Ultracold atoms as a new tool in condensed matter physics

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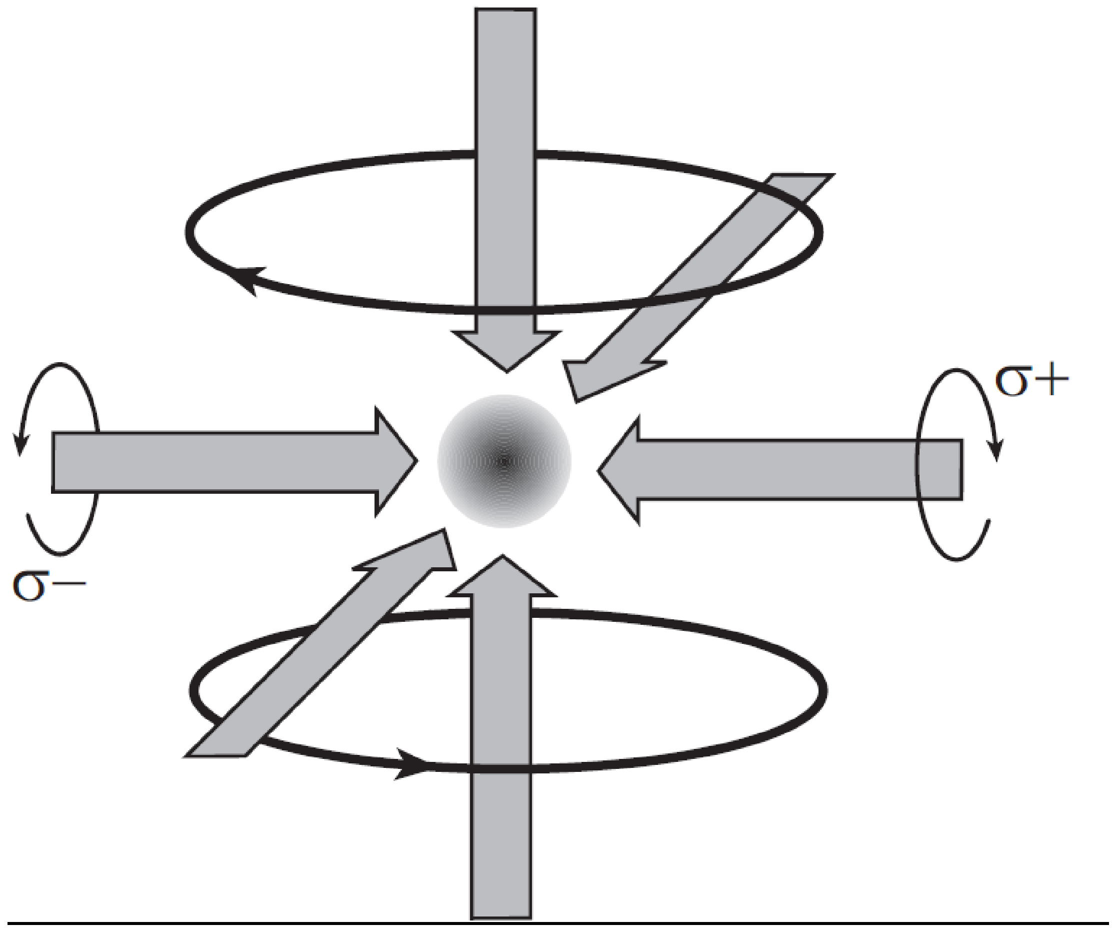
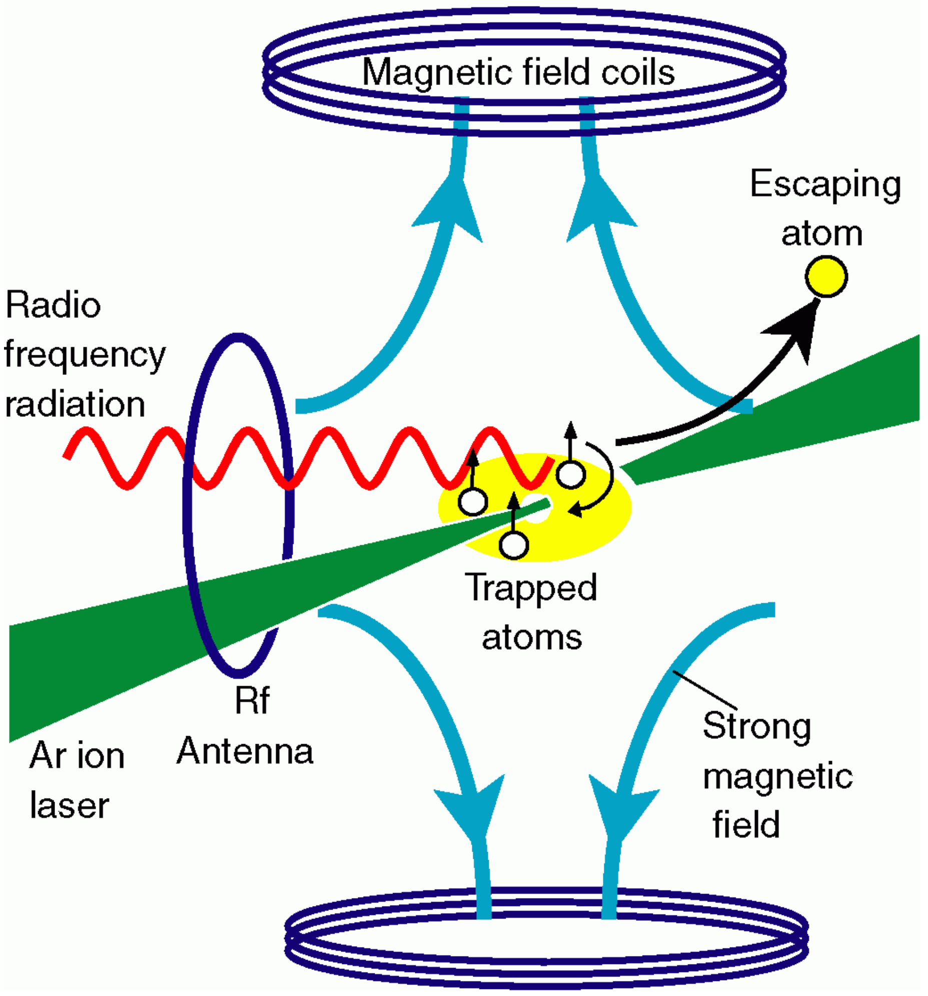
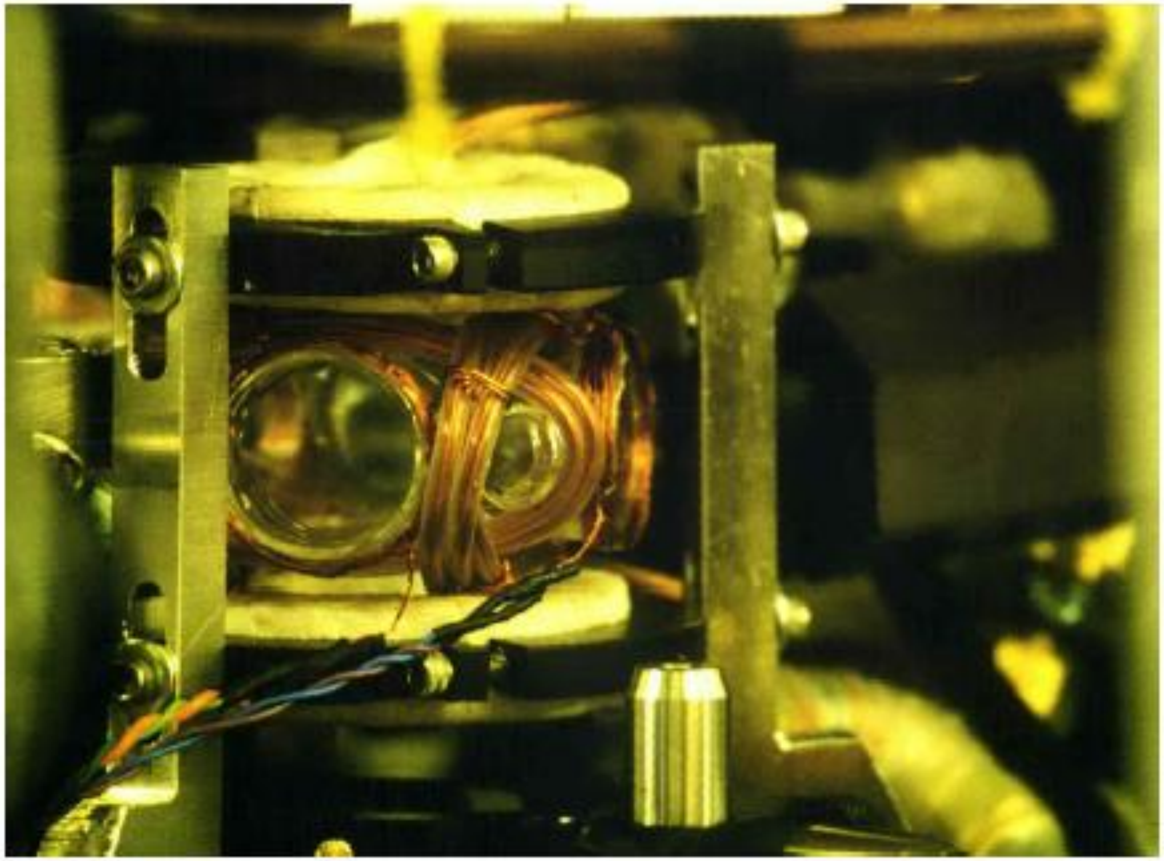
Trapping and cooling



