Quantum Metrology with Spatially Resolved Atom Detection

Technische Universität München

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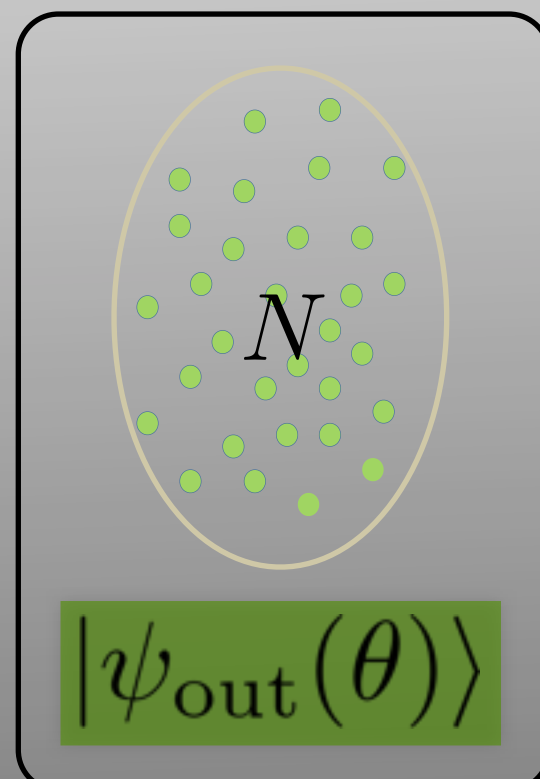
Outline

1. Metrology and Atom Detection
2. Measuring the average density - Sensitivity bounds
3. Application to Spatially Interfering BECs
    - Double-Well trap - Two Mode Interferometer
    - Optical Lattice - Multimode Interferometer
4. Summary & Outlook

Metrology with atom position measurements

Quantum Metrology :

Estimation of a physical parameter θ on which the (quantum)state depends



Sensitivity Bound

$$\Delta\theta = \frac{1}{\sqrt{mN}} \quad \frac{1}{\sqrt{N}}$$

Metrology with atom position measurements

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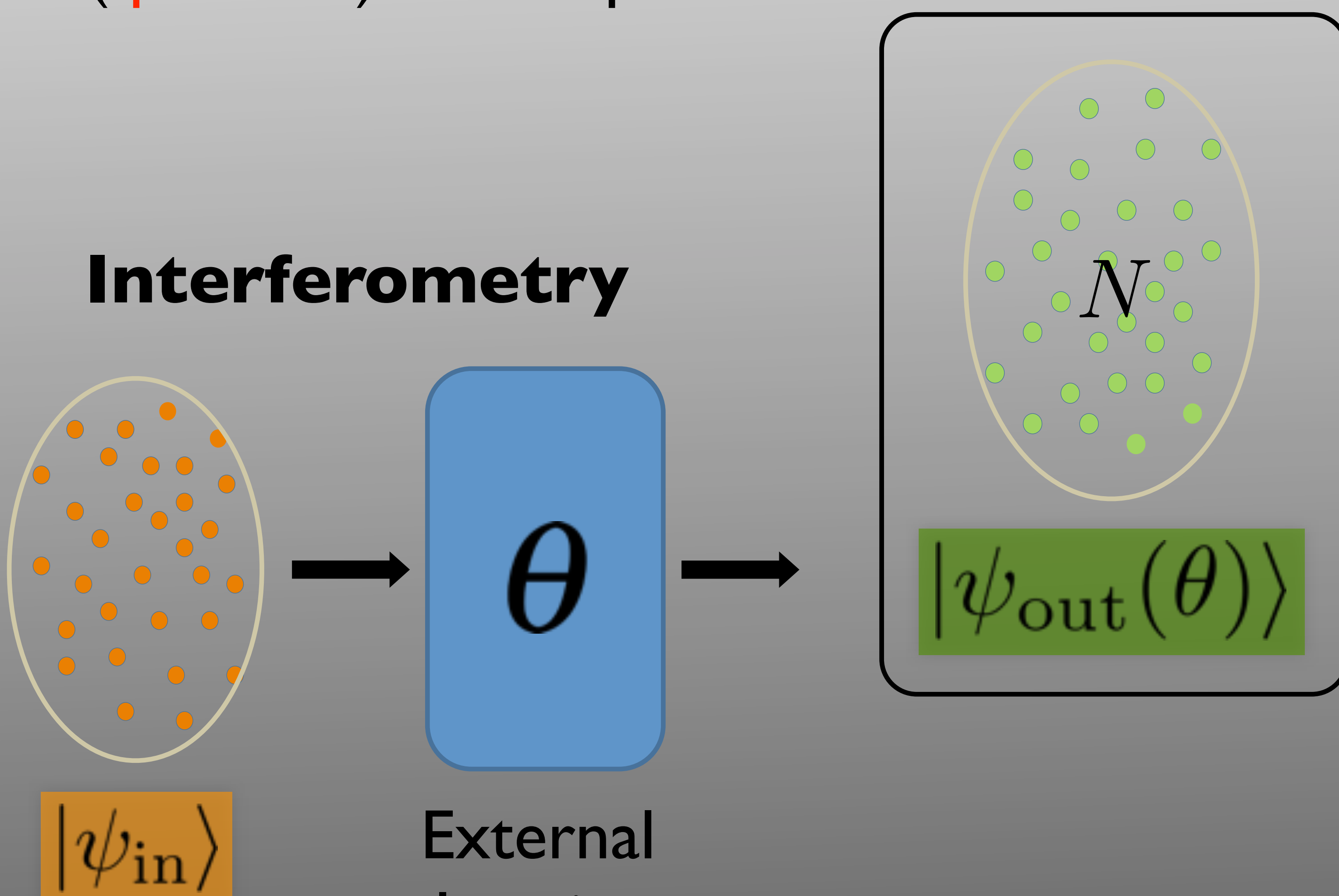
Interferometry



