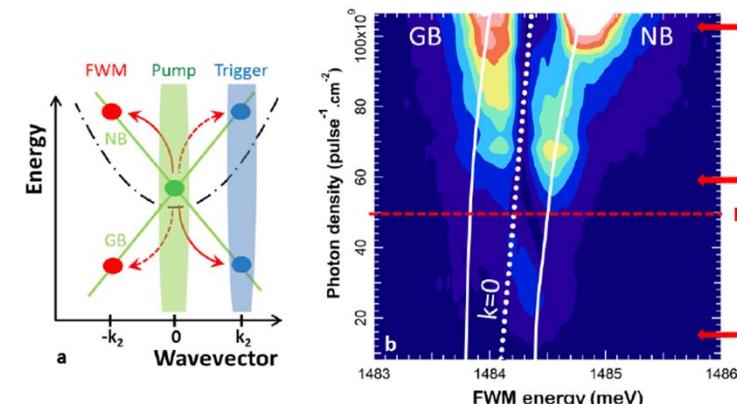
Cold atoms



J. Steinhauer et al., PRL **88**, 120407 (2002)

“Negative-energy” branch directly observable

- M. Wouters and I. Carusotto, PRL **99**, 140402 (2007)
- J. Keeling et al., PRB **72**, 115320 (2005)
- T. Byrnes et al., PRB **85**, 075130 (2012)



V. Kohnle et al., PRL **106**, 255302 (2011)

Symmetry-breaking many-body perturbation theory (thermal equilibrium)

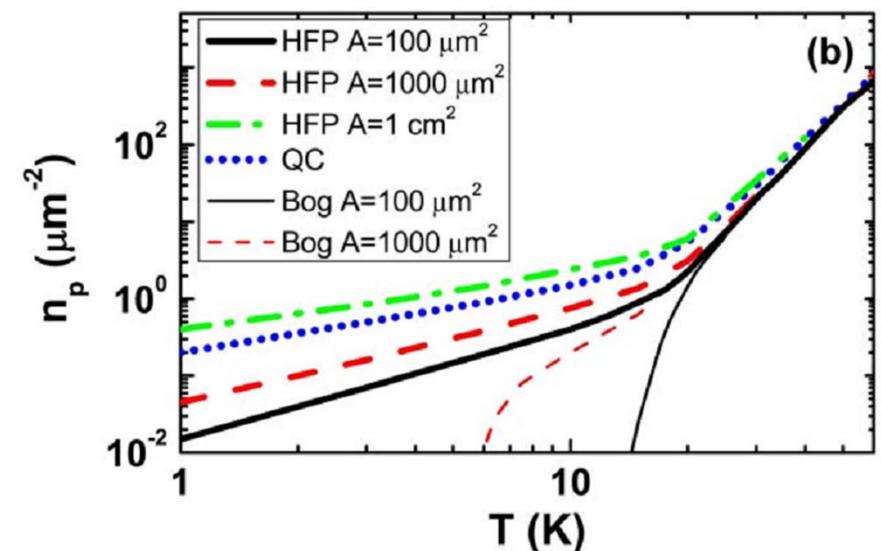
Generalized symmetry breaking ansatz:

$$\left. \begin{aligned} \hat{\Psi}_X(\mathbf{r}) &= \Phi_X + \tilde{\psi}_X(\mathbf{r}) \\ \hat{\Psi}_{Ph}(\mathbf{r}) &= \Phi_{Ph} + \tilde{\psi}_{Ph}(\mathbf{r}) \end{aligned} \right\} \Rightarrow \Phi_{Pol} = \Phi_X + \Phi_{Ph}$$

Example: Hartree-Fock-Popov diagrams

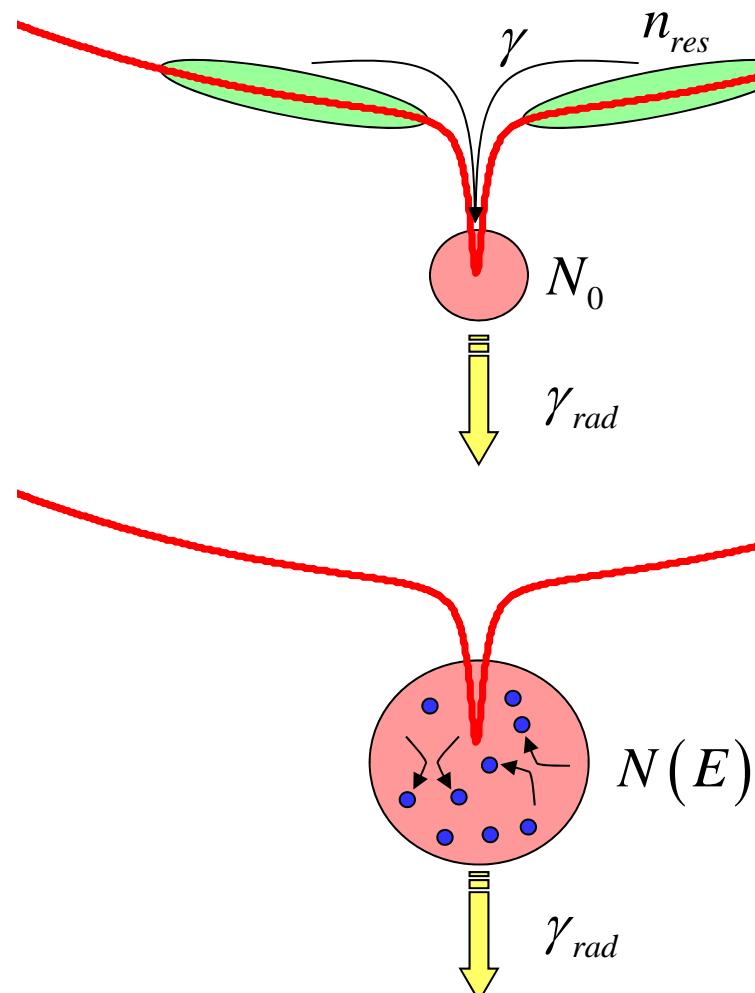
$$\begin{aligned} \Sigma_{11}^{\chi\xi} &= \circ(1 - \delta_{\chi\xi}) \\ &+ \text{Diagram with } \Phi_\nu \text{ and } \Phi_\xi \text{ (wavy lines)} + \text{Diagram with } \Phi_\nu \text{ and } \Phi_\xi \text{ (dashed loop)} \\ &+ \text{Diagram with } g_{11}^{\nu\xi} \text{ (circle)} + \text{Diagram with } g_{11}^{\nu\xi} \text{ (dashed loop)} \\ \Sigma_{12}^{\chi\xi} &= \text{Diagram with } \Phi_\nu \text{ and } \Phi_\xi \quad (\chi, \xi, \nu, \varsigma = X, Ph) \end{aligned}$$