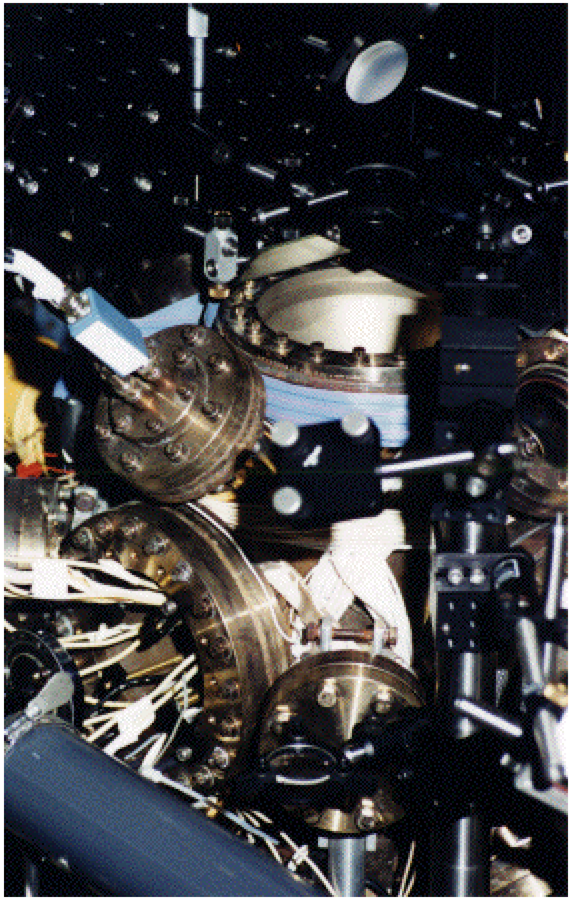
Magnetic trap with optical plug



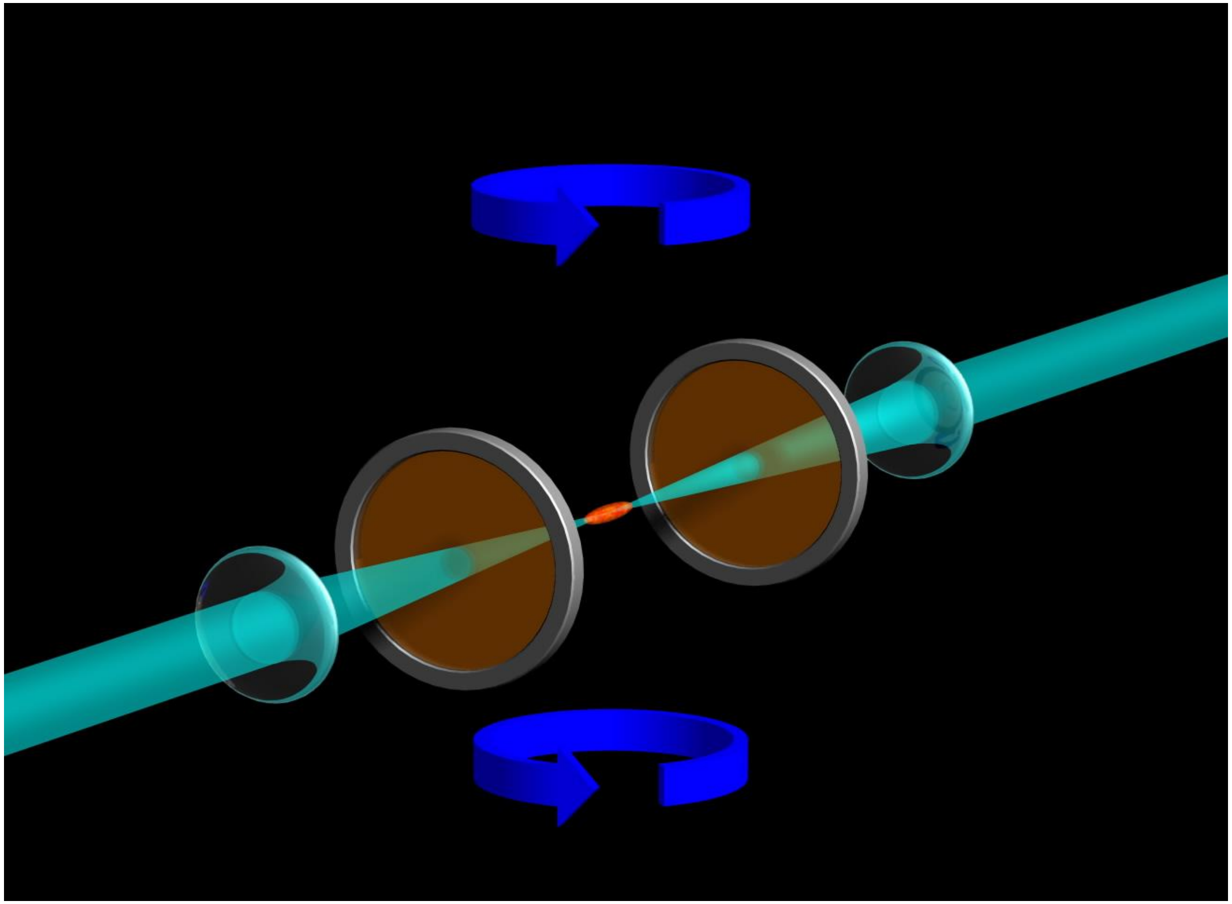
MOT (Boulder)



MOT (MIT)



Fermions in optical trap (Duke University)



Typical parameters

$$N \sim 3\times10^{6} - 10^{7}$$

$$n \sim 2\times10^{12}\,cm^{-3}, n^{-1/3} \sim 0.3\mu m$$

$$\nu_{\perp} \sim 60 - 300\,Hz, \nu_{z} \sim 20\,Hz$$

$$T_{Deg} \sim 200 - 500\,nK - 1.7\,\mu K$$

$$T/T_{Deg} < 0.06, T_{Min} \sim 20\,pK$$

Two BEC on atom chip. (MIT)

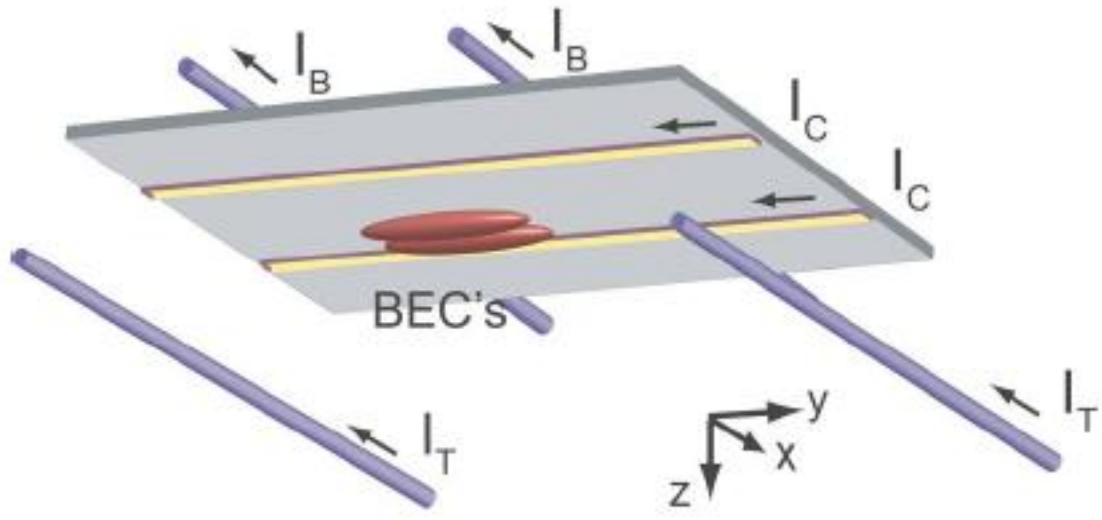


FIG. 1. (Color online) Schematic diagram of the atom chip. A magnetic double-well potential was created by two chip wires with a current I_C in conjunction with an external magnetic field. The distance between the two chip wires was 300 μm.

New experimental possibilities

Strong effects of extermal fields.

Force in a.c. electric field:

$$U(\mathbf{r}) = -\frac{1}{2}\alpha(\omega)E^2(\mathbf{r},t)$$

Near the absorption line

$$\alpha(\omega) = \frac{\left|\langle n|\mathbf{d}\bullet\mathbf{e}|0\rangle\right|^2}{\omega_n - \omega}$$

Possibility to change interaction.

$$f = -\frac{1}{a^{-1} + ik}$$

a - scattering length

1) $a > 0 : a = \hbar / \sqrt{m|\varepsilon|}.$

Weakly bound state

with energy $\varepsilon < 0.$

2) $a < 0$ - "virtual level"