External Imprint

Metrology with atom position measurements

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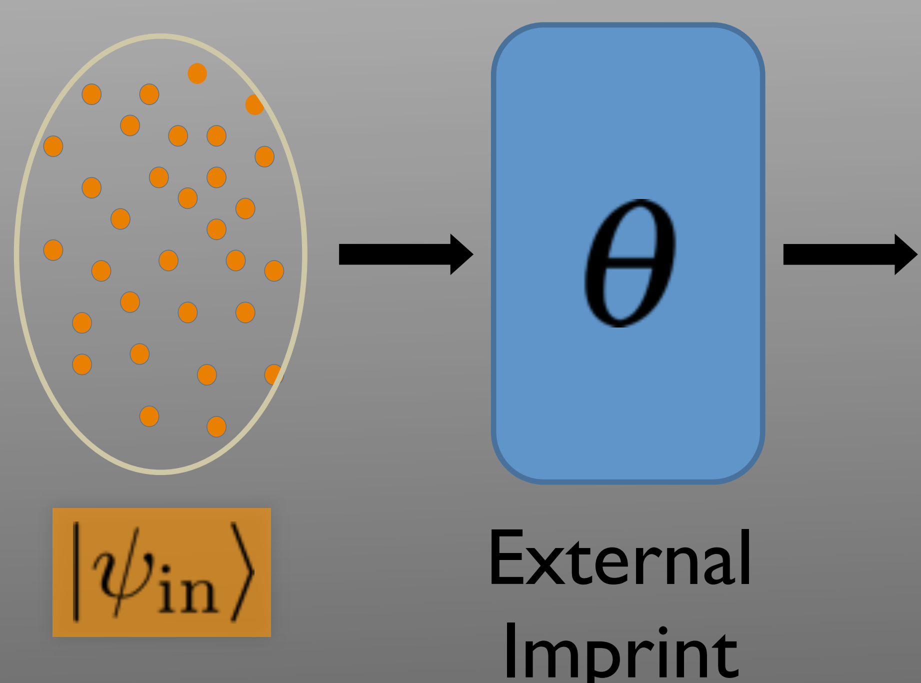
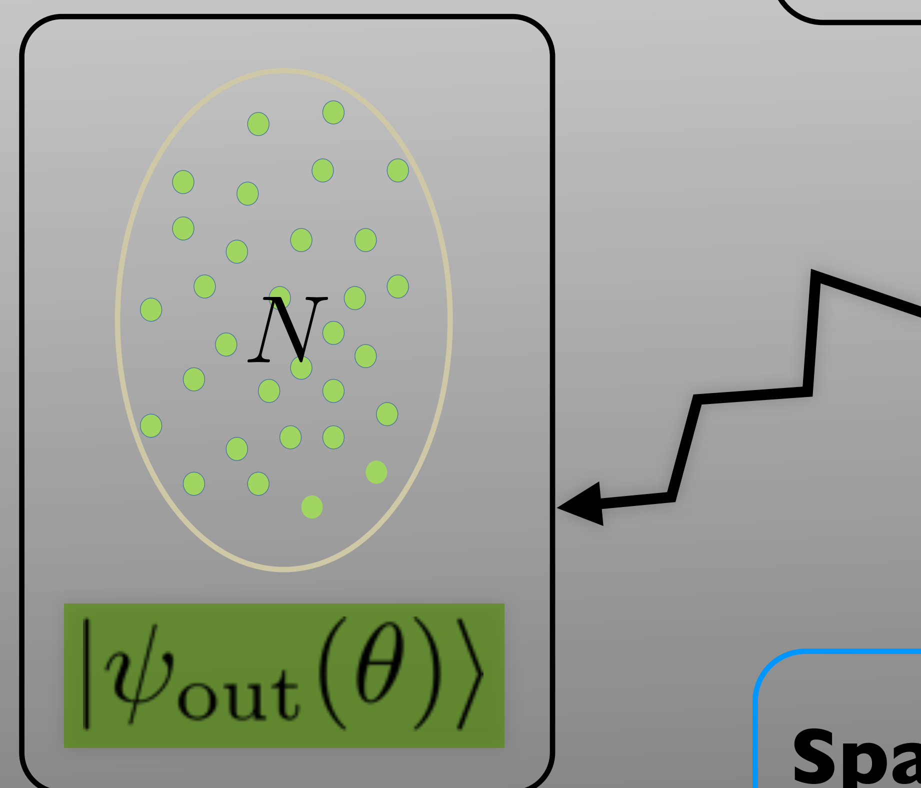
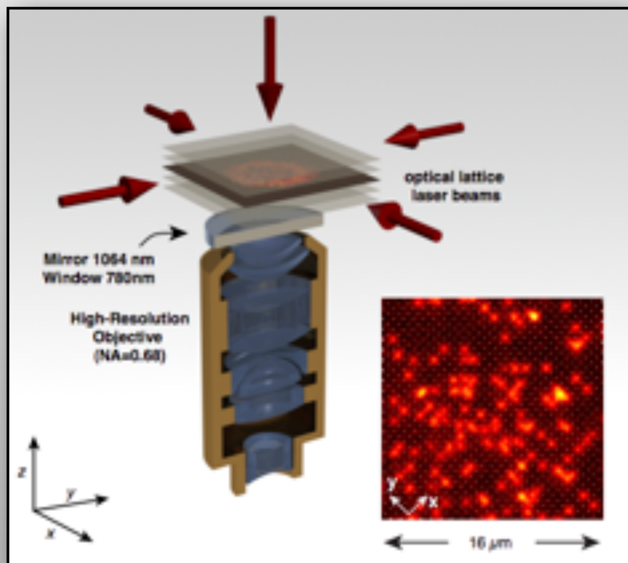