Quasicondensate phase diagram



VS and D. Sarchi, pss (b) **242**, 2290 (2005)
 D. Sarchi and VS, SSC **144**, 371 (2007)
 D. Sarchi and VS, PRB **77**, 045304 (2008)

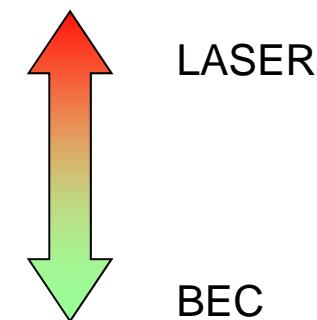
Polariton Laser vs polariton BEC



$$\gamma \propto n_{res} (N_0 + 1) \quad \text{gain}$$

γ_{rad} radiative decay

$$\gamma_{coll} \ll \gamma, \gamma_{rad}$$



$$\gamma_{coll} \gg \gamma_{rad}$$

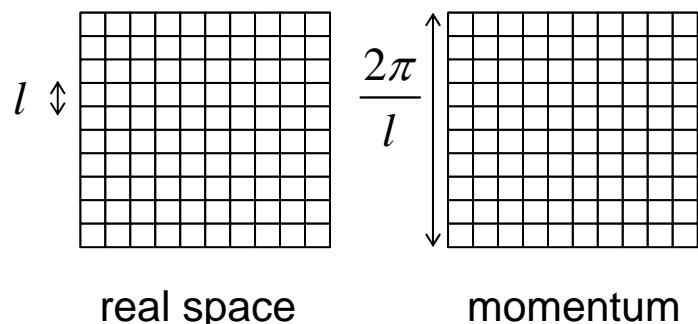
Partial thermalization a polariton condensate
H. Deng, et al., PRL 97, 146402 (2006)

Classical field approach to the kinetics of BEC

A quantum degenerate Bose gas is described as a classical field, in 1st approximation

Planck	vs	Rayleigh-Jeans	
$n_k = \frac{1}{\exp\left(\frac{E_k}{k_B T}\right) - 1}$	\approx	$\frac{k_B T}{E_k}$	UV catastrophe: $\int n_k dk = \infty$

Introduce a real-space discretization, in order to cut off high momenta



Assume $N(\mathbf{r}_i) \gg 1 \Rightarrow$ momentum cutoff at

$$\frac{\hbar^2 k_c^2}{2m} \sim k_B T$$

Full field is now described classically:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\mu - \frac{\hbar^2 \nabla^2}{2m} + V(\mathbf{r}) + g |\psi|^2 \right) \psi$$

See e.g.: A. Sinatra, J. Lobo, and Y. Castin, J. Phys. B **35**, 3599 (2002)

Classical kinetic theory of the polariton condensate

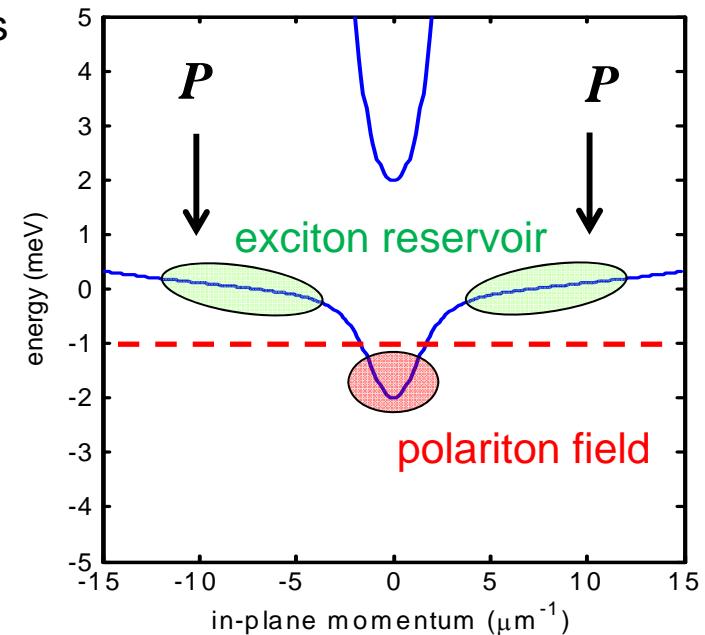
For polaritons, separate into polariton and exciton fields

Generalized Gross-Pitaevskii equation:

$$i \frac{\partial \psi}{\partial t} = \left\{ -\frac{\hbar \nabla^2}{2m_{LP}} + \frac{i}{2} [R(n_R) - \gamma] + g |\psi|^2 + 2\tilde{g} n_R \right\} \psi$$

Rate-diffusion equation for reservoir:

$$\frac{\partial n_R}{\partial t} = P - \gamma_R n_R - R(n_R) |\psi(x)|^2 + D \nabla^2 n_R$$



M. Wouters, I. Carusotto, PRL **99**, 140402 (2007)

Universally used today

Phase fluctuations: Truncated Wigner approximation

Wigner quasi-probability distribution

$$\frac{1}{2} \langle \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}') + \hat{\psi}(\mathbf{r}') \hat{\psi}^\dagger(\mathbf{r}) \rangle = \int d^2\psi(\mathbf{r}) P_W [\psi(\mathbf{r}), \psi^*(\mathbf{r})] \psi^*(\mathbf{r}) \psi(\mathbf{r}')$$

3rd order differential equation for $P_W [\psi(\mathbf{r}), \psi^*(\mathbf{r})]$

“Truncate” to 2nd order: Fokker-Planck equation

Solve corresponding Langevin equation with noise sources (fluct.-diss. theorem)

Polariton $d\psi(\mathbf{r}) = -i \left[-\frac{\hbar^2 \nabla^2}{2m} + \frac{i(\mathcal{R}_{in} - \mathcal{R}_{out} - \gamma)}{2} + \frac{g}{\Delta V} |\psi(\mathbf{r})|^2 \right] \psi(\mathbf{r}) dt + dW(\mathbf{r})$