$$|\varepsilon| << \hbar^2 / m r_0^2, |a| >> r_0.$$

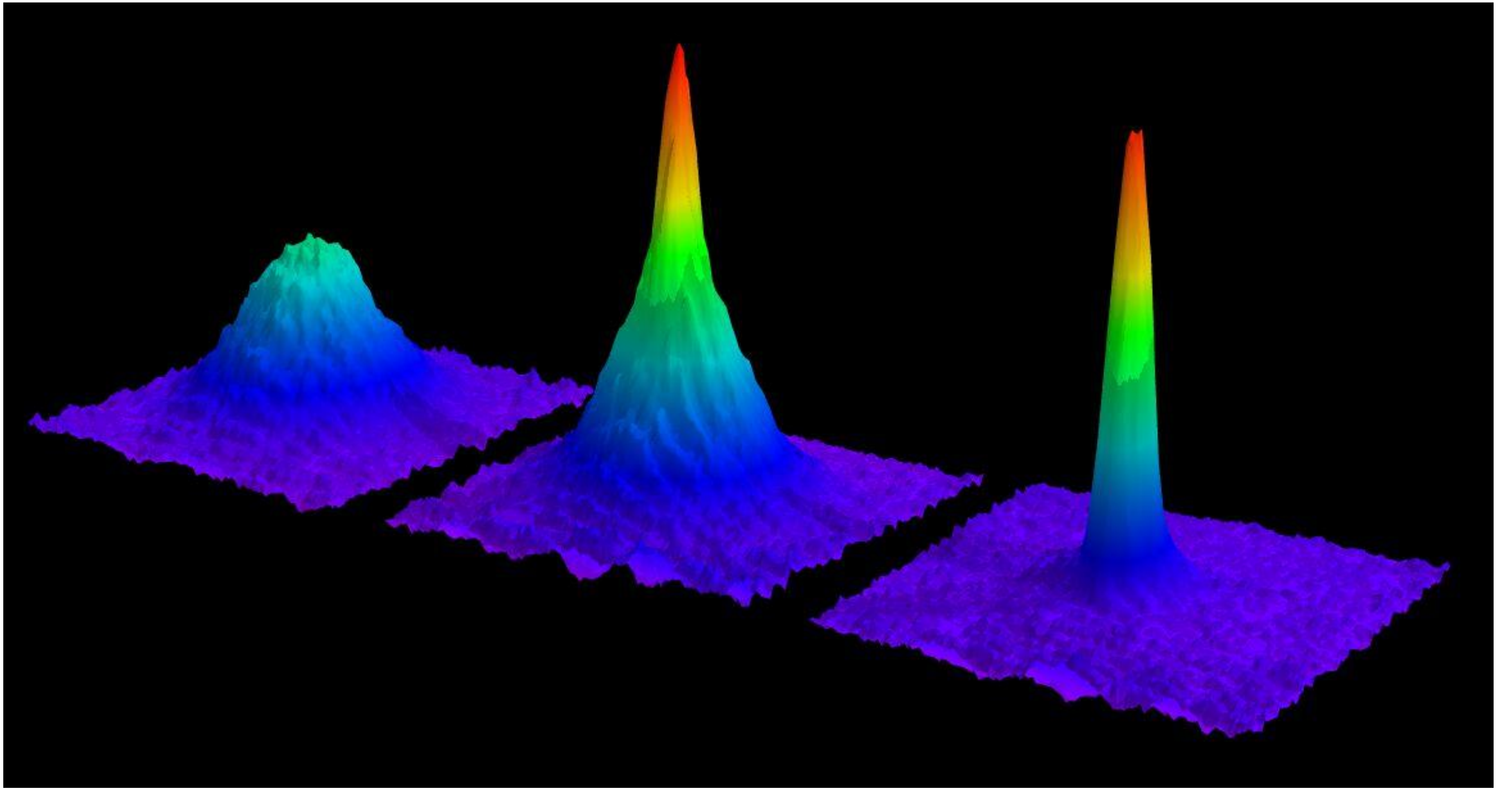
Feshbach resonance

$$a = a_{bg}\left(1 - \frac{\Delta_B}{B - B_0}\right)$$

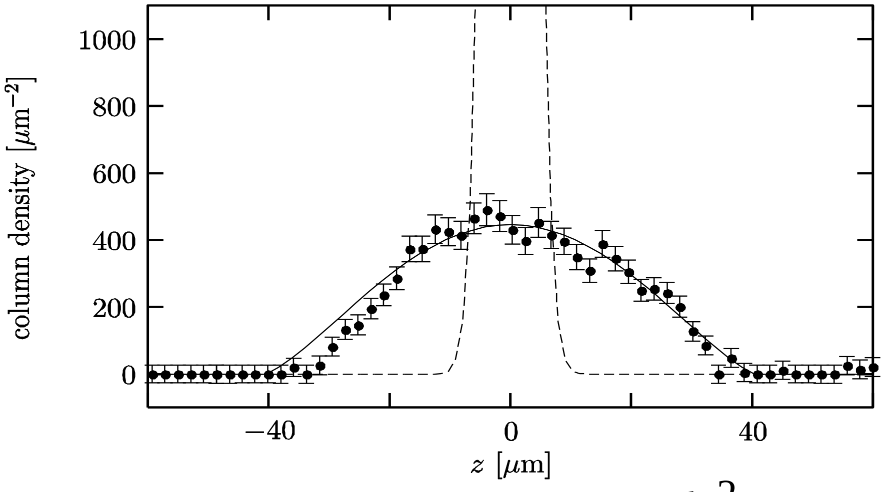
$$B \to B_0,\, a \to \pm\infty$$

$$f = 1/ik$$

Bose-Einstein Condensation



Density distribution

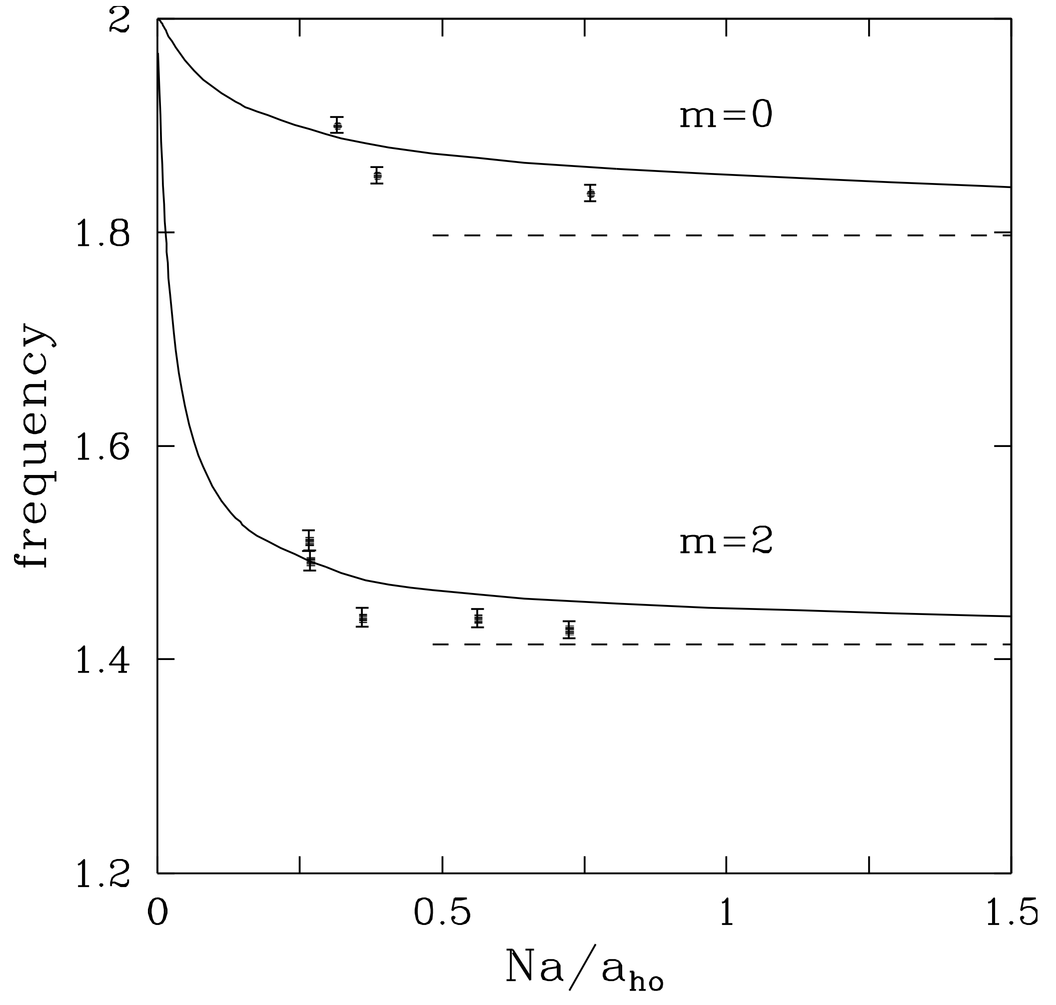


$$\mu = gn,\ g = \frac{4\pi\hbar^2}{m}a$$

N.N. Bogoljubov , 1947

$$gn + \frac{m\omega^2 r^2}{2} = \mu_0$$

Collective oscillations. Stringari prediction and experiment



Lee-Huang-Yang correction

$$\delta E \sim \sum_{p}^{mu} \frac{\hbar\omega}{2}$$

$$\delta\mu / \mu \propto \sqrt{na^3}$$

Lee, Huang, Yang, 1957

LHY correction. N. Navon et. all, 2011

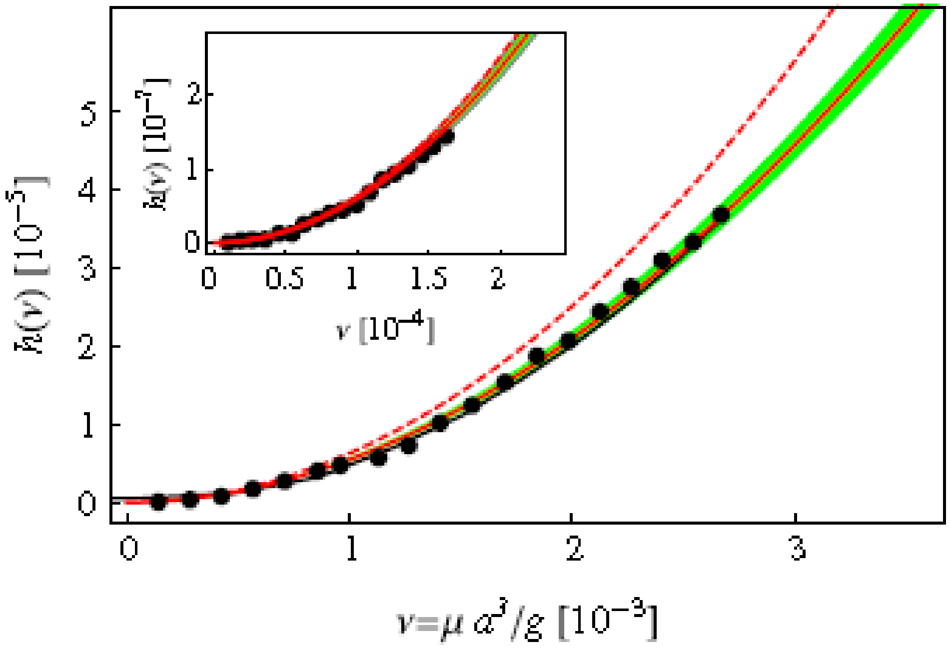


FIG. 2 (color online). Equation of state of the homogeneous Bose gas expressed as the normalized pressure h as a function of the gas parameter ν. The gas samples for the data shown in the main panel (inset) have been prepared at scattering lengths of $a/a_0 = 1450$ and 2150 ($a/a_0 = 700$). The gray (red online) solid line corresponds to the LHY prediction, and the gray (red online) dashed line to the mean-field EOS $h(\nu) = 2\pi\nu^2$.