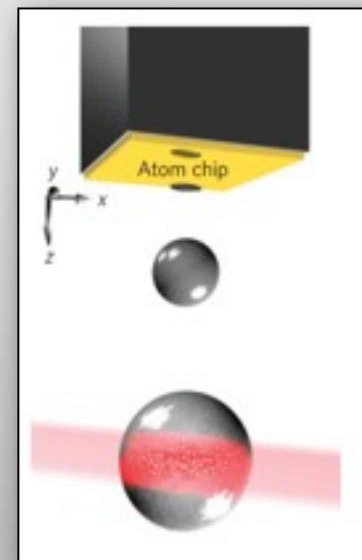
Measurement

Spatially Resolved Atom Detection

Atom detection techniques



J. F. Sherson, et al., Nature 467(2010)

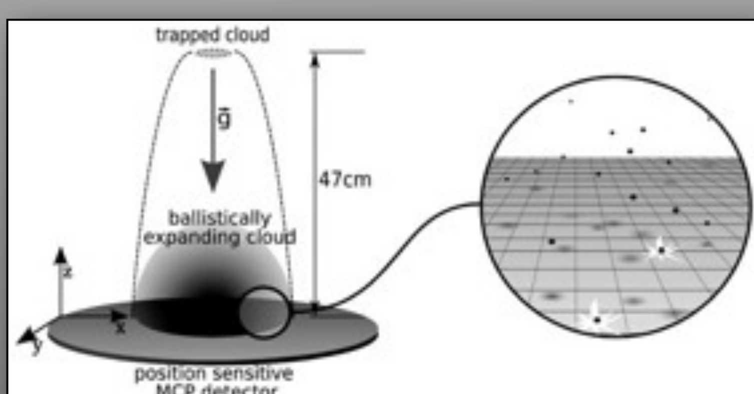


A. Perrin, et al., Nat. Phys. 8(2012)

Fluorescence

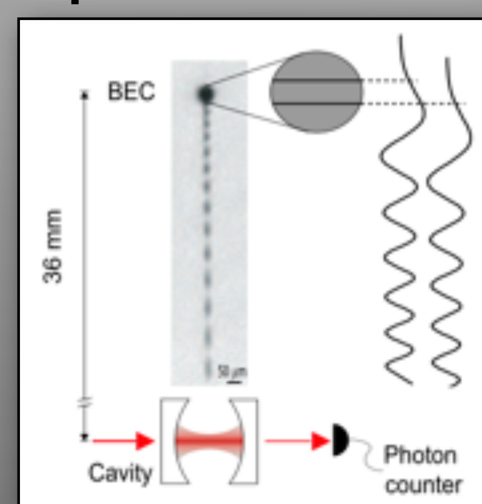
Schlosser, et al., Nature 411(2001)
S. Kuhr, et al., Science 293(2001)
K. D. Nelson, et al., Nat. Phys. 3(2007)
T. Bondo, et al., Optics Comm. 264(2006)
W.S Bakr, et al., Nature 462(2009)
R. Buecker, at al., NJP 11(2009)
D. Heine, et al., NJP 12(2010)

Microchannel Plate



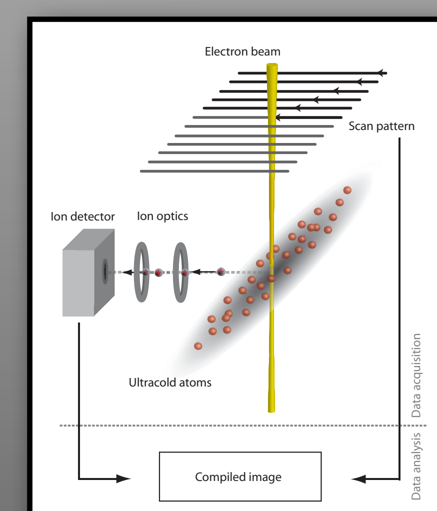
M. Schellenkens, et al., Science 310(2005)

Optical Cavities



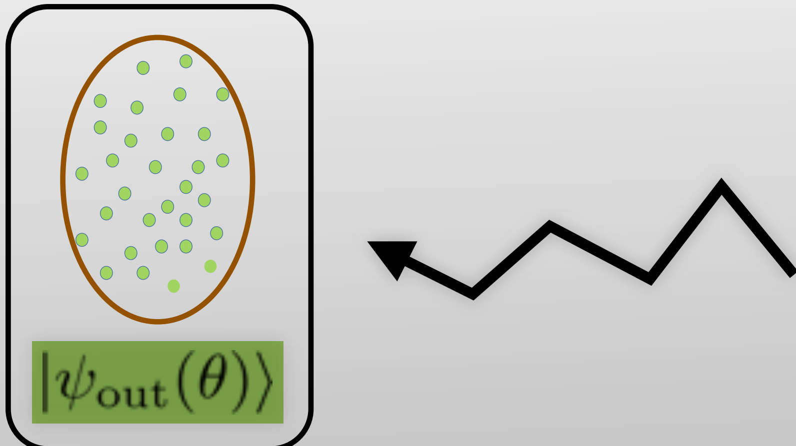
A. Oettl, et al., Phys. Rev. Lett. 95(2005)

Electron Microscopy



T. Gericke, et al., Nat. Phys. 4(2008)

Estimation from the average density



Measurement

$$\rho(x|\theta) = \langle \psi_{\mathrm{out}}(\theta)|\hat{\Psi}^{\dagger}(x)\hat{\Psi}(x)|\psi_{\mathrm{out}}(\theta)\rangle$$