Langevin $\langle dW(\mathbf{r}) dW(\mathbf{r}') \rangle = \frac{dt}{2\Delta V} (\langle \mathbf{r} | \mathcal{R}_{in} + \mathcal{R}_{out} | \mathbf{r}' \rangle + \gamma \delta_{\mathbf{r}, \mathbf{r}'})$

Reservoir $\frac{dn_R}{dt} = P - \gamma_R n_R - \alpha \frac{d}{dt} \text{Re} [\psi^* (\mathcal{R}_{in} - \mathcal{R}_{out}) \psi] - \frac{\alpha}{2\Delta V} (R_{in} + R_{out})$

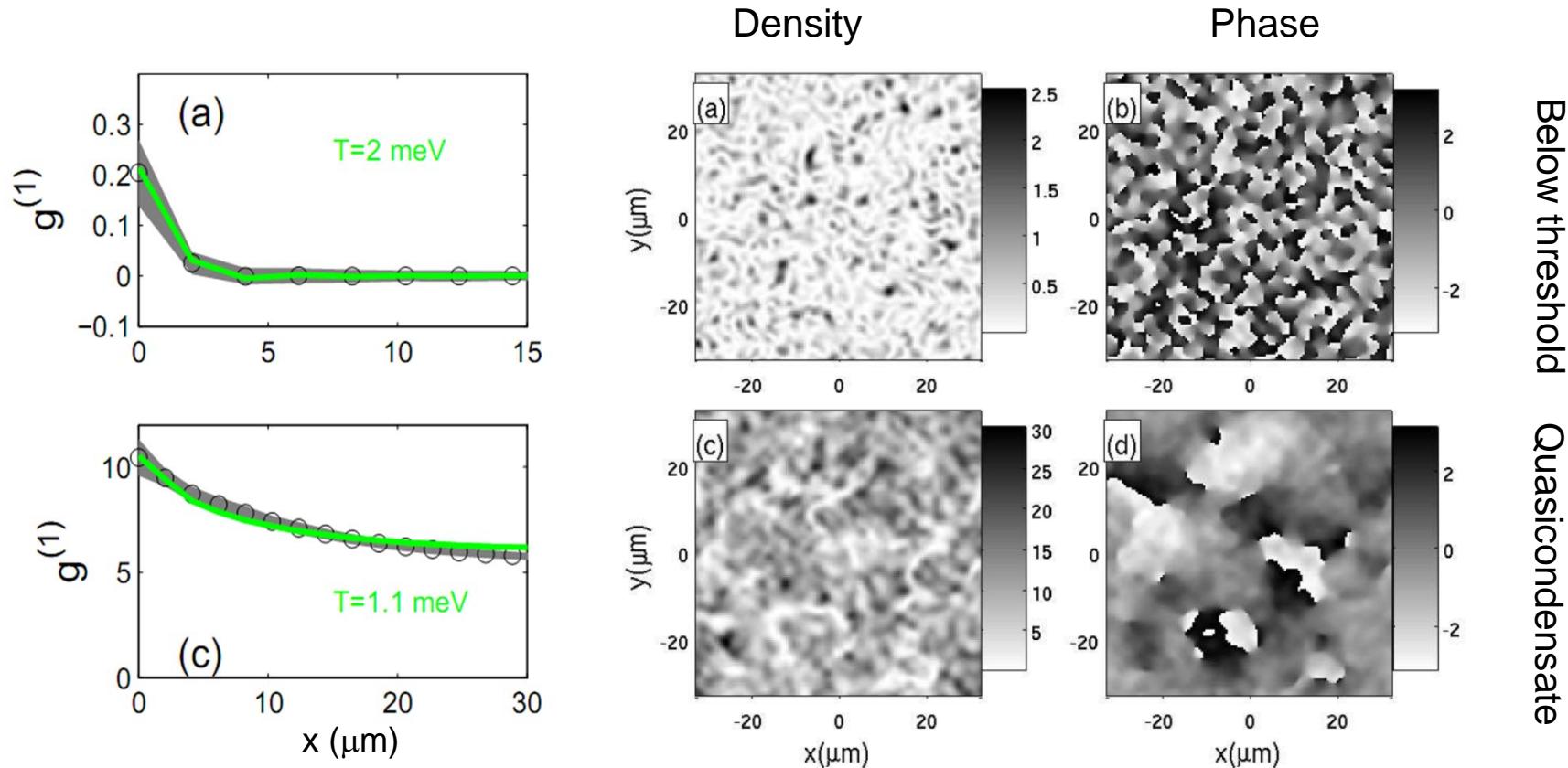
M. Wouters, V. Savona, Phys. Rev. B **79**, 165302 (2009)