Rotation. Vortex lines

Ordinary liquid :

$$v_\varphi = \Omega r, \quad \mathbf{curl}\ \mathbf{v} = 2\Omega \neq 0$$

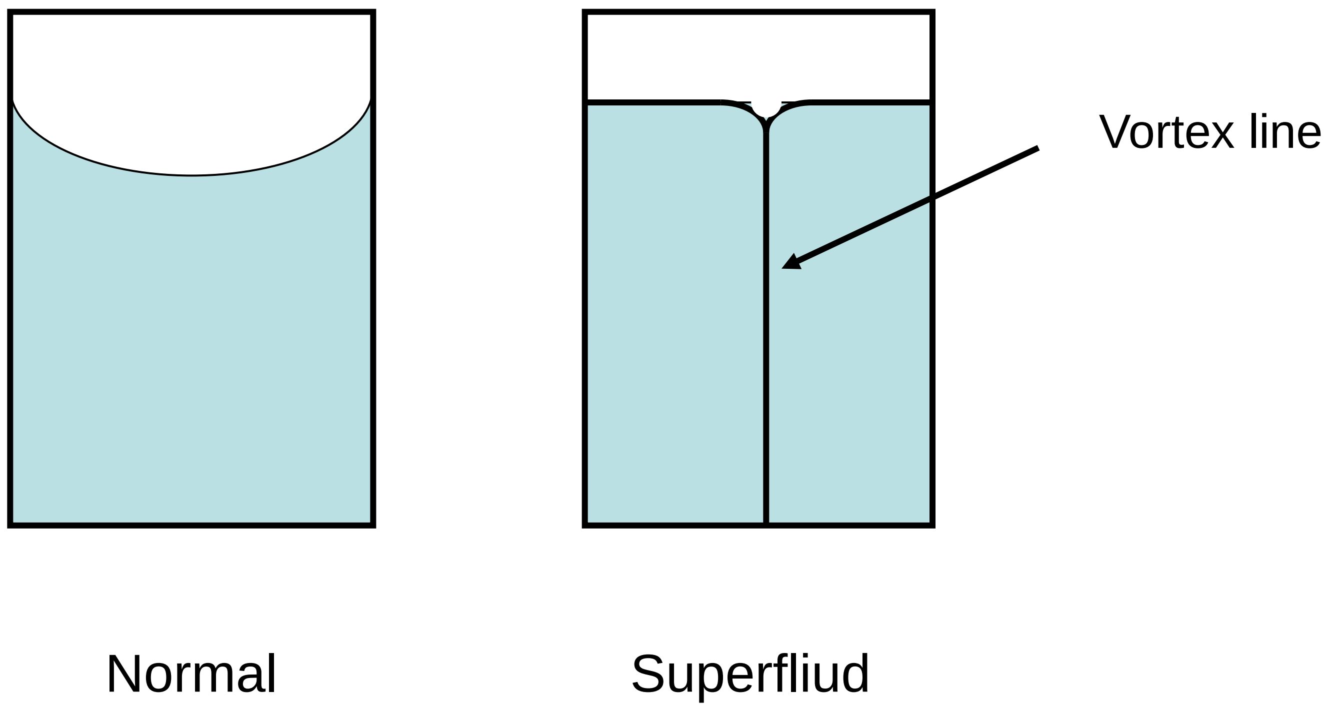
Condensate :

$$\Psi = |\Psi| e^{i\varphi}, v_\varphi = \frac{\hbar}{m}\frac{1}{r}, n_v = \frac{m\Omega}{\pi\hbar}$$

Vortex line.

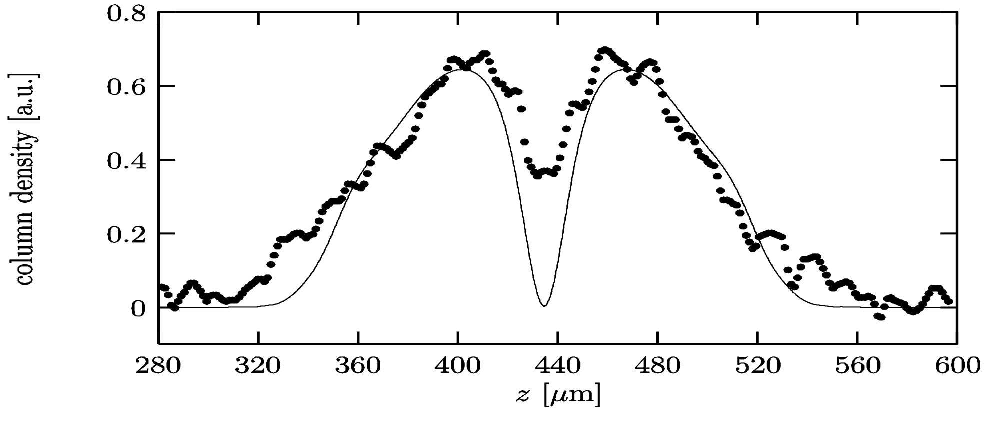
L. Onsager, 1949; R. Feynman, 1954.

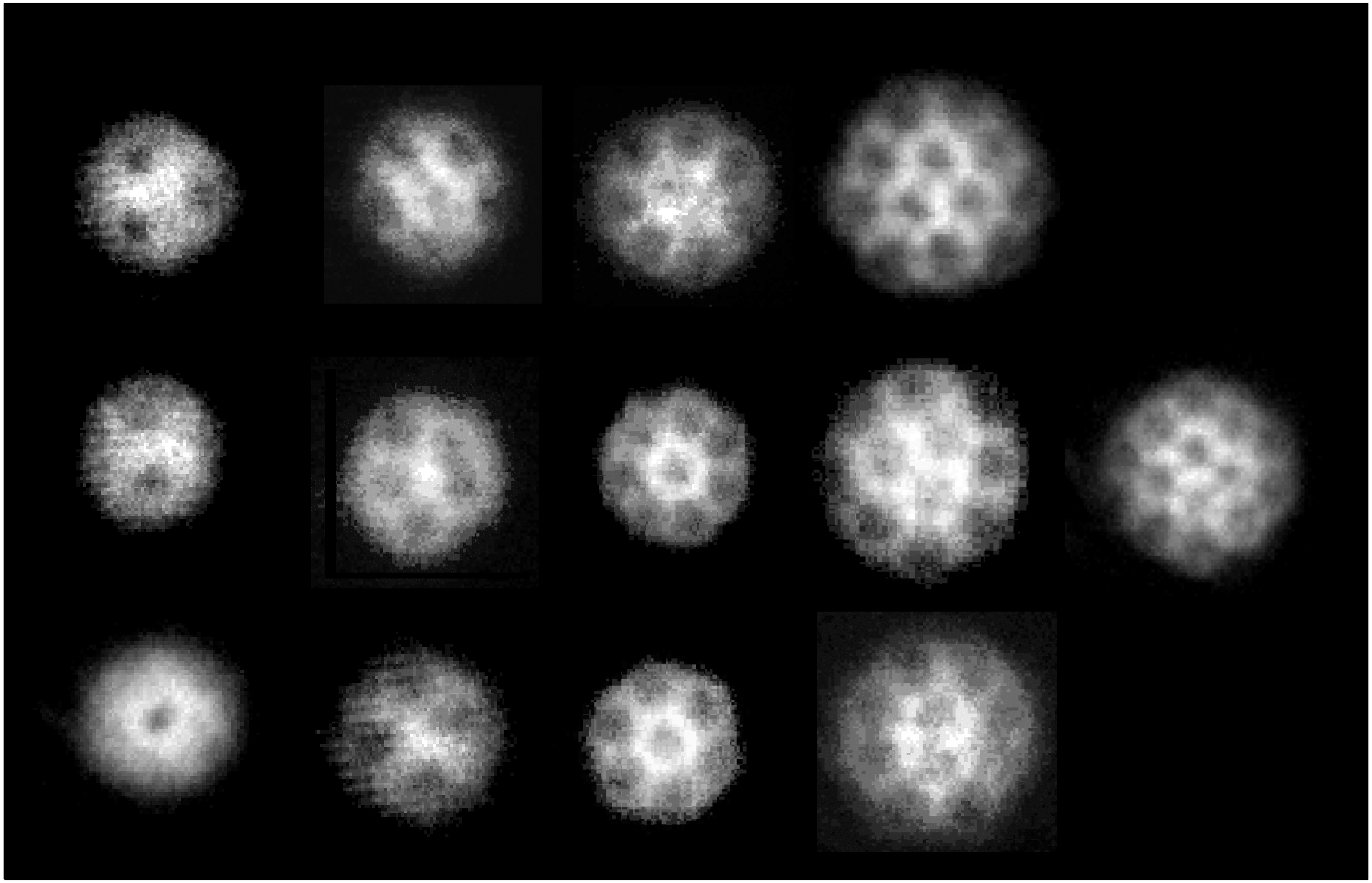
Rotation of normal and superfluid liquids