Estimation Protocol

i) Imprint the **true value** θ

ii) Measure the density **averaged over m** repetitions

iii) Infer by a **least-squares fit**

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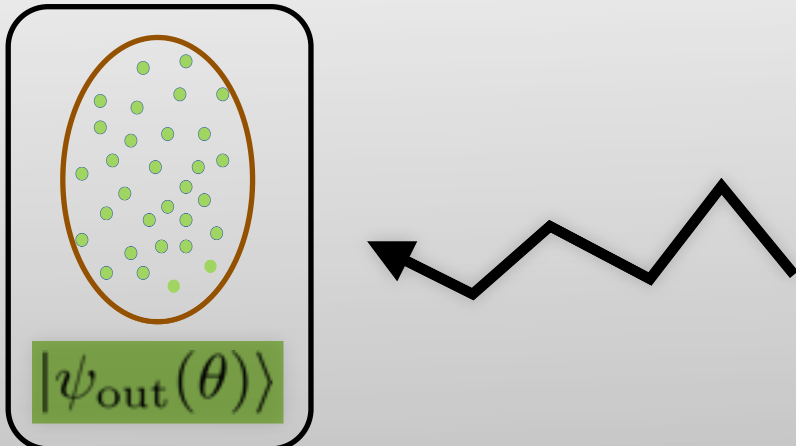

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$$\boxed{\Delta^2\theta_{\rm ML} = \frac{1}{m}\left(\frac{1}{F_1} + \frac{C}{F_1^2}\right)}$$

Estimation sensitivity

$$\boxed{\Delta^2\theta_{\mathrm{ML}} = \frac{1}{m}\left(\frac{1}{F_1} + \frac{C}{F_1^2}\right)}$$

$$F_1 = \int dx \, \frac{1}{\rho(x|\theta)}\left(\frac{\partial\rho(x|\theta)}{\partial\theta}\right)^2$$

Fisher Information - single particle

$$\boxed{C = \int dxdy \; g_2(x,y|\theta) \; \partial_\theta\rho(x|\theta)\partial_\theta\rho(y|\theta)}$$

$$g_2(x,y|\theta) = \frac{\langle\hat{\psi}^\dagger(x)\hat{\psi}^\dagger(y)\hat{\psi}(y)\hat{\psi}(x)\rangle}{\langle\hat{\psi}^\dagger(x)\hat{\psi}(x)\rangle\langle\hat{\psi}^\dagger(y)\hat{\psi}(y)\rangle}$$

Two-particle correlation

J. Chwedenczuk, P. Hyllus, FP, A. Smerzi, arXiv :1108.2785

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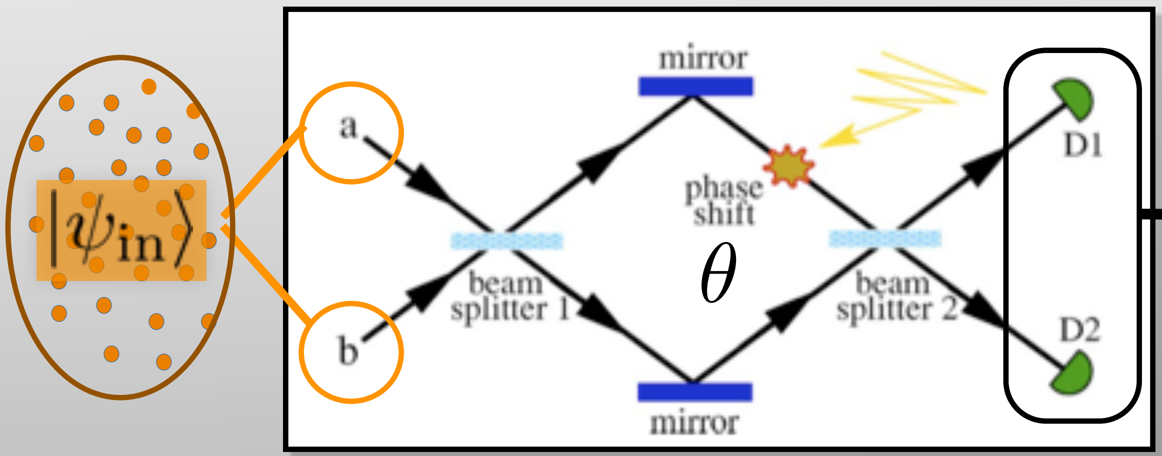
Two-particle correlation

Quite **general** result :

- two and many modes
- bosons and fermions
- T=0 and T>0
- any interferometer

J. Chwedenczuk, P. Hyllus, FP, A. Smerzi, arXiv :1108.2785

Two-mode interferometry - Mach-Zehnder



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$$\langle \hat{n}_1 - \hat{n}_2 \rangle$$

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$$\boxed{\xi_n^2 = N \frac{\Delta^2 \hat{J}_z}{\langle \hat{J}_x \rangle^2}}$$