V. Savona, Lyon BEC 2012, Lyon, June 2012



Wigner Monte Carlo approach to driven-dissipative BEC

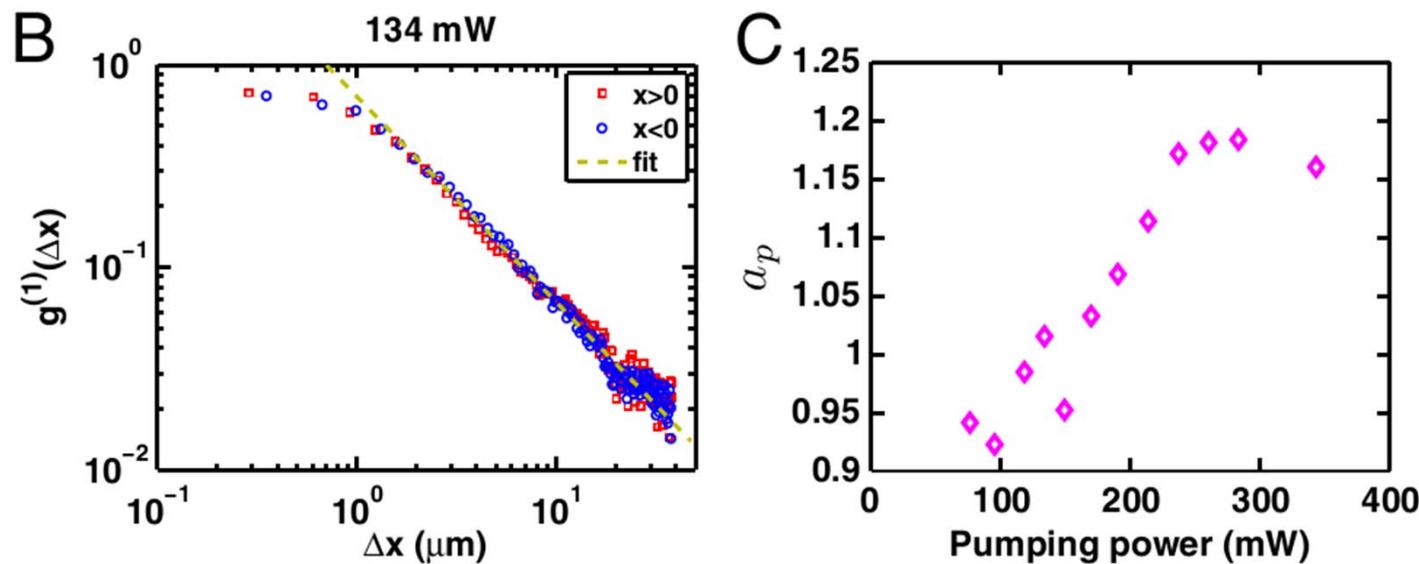


M. Wouters and VS, Phys. Rev. B **79**, 165302 (2009)

Alternative approach: wave function Monte Carlo

M. Wouters, Phys. Rev. B **85**, 165303 (2012)

Observation of 2-D character of fluctuations



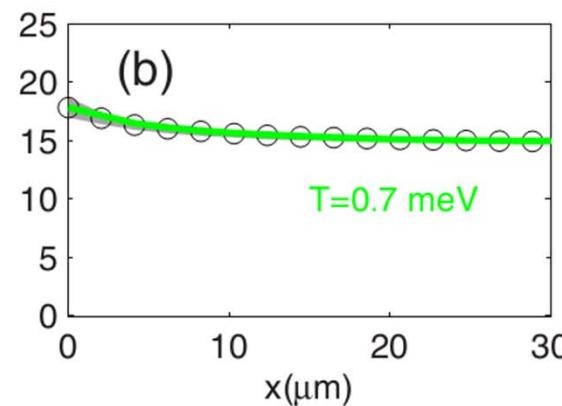
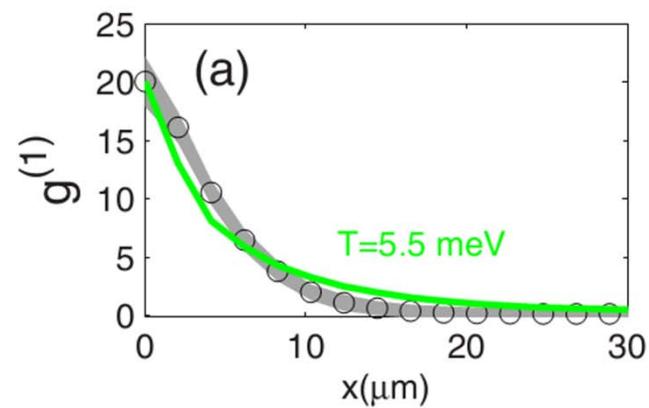
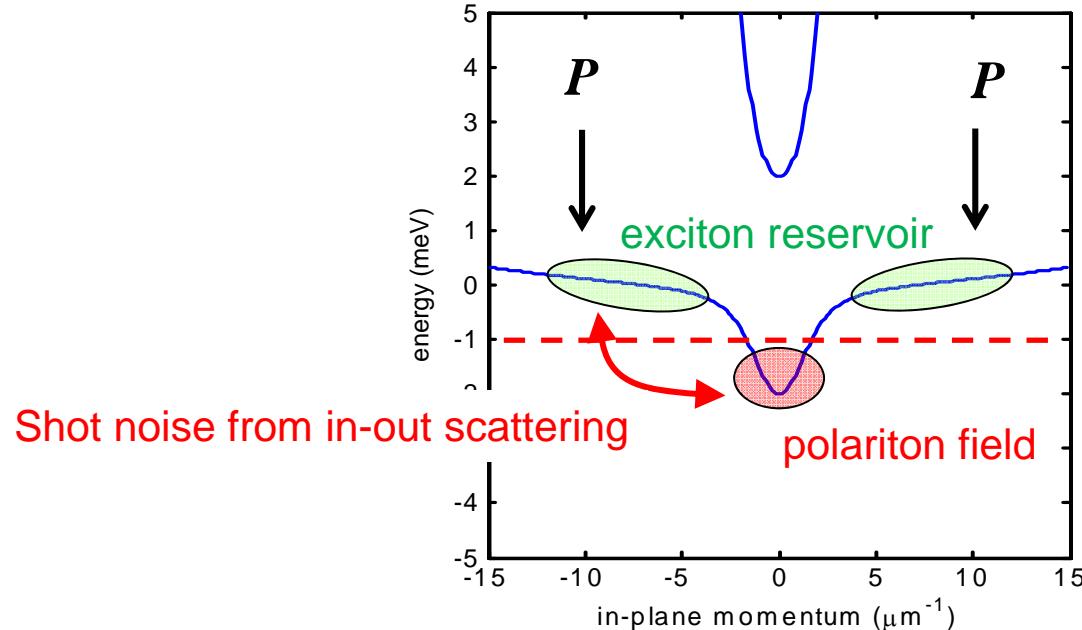
G. Roumpos et al., PNAS **109**, 6467 (2012)

However... no BKT (yet!). Fluctuations determined by gain saturation mechanism.

M. Wouters and VS, Phys. Rev. B **79**, 165302 (2009)

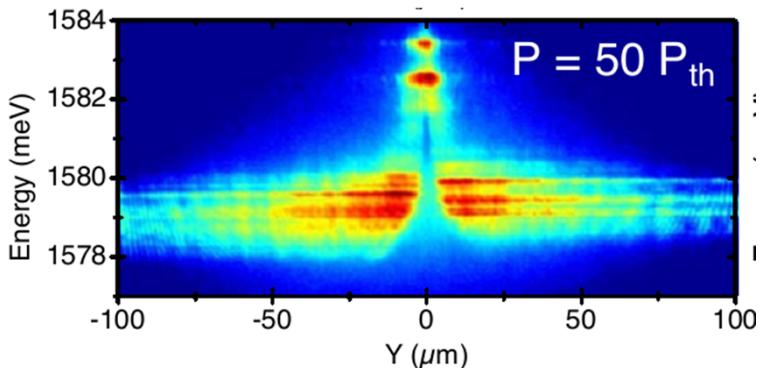
(remember yesterday's talk by L. Mathey)