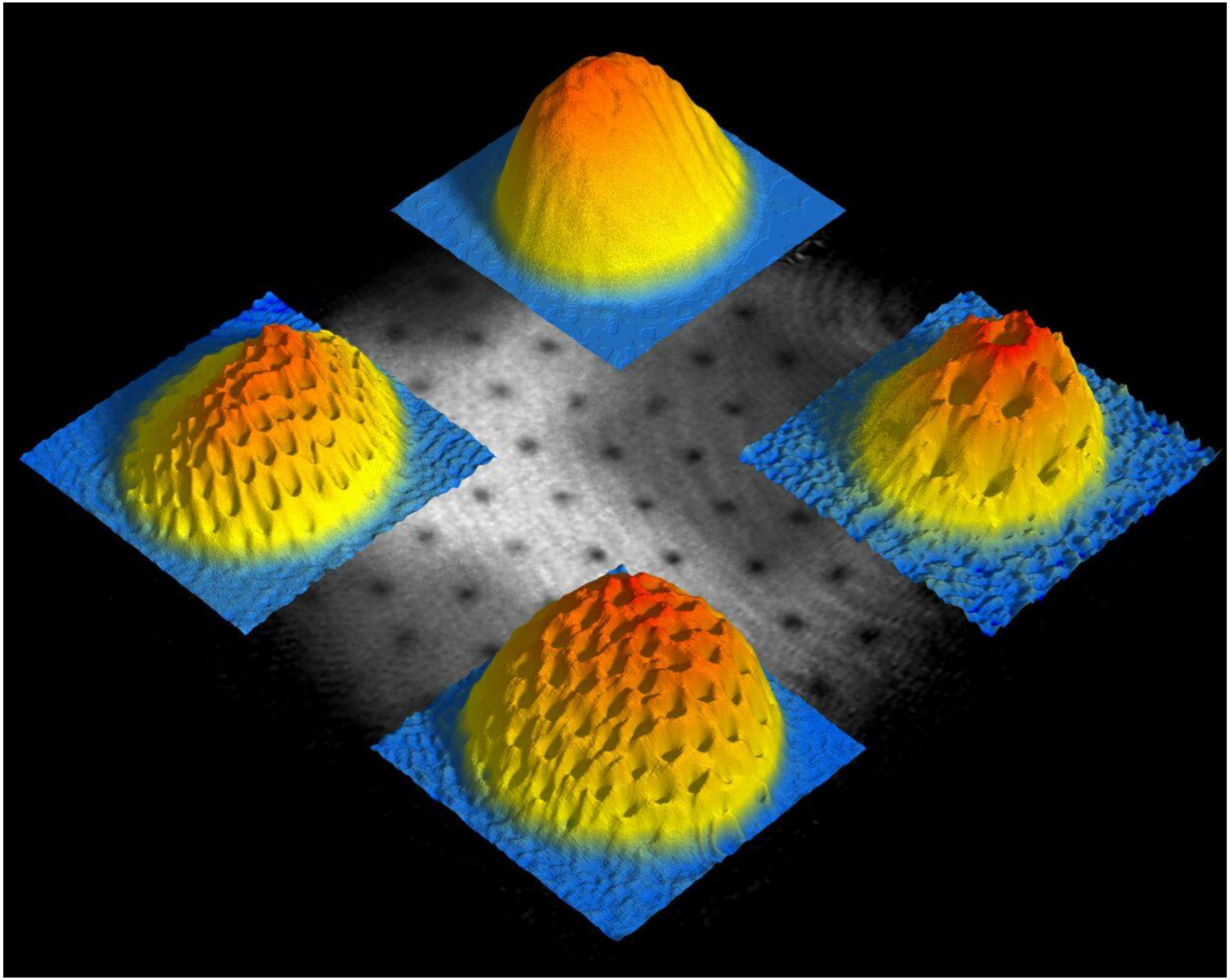


Structure of a vortex line

$$|\Psi| = f(r/\xi), \xi = \hbar/\sqrt{2ngm}, n = |\Psi|^2$$







Two classical limits of QM

1. Classical body: $m \to \infty$

2. Classical electromagnetic waves:

Number of photons $N_{ph} = \dfrac{E}{\hbar\omega} \to \infty$

From quantum electrodynamics to classical Maxwell equations

$$\text{Commutation relations for the vector-potential:}$$

$$\left[\hat{A}_i, \frac{\partial \hat{A}_k}{\partial t}\right] \sim \hbar, \quad \hat{A}_i(\mathbf{r}, t) \to A_i(\mathbf{r}, t)$$

$$\hat{\mathbf{E}}(\mathbf{r}, t) \to \mathbf{E}(\mathbf{r}, t), \hat{\mathbf{B}}(\mathbf{r}, t) \to \mathbf{B}(\mathbf{r}, t)$$

$$\text{The Maxwell equations:}$$

$$\text{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \quad \text{etc.}$$

Why Maxwell equations do not contain *h* ?

Energy - momentum relation for photons :

$$\varepsilon = cp$$

Transition from particles to waves :

$$\varepsilon,\ p \rightarrow \omega, k: \quad \varepsilon = \hbar\omega,\ p = \hbar k$$

Frequency – wave vector relation

$$\omega = ck$$

does not contain $\hbar$.

Bose-Einstein Condensation - classical limit for the Broglie waves

$$\left[\hat{\Psi}(\mathbf{r},t)\,\hat{\Psi}^{+}(\mathbf{r}',t)\right] = \hbar\delta(\mathbf{r}-\mathbf{r}')$$

$$\Psi \sim \sqrt{N}\ , \quad \hat{\Psi}(\mathbf{r},t) \rightarrow \Psi(\mathbf{r},t)$$

$$\text{In an uniform condensate } \hat{\Psi} \rightarrow \sqrt{N}$$

N.N. Bogoliubov, 1947

From particles to classical waves

Energy - momentum relation for atoms:

$$\varepsilon = \frac{p^2}{2m}$$

Transition from particles to waves:

$$\varepsilon,\ p \to \omega, k: \quad \varepsilon = \hbar\omega,\ p = \hbar k$$

Frequency – wave vector relation

$$\omega = \frac{\hbar k^2}{2m}$$

This relation does contain $\hbar$.

Equation for the classical function $\Psi(\mathbf{r}, t)$

will contain $\hbar$.

Equation for $\Psi(\mathbf{r},t)$

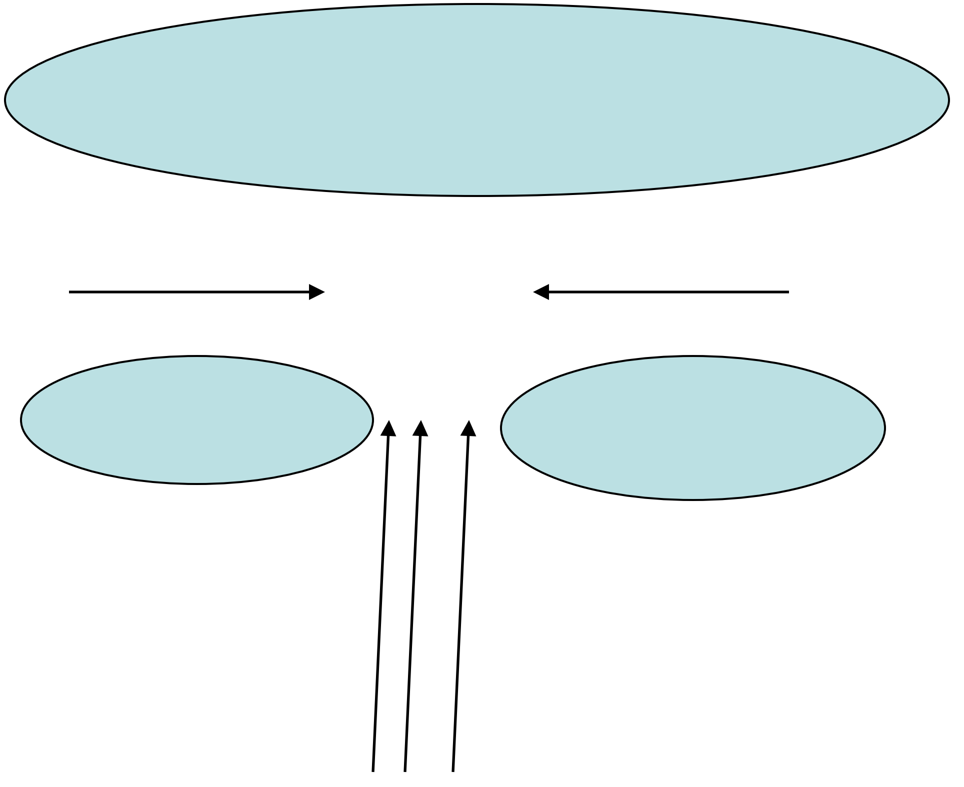
$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\Delta\Psi + g\Psi\left|\Psi\right|^2$$

E.P. Gross, 1961; L.P. Pitaevskii, 1961

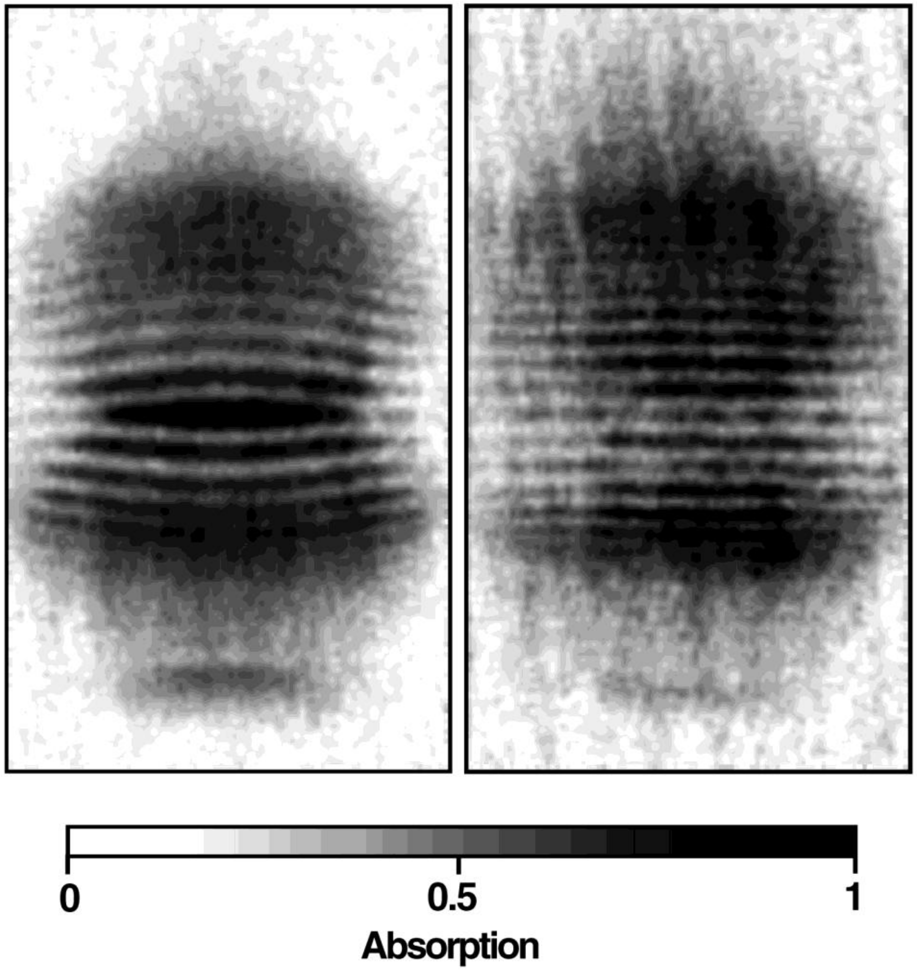
$g = \dfrac{4\pi\hbar^2}{m}a$, a is s-wave scattering length