Heisenberg limit
for spin-squeezed states

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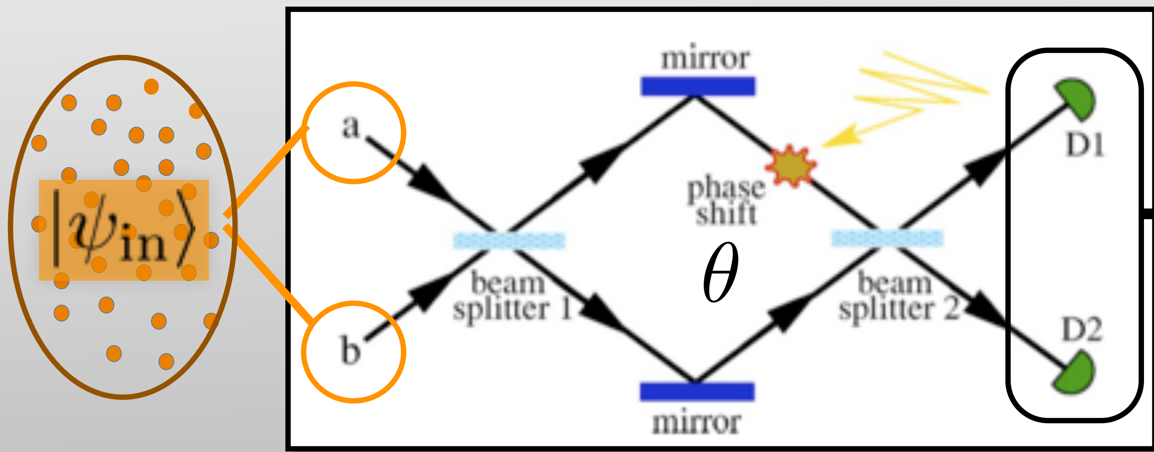

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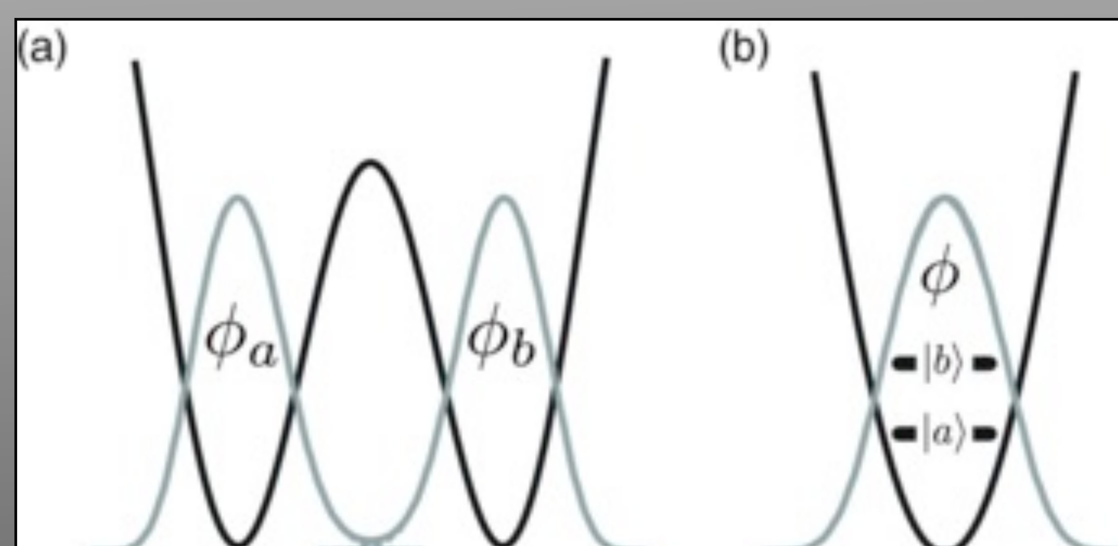
Implementation with **Ultracold Bosons**



C. Gross, J. Phys. B 45(2012)

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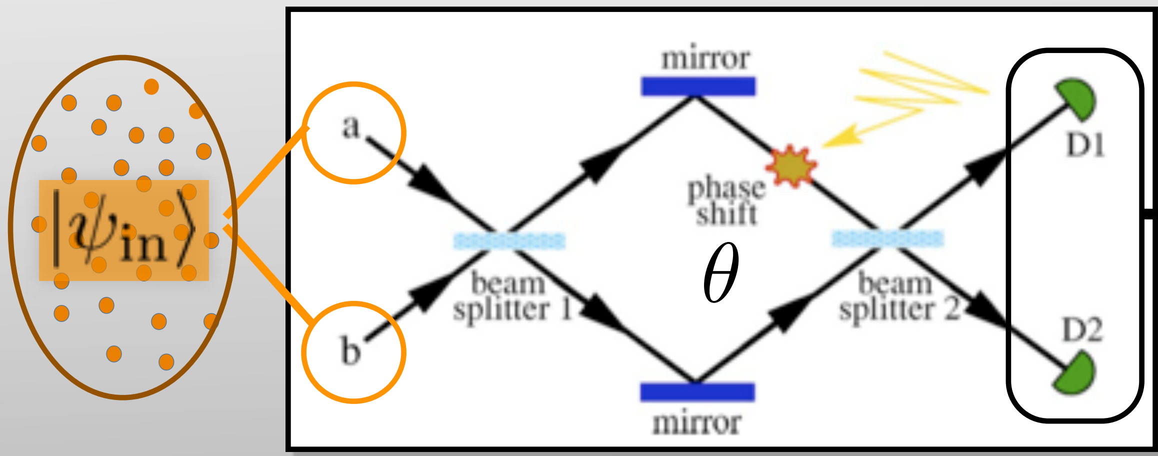