Effect of kinetics on phase fluctuations



M. Wouters and VS, Phys. Rev. B **79**, 165302 (2009)

Intra-condensate relaxation and multimode condensation



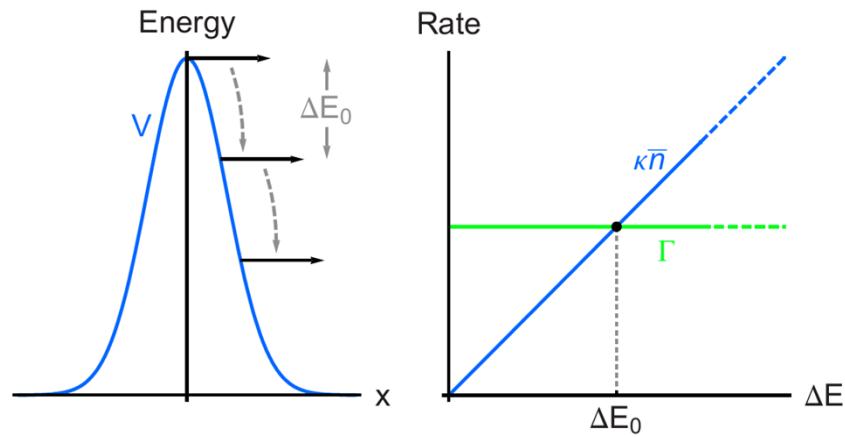
Interplay of gain and losses can produce multimode condensation

E. Wertz et al., Nat. Phys. **6**, 860 (2010)
M. Richard et al., PRL **94**, 187401 (2005)

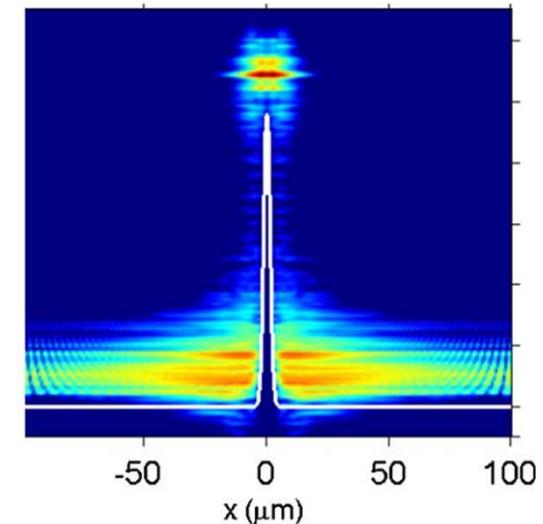
More pronounced in spatially inhomogeneous systems

Mean-field model for intra-condensate relaxation

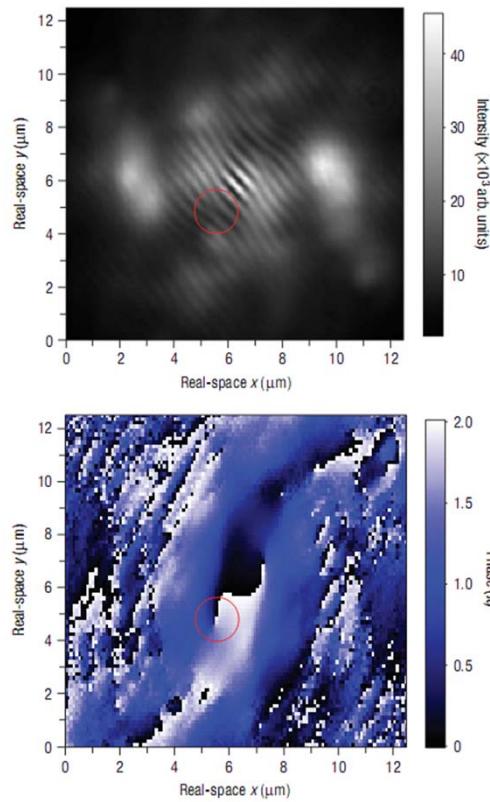
$$\frac{\partial \psi(x,t)}{\partial t} \Big|_{\text{rel}} = \frac{\kappa \bar{n}(x,t)}{2} \left[\bar{\mu}(x,t) - \frac{i\partial}{\partial t} \right] \psi(x,t)$$



M. Wouters, T. C. H. Liew, and VS, PRB **82**, 245315 (2010)



Quantized vortices and half-vortices in a polariton superfluid

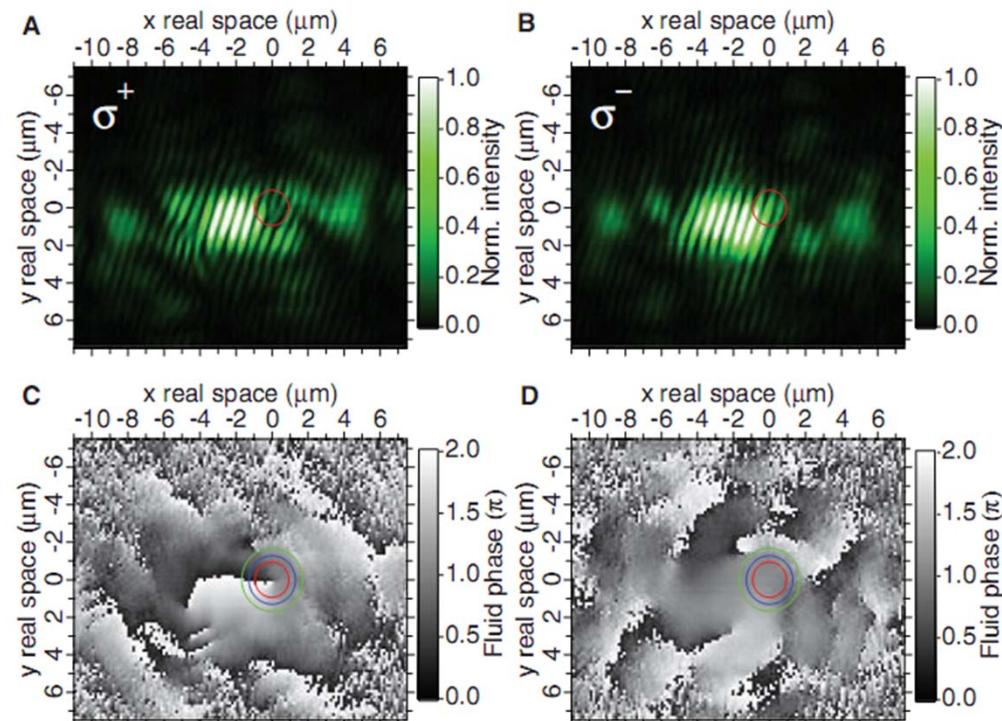


K. G. Lagoudakis, et al.,
Science **326**, 974 (2009)

K. G. Lagoudakis, et al., Nature Physics **4**, 706 (2008)
D. Sanvitto et al., Nat Phys. **6**, 527 (2010)
G. Roumpos et al., Nat. Phys **7**, 129 (2011)