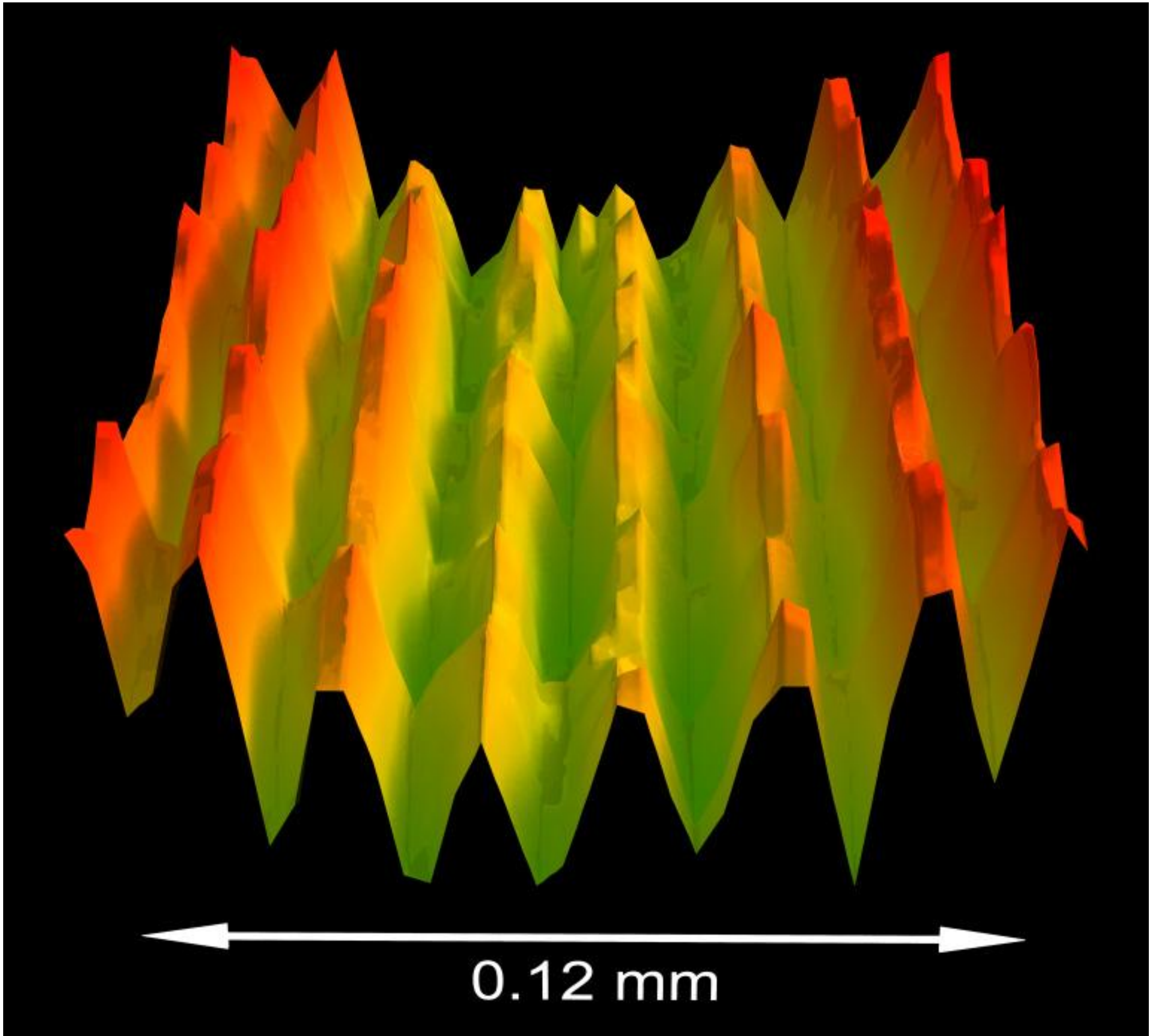
Plays role of the Maxwell equations in this problem.