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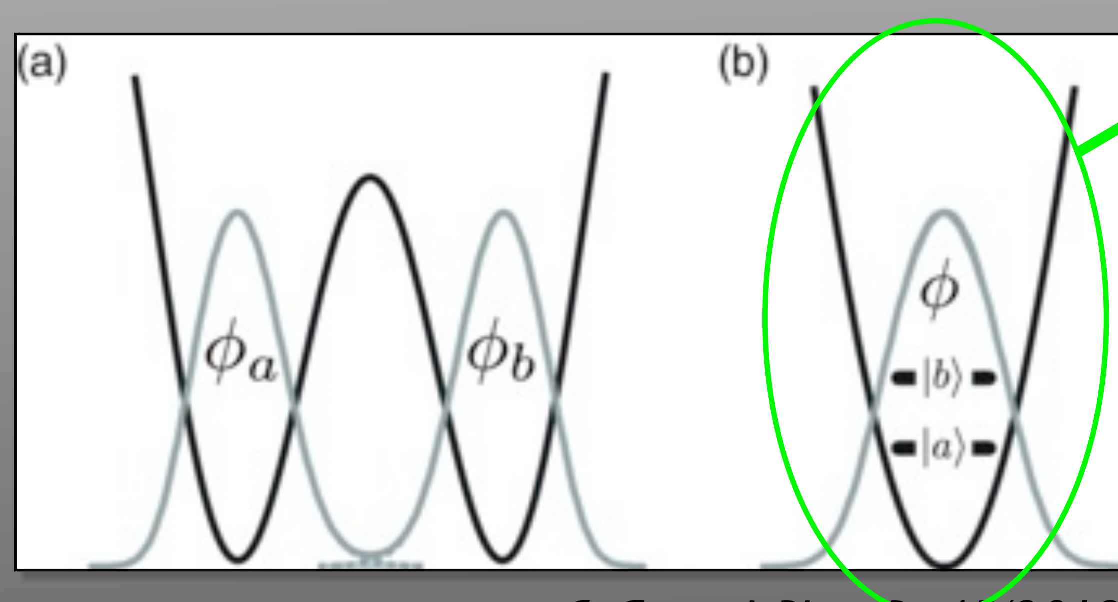
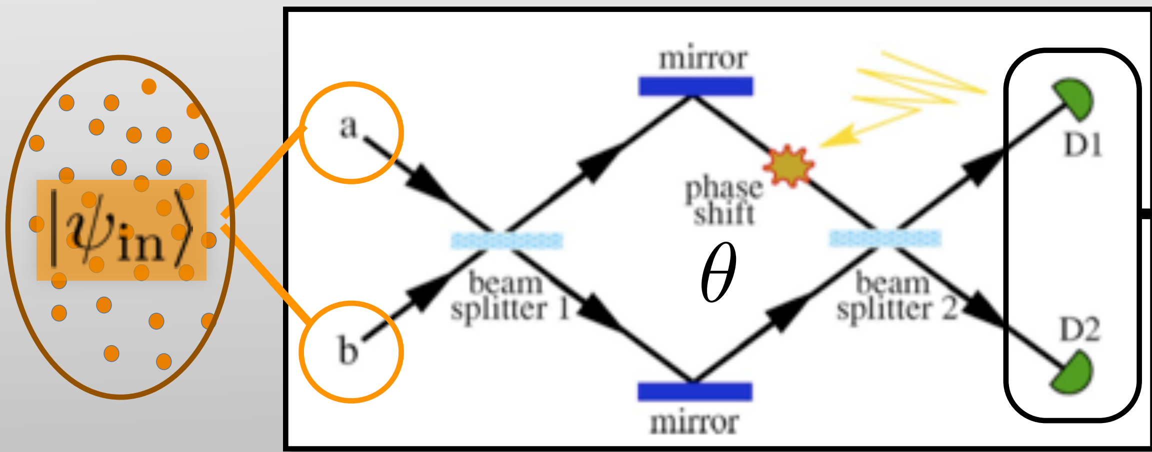


C. Gross, J. Phys. B 45(2012)

Experiments
Ramsey-spectroscopy

Two-mode interferometry - Mach-Zehnder



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Heisenberg limit
for spin-squeezed states