Polaritons are spinor-bosons: $\psi_{lin}(\mathbf{r}) = \sqrt{n} e^{i\theta(\mathbf{r})} \begin{pmatrix} \cos \eta(\mathbf{r}) \\ \sin \eta(\mathbf{r}) \end{pmatrix}$

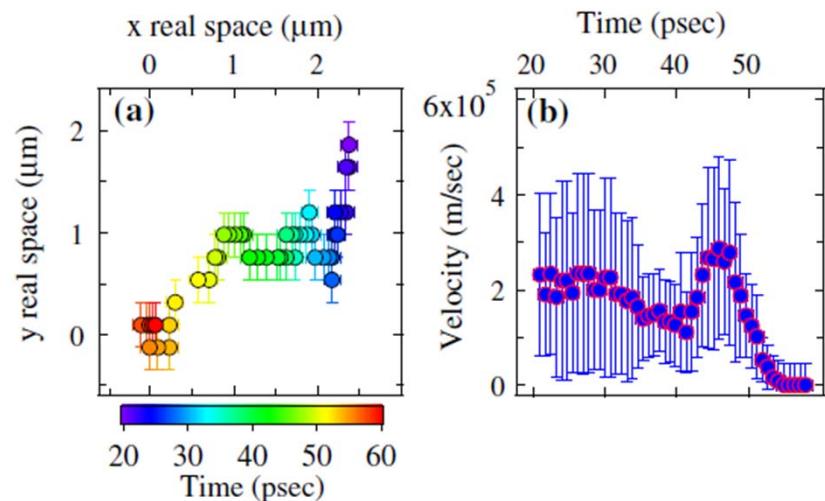
Direct observation of polariton half-vortices



Physics of the polariton quantum fluid

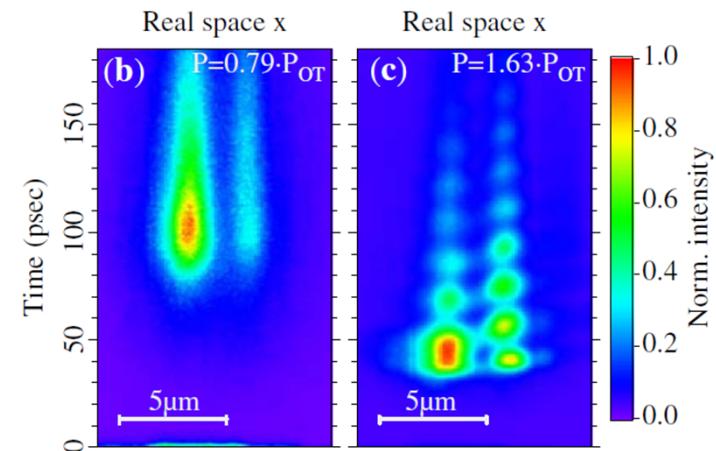
Migrating vortices

K. Lagoudakis et al., PRL **106**, 115301 (2011)

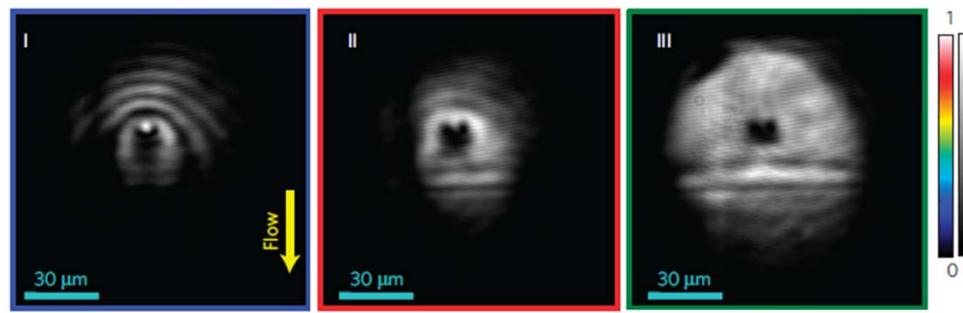


Josephson oscillations

K. Lagoudakis et al., PRL **105**, 120403 (2010)



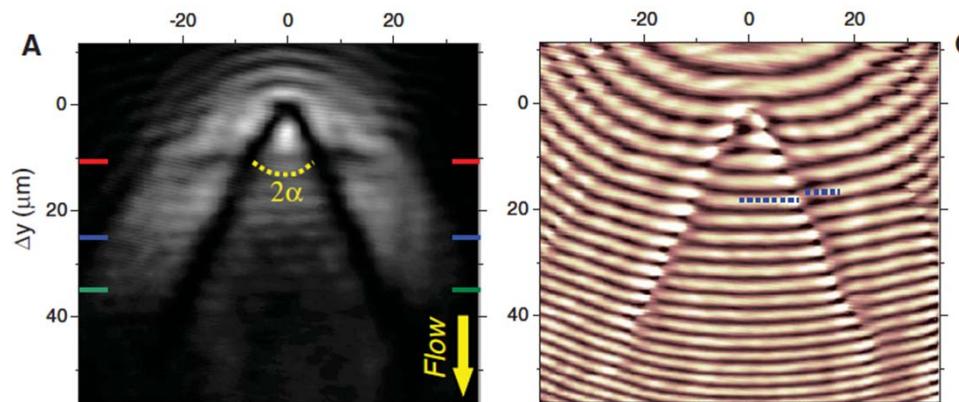
Polariton Superfluid under resonant excitation