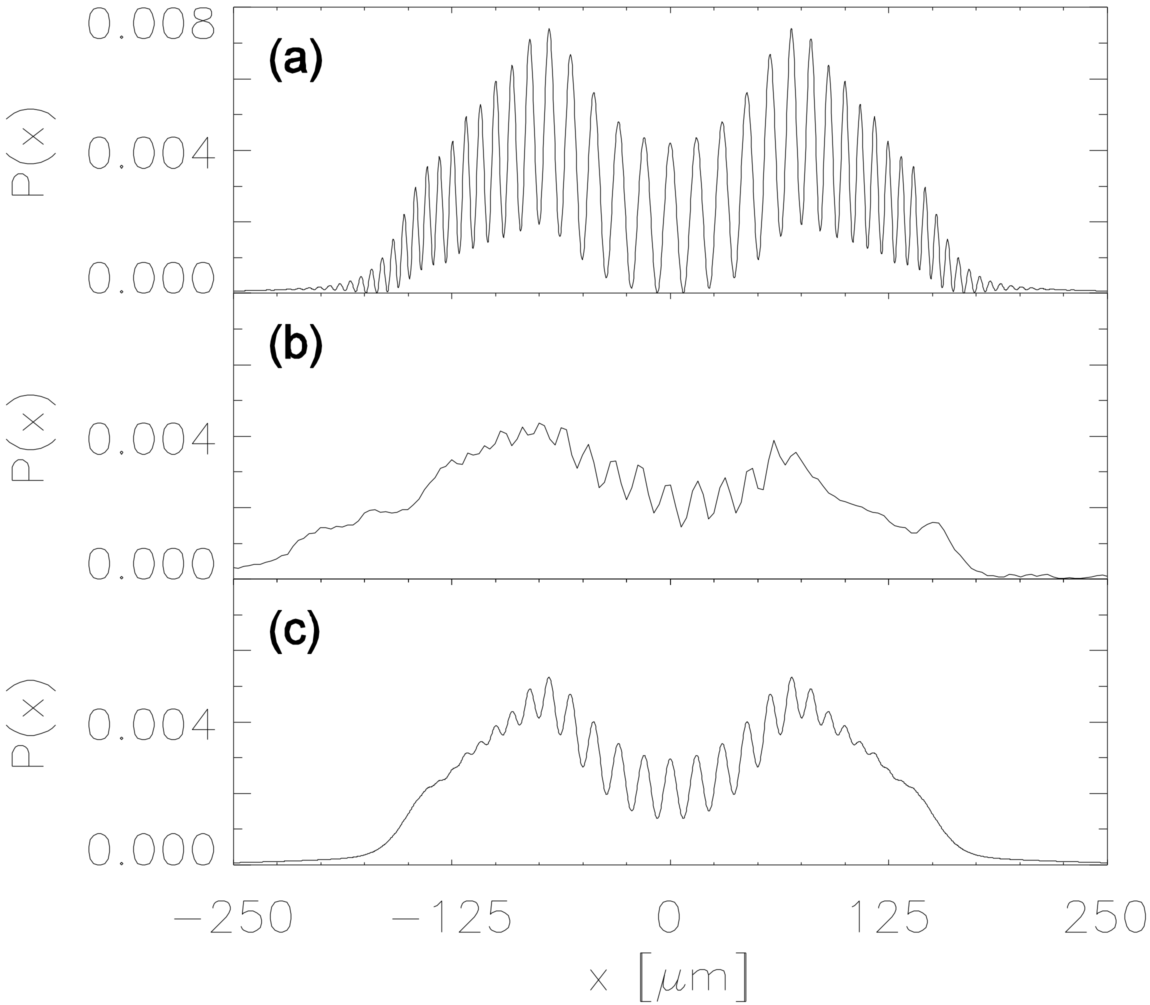
Interference experiment



Laser knife

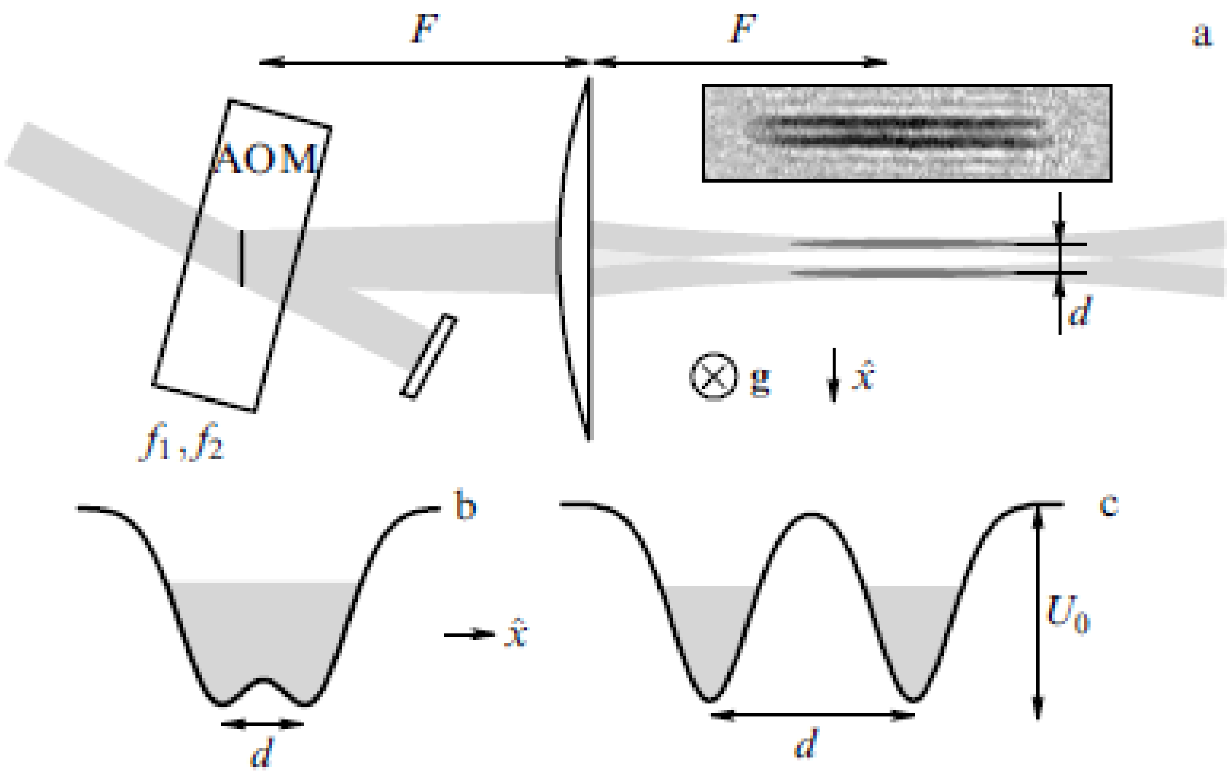

0
0.5
1
Absorption


0.12 mm

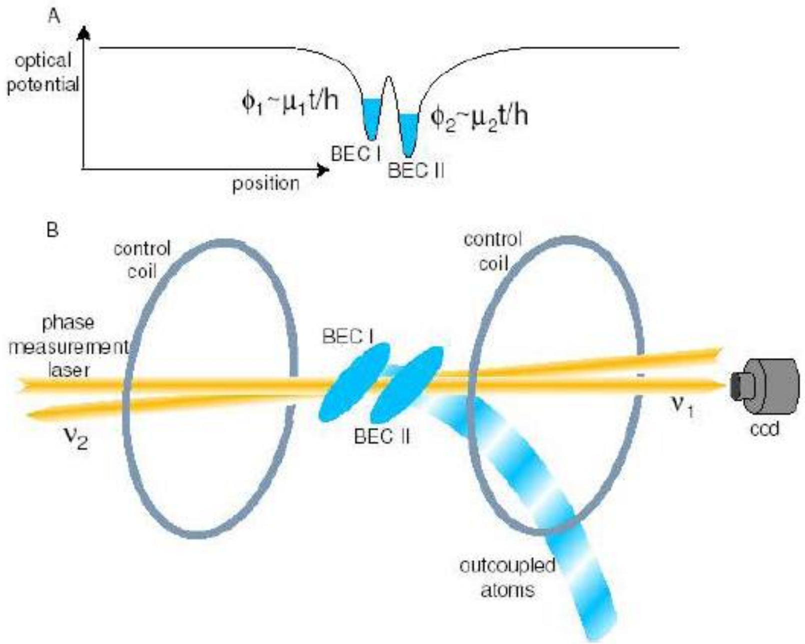


(a)
(b)
(c)
P(x)
P(x)
P(x)
0.008
0.004
0.000
0.004
0.000
0.004
0.000
−250
−125
0
125
250
x [μm]

Interference independent condensates



Continuous phase measurement



Setup for continuous phase measurement.
M. Saba, T. Pasquini, C. Sanner, Y. Shin, W. Ketterle, and D. Pritchard, Science (2005).

Mott transition in optical lattice

$$U(z) = sE_r \cos^2(qz),\ E_r = \hbar^2 q^2 / 2m$$

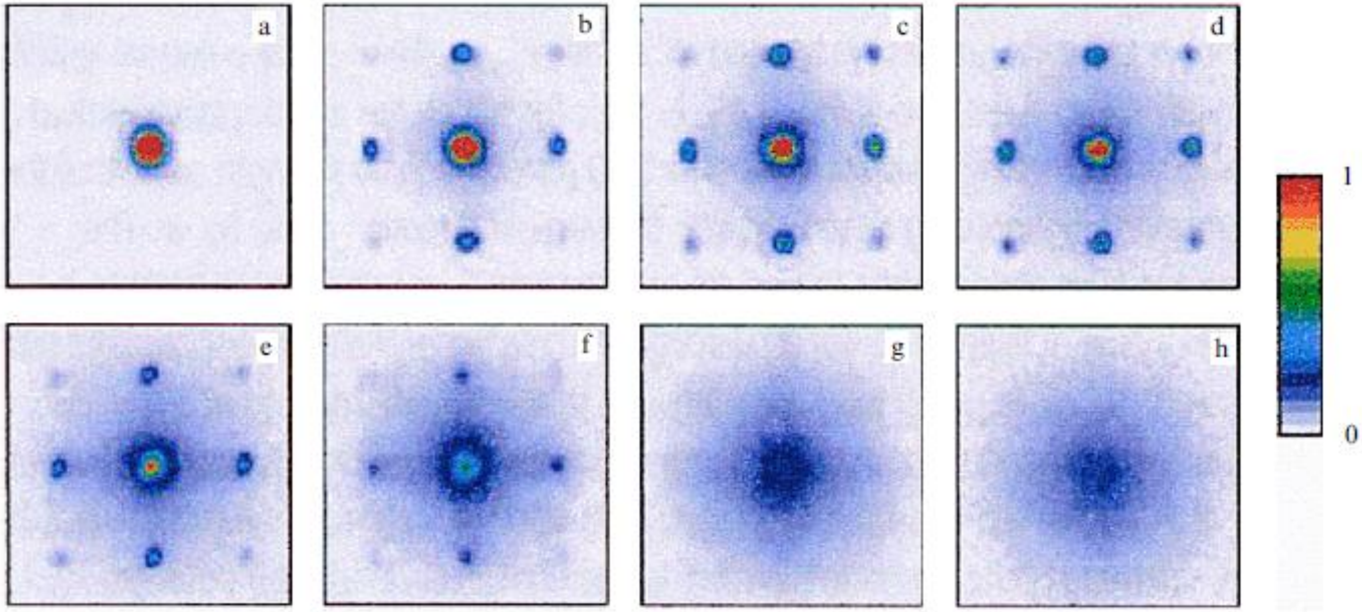


Figure 16. Interference pattern upon Bose-gas expansion from a three-dimensional lattice for different values of the parameter s: (a) $s = 0$, (b) 3, (c) 7, (d) 10, (e) 13, (f) 14, (g) 16, (h) 20. The disappearance of diffraction spots for $s > 13$ signifies the Mott transition to the dielectric phase [48].

Strongly interacting dilute liquid

$$r_0 << n^{-1/3}$$

$$\mathrm{BUT:}$$

$$|f| \sim n^{-1/3}$$

Universal liquid

$$r_0 << n^{-1/3}, |a| \sim n^{-1/3}$$

Properties of the liquid are defined by a unique parameter a.

$$|a| = \infty \text{ - "universal liquid"}$$

Weakly bound dimers of fermions

$$a > 0 \ \text{Binding energy} \ |\varepsilon| = \hbar^2 / ma^2.$$

Dimer - dimer scatering length :

$$a_{dd} = 0.6a > 0. \quad \text{(A)}$$

Recombinat ion rate

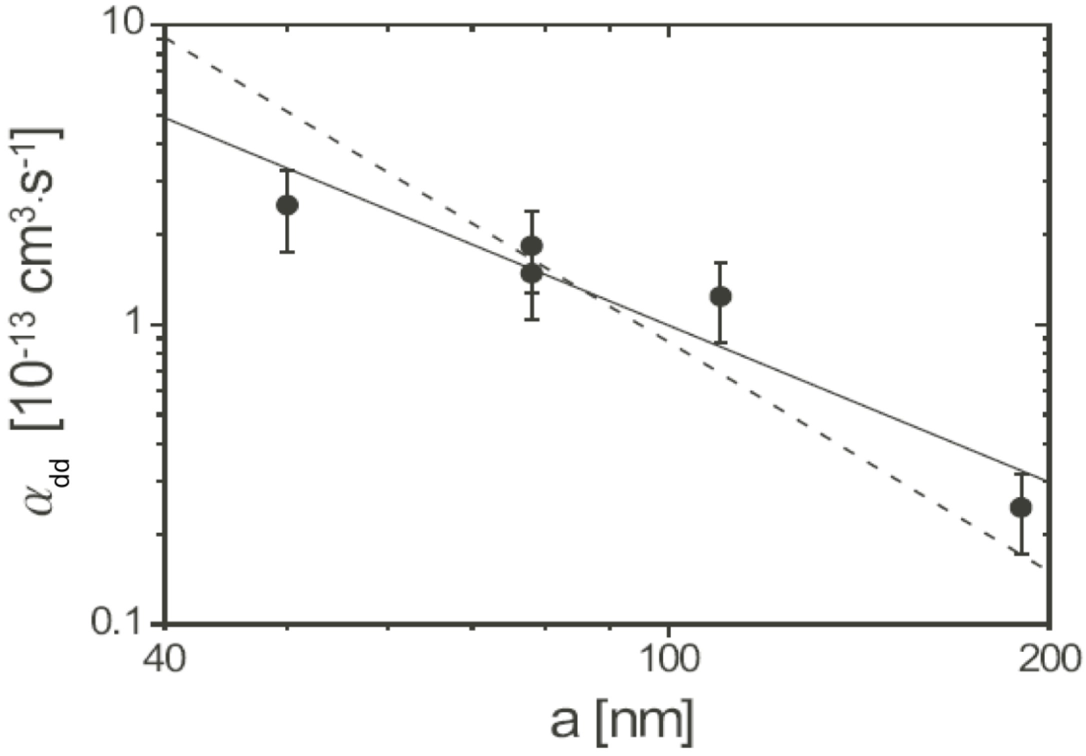
$$dn_d / dt = -\alpha_{dd} n_d^2, \alpha_{dd} \propto a^{-2.25} \quad \text{(B)}$$

$$a \to \infty, \alpha_{dd} \to 0 \, !!!$$

(A) и (B) :