Implementation with **Ultracold Bosons**

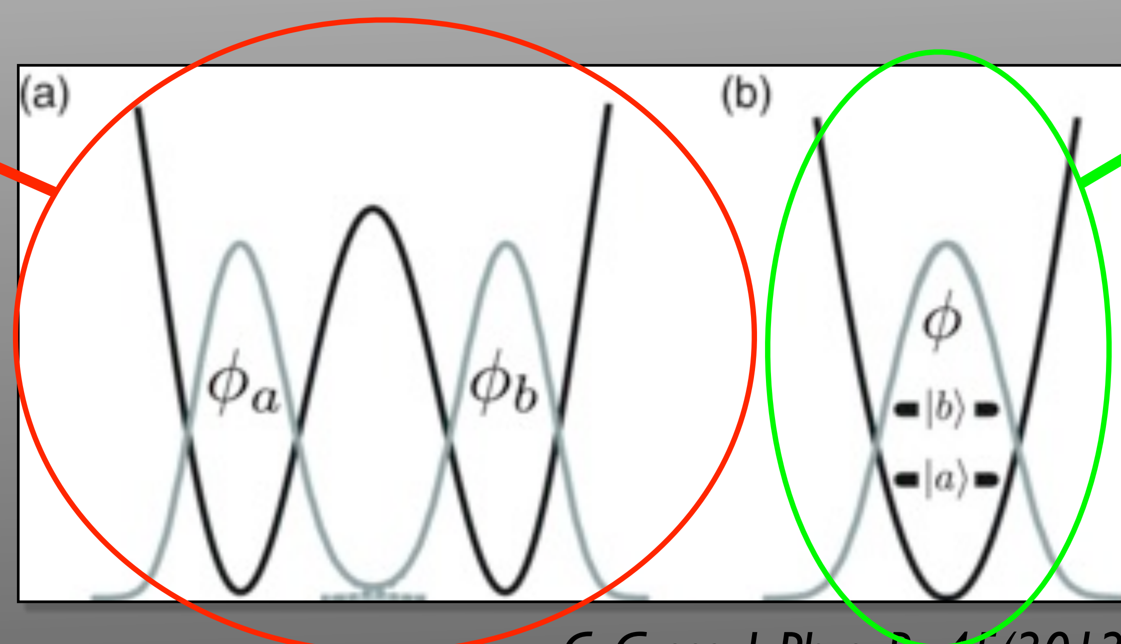
No MZI yet

Double-well

J. Esteve, et al., Nature 554(2008)

Twin-atom beams

R. Buecker, et al., Nat. Phys. 7(2011)



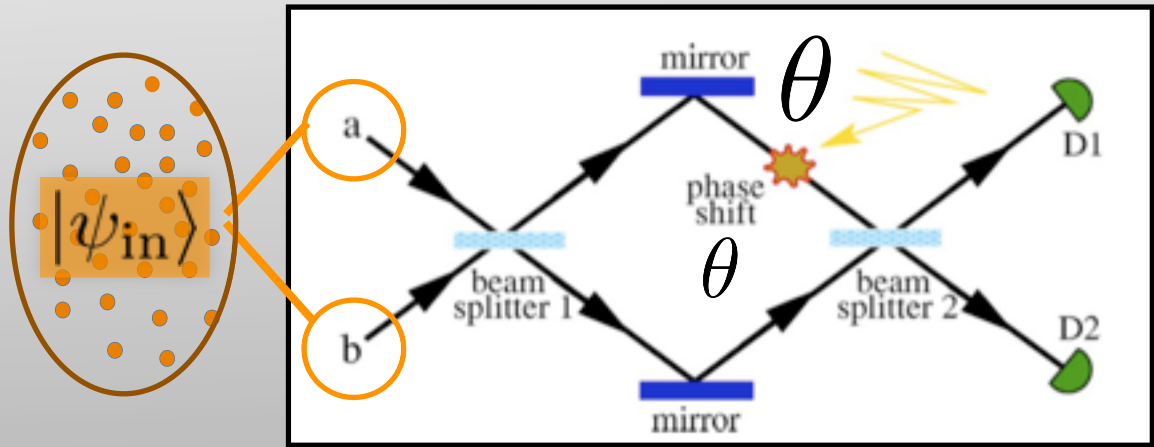
C. Gross, J. Phys. B 45(2012)

Experiments

Ramsey-spectroscopy

Tuesday, June 12, 12

Two-mode interferometry - Interference pattern



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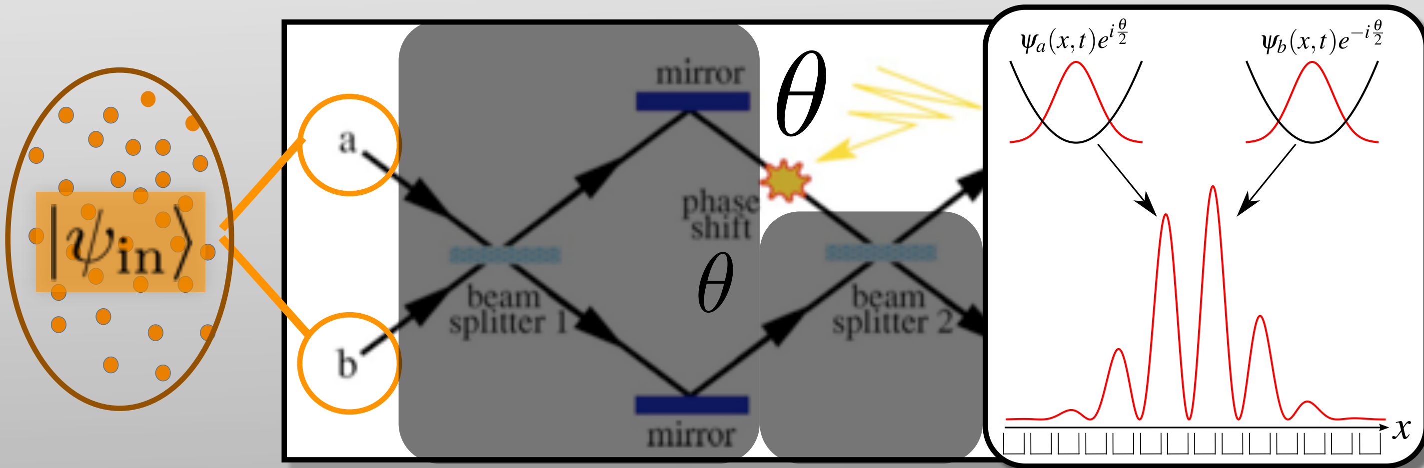

Phase Shift + Expansion

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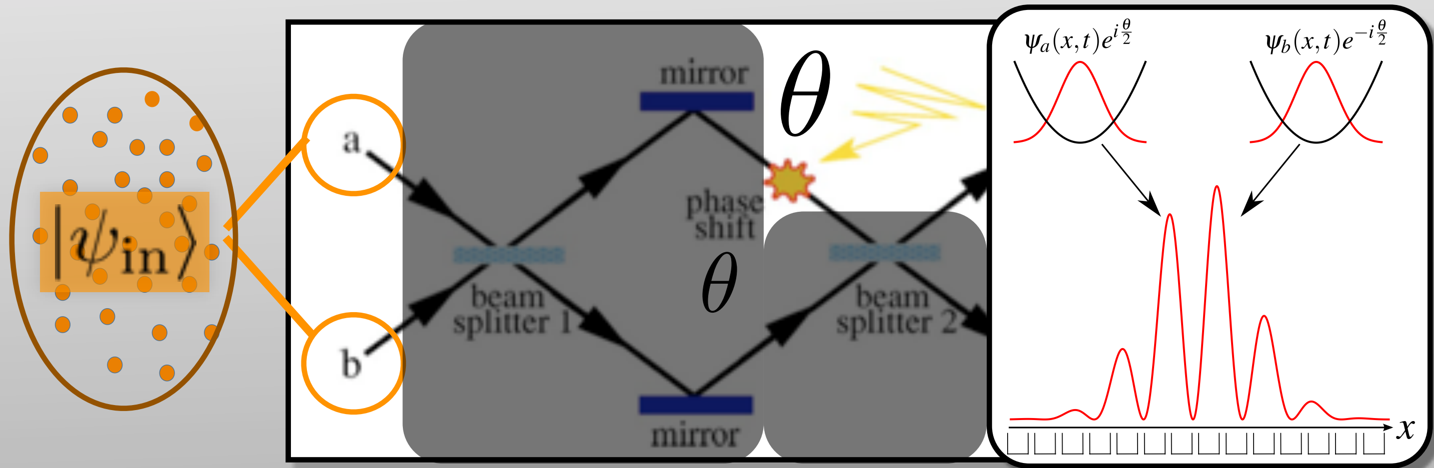