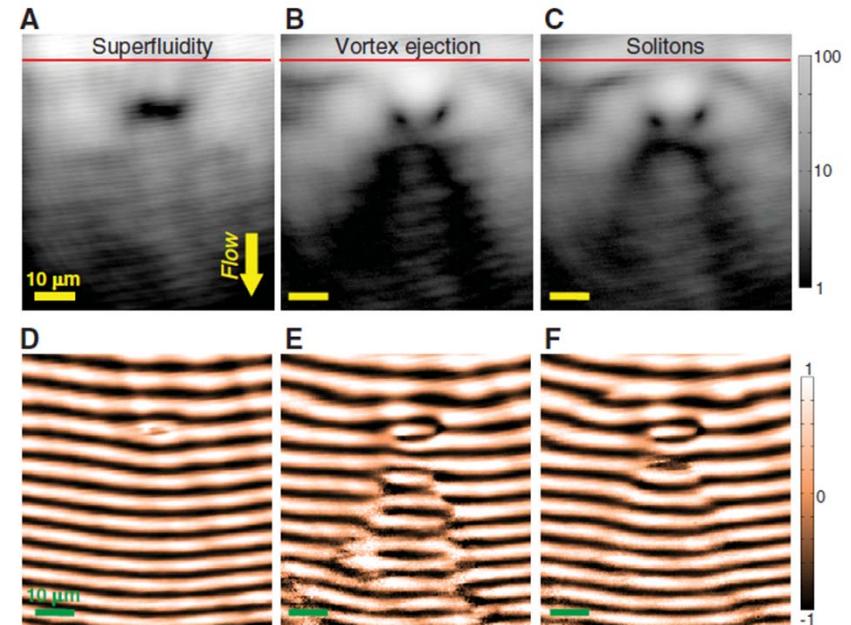
Superfluid flow without scattering

A. Amo et al., Nat. Phys. **5**, 805 (2009)

Dark solitons and quantum turbulence



A. Amo et al., Science **332**, 1167 (2011)
S. Pigeon et al., PRB **83**, 144513 (2011)



(non exhaustive) List of recent experiments

Polariton BEC in various materials and at room temperature

S. Kéna-Cohen and S. R. Forrest, *Nature Photon.* **4**, 371 (2010)
T. Guillet et al., *APL* **99**, 161104 (2011)
S. Christopoulos et al., *PRL* **98**, 126405 (2007)

Study of various patterns in the superfluid flow of a polariton BEC

G. Tosi, et al., *PRL* **107**, 036401 (2011)
M. D. Fraser, et al., *New J. Phys.* **11**, 113048 (2009)
F. Manni, et al., *PRL* **107**, 106401 (2011)
M. Sich, et al., *Nature Photonics* **6**, 50 (2012)
G. Tosi, et al., *Nature Physics* **8**, 190 (2012)
G. Christmann, et al., *arXiv:1201.2114* (2012)

Trapped condensates

R. Idrissi Kaitouni et al., *PRB* **74**, 155311 (2006)
E. Wertz et al., *Nat. Phys.* **6**, 860 (2010)
R. Balili, et al., *Science* **316**, 1007 (2007)

Polariton BEC and disorder

V. Savona, *J. Phys.: Cond. Mat.* **19**, 295208 (2007)
A. Baas et al., *PRL* **100**, 170401 (2008)
F. Manni et al., *PRL* **106**, 176401 (2011)

Study of fluctuations

P.-E. Larré et al., *Phys. Rev. A* **85**, 013621 (2012)
K. G. Lagoudakis et al., *PRL* **106**, 115301 (2011)

Etc...



V. Savona, Lyon BEC 2012, Lyon, June 2012



Conclusions

- Polaritons would be an ideal system to study the fundamental properties of the Bose gas in 2-D
- Truly 2-D system. Disorder almost always present. Trapping and spatial patterning possible. Direct experimental readout of order parameter (amplitude and phase)
- Driven-dissipative system. Non-equilibrium makes the system non-universal
- Several very nice studies of the dynamics of the polariton quantum fluid, but...

Outlook

- Need for a new generation of samples with longer lifetime and controlled disorder
- Polaritons are a great opportunity to study the fundamental physics of a quantum gas in low dimensions

For a recent review on polariton BEC see:

H. Deng, H. Haug, and Y. Yamamoto, Rev. Mod. Phys. **82**, 1489 (2010)

Thanks to: M. Wouters, T. C. H. Liew, and D. Sarchi

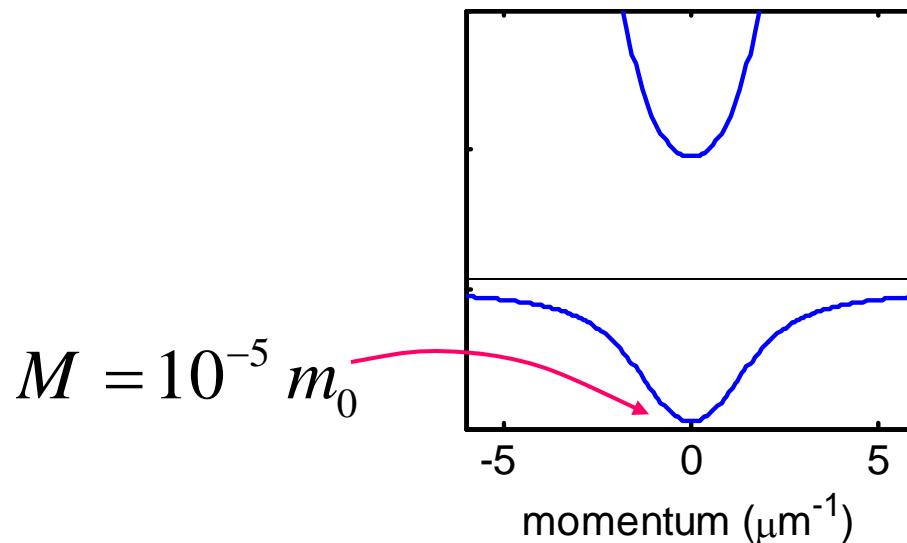




V. Savona, Lyon BEC 2012, Lyon, June 2012



Is the polariton gas a “quantum” gas?