Petrov, Salomon and Shlyapniko v, 2004

Recombination rate against scattering length



Limiting cases at *T=0*

$$n^{-1/3} \sim k_F$$

$$1) \quad a > 0, ak_F << 1$$

$$\text{Superfluid dilute gas of dimers.}$$

$$\text{Bogoliubov theory.}$$

$$a_{dd} = 0.6a > 0$$

$$2) \, a < 0, |a|k_F << 1$$

$$\text{Superfluid dilute BCS-gas}$$

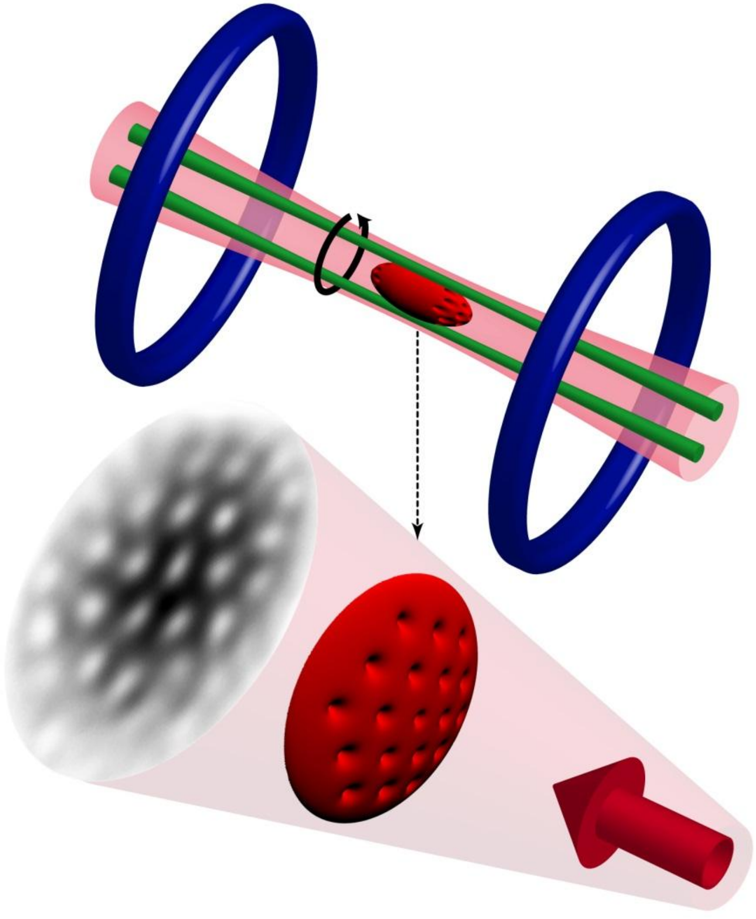
$$3) \, a = \infty - \text{ universal liquid}$$

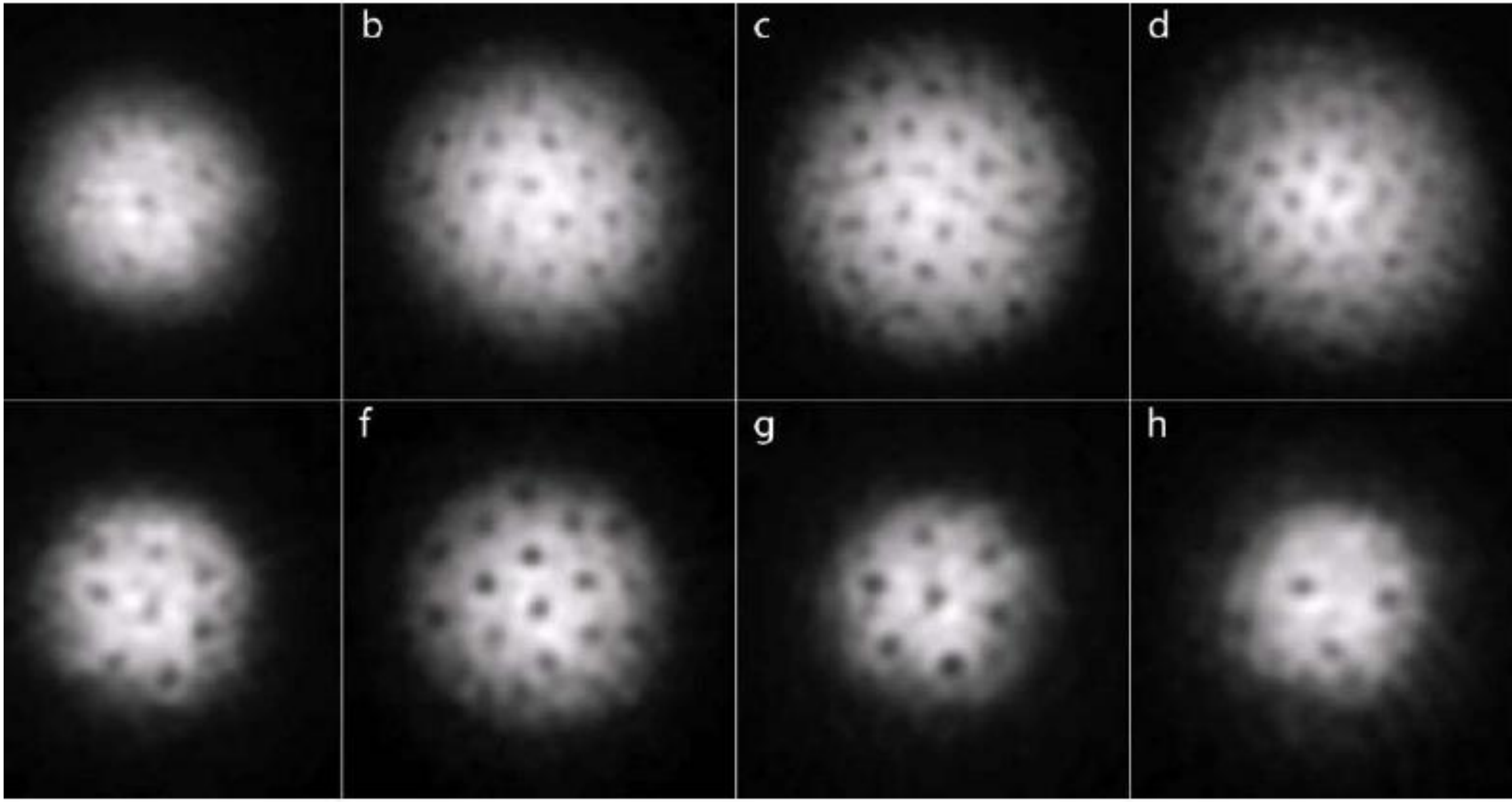
Vortexes in superfluid Fermi-liquid

$$\text{Velocity circulation } v = \frac{\hbar}{2m}\frac{1}{r}$$

$$\text{Density of vortexes } n_v = \frac{2m\Omega}{\pi\hbar}$$

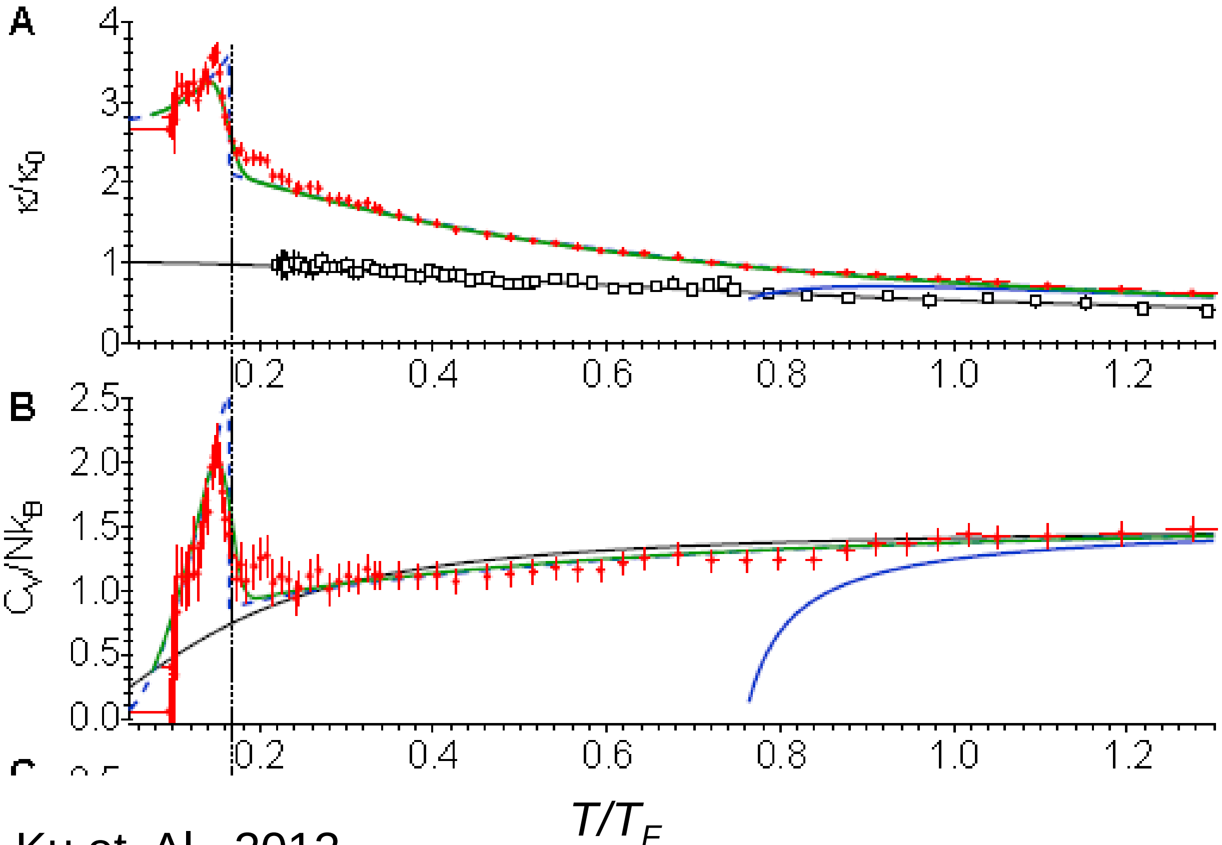
Experiment at MIT





. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic ansion, the magnetic field was ramped to 735 G for imaging (see text for details). The gnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G l (h) 863 G. The field of view of each image is $880\ \mu m \times 880\ \mu m$.

Transition point singularity in the unitary Fermi gas.



M. Ku et. Al., 2012