Phase Shift + Expansion

$$\Delta^2\theta_{\text{ML}} = \frac{1}{m}\frac{1}{N}\left(\xi_\phi^2 + \frac{\sqrt{1-\nu^2}}{\nu^2}\right)$$

Phase squeezing

$$\xi_\phi^2 = N\frac{\Delta^2 \hat{J}_y}{\langle \hat{J}_x\rangle^2}$$

Contrast

$$\nu = \frac{2}{N}\langle \hat{J}_x\rangle$$

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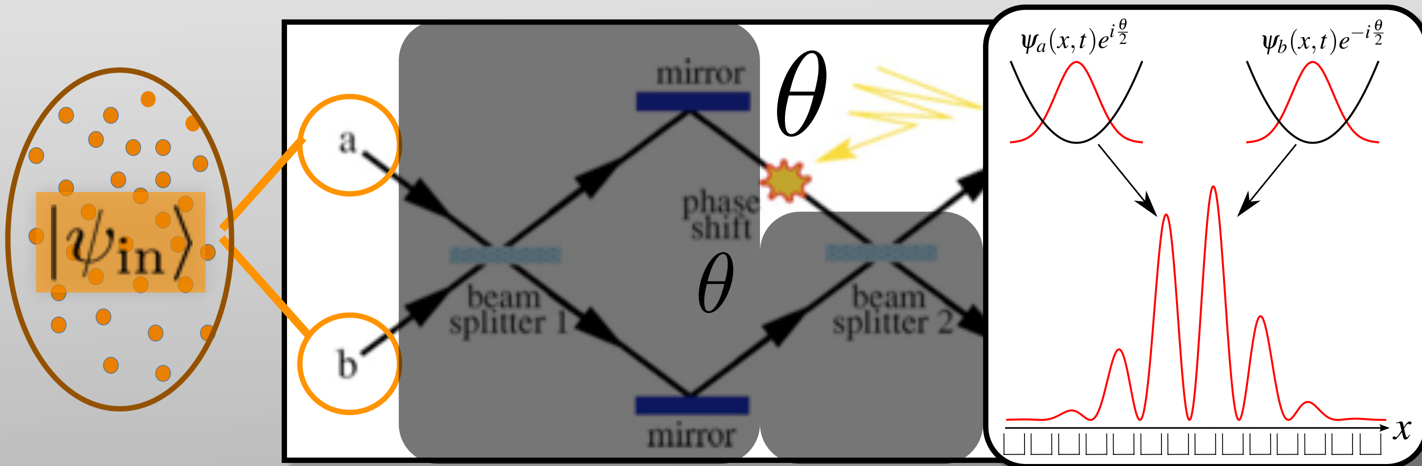

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Realistic phase squeezing

J. Grond, et al., NJP 12(2010)

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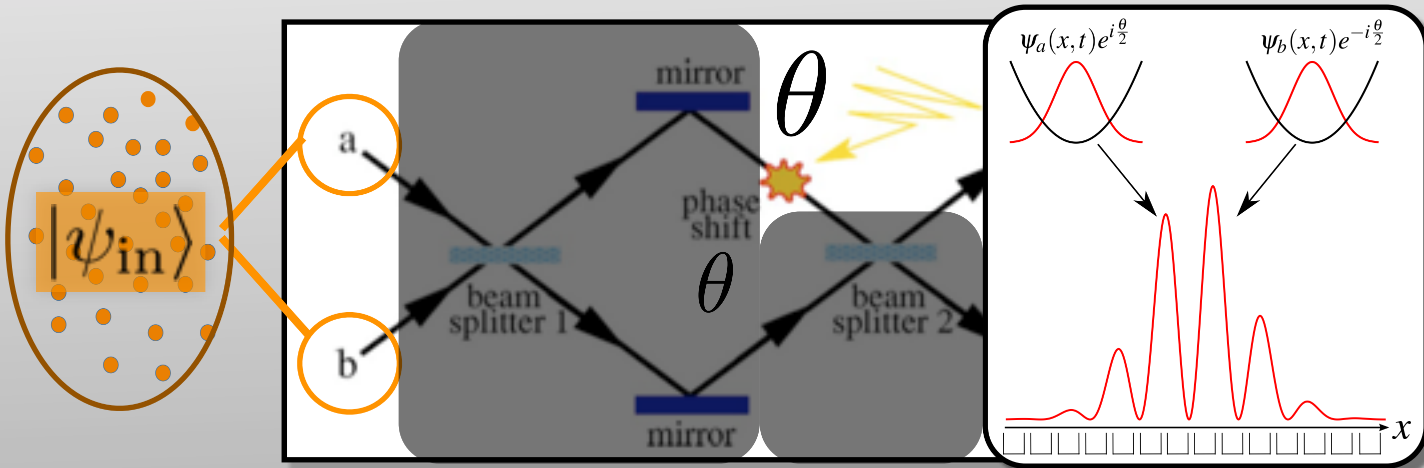

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Include **technical noise**

Least-squares **fit** to **fluorescence** density

5 bins / fringe