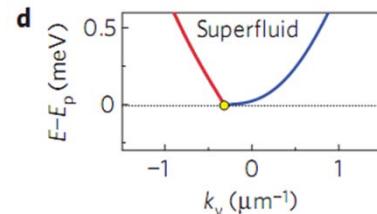
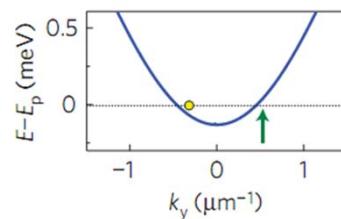


But...
$$\frac{E_{\mathbf{k}}}{\hbar} = \frac{c}{n} \sqrt{k^2 + \left(\frac{2\pi}{\lambda}\right)^2}$$
 not of the form
$$\frac{\hbar^2 k^2}{2m}$$

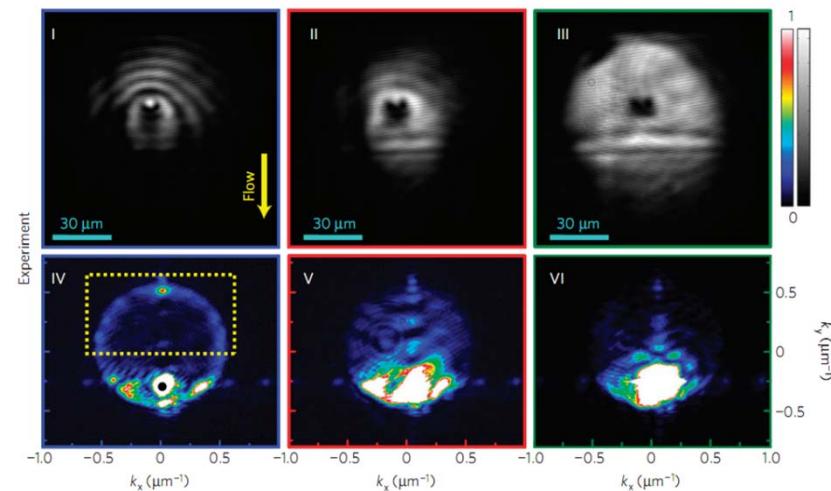
Thermal wavelength
$$\lambda_T = \lambda \sqrt{\frac{E_{ph}}{2\pi k_B T}}$$

Polariton effective mass is not a kinetic mass. It is enforced by Maxwell equations

Polariton superfluidity



A. Amo, et al., Nature Physics **5**, 805 (2009)



Is a driven-dissipative system superfluid?

M. Wouters and I. Carusotto, PRL **99**, 140402 (2007)

Diffusive Goldstone mode is predicted

A. Amo, et al., Nature **457**, 291 (2009)

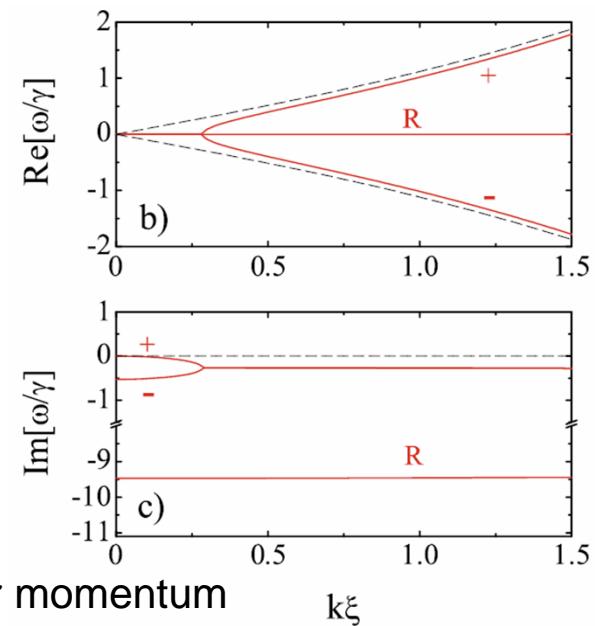
D. Sanvitto, et al., Nature Physics (online) (2010)

Persistent current – vorticity is observed

M. Wouters and VS, PRB **81**, 054508 (2010)

M. Wouters and I. Carusotto arXiv:1001.0660

Generalized superfluidity: diffusive modes do not scatter momentum



Effective bosonic Hamiltonian for excitons and photons

Can we map the composite fermion problem onto an effective Bose Hamiltonian with interactions?

$$\hat{H} = \hat{H}_{lin} + \hat{H}_x + \hat{H}_s$$

X-ph linear interaction:

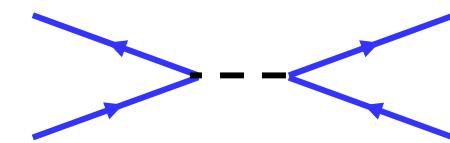
$$\hbar\Omega$$

$$\hat{H}_{lin} = \sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}}^x b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \varepsilon_{\mathbf{k}}^c c_{\mathbf{k}}^\dagger c_{\mathbf{k}}) + \hbar\Omega \sum_{\mathbf{k}} (c_{\mathbf{k}}^\dagger b_{\mathbf{k}} + b_{\mathbf{k}}^\dagger c_{\mathbf{k}})$$



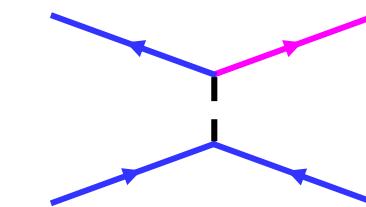
$$\hat{H}_x = \frac{1}{2A} \sum_{\mathbf{kk}'\mathbf{q}} v_x(\mathbf{k}, \mathbf{k}', \mathbf{q}) b_{\mathbf{k}+\mathbf{q}}^\dagger b_{\mathbf{k}'-\mathbf{q}}^\dagger b_{\mathbf{k}'} b_{\mathbf{k}}$$

X-X interaction:



$$\hat{H}_s = \frac{1}{A} \sum_{\mathbf{kk}'\mathbf{q}} v_s(\mathbf{k}, \mathbf{k}', \mathbf{q}) c_{\mathbf{k}+\mathbf{q}}^\dagger b_{\mathbf{k}'-\mathbf{q}}^\dagger b_{\mathbf{k}'} b_{\mathbf{k}} + h.c.$$

X- oscillator strength saturation:



$$[c_{\mathbf{k}}, c_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{kk}'}$$

photons

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{kk}'}$$

excitons

Rochat, et al., PRB **61**, 13 856 (2000),
 Ben-Tabou de-Leon, et al., PRB **63** 125306 (2001),
 Okumura, et al., PRB **65**, 035105 (2001),
 Ch. Schindler and R. Zimmermann, PRB **78**, 045313 (2008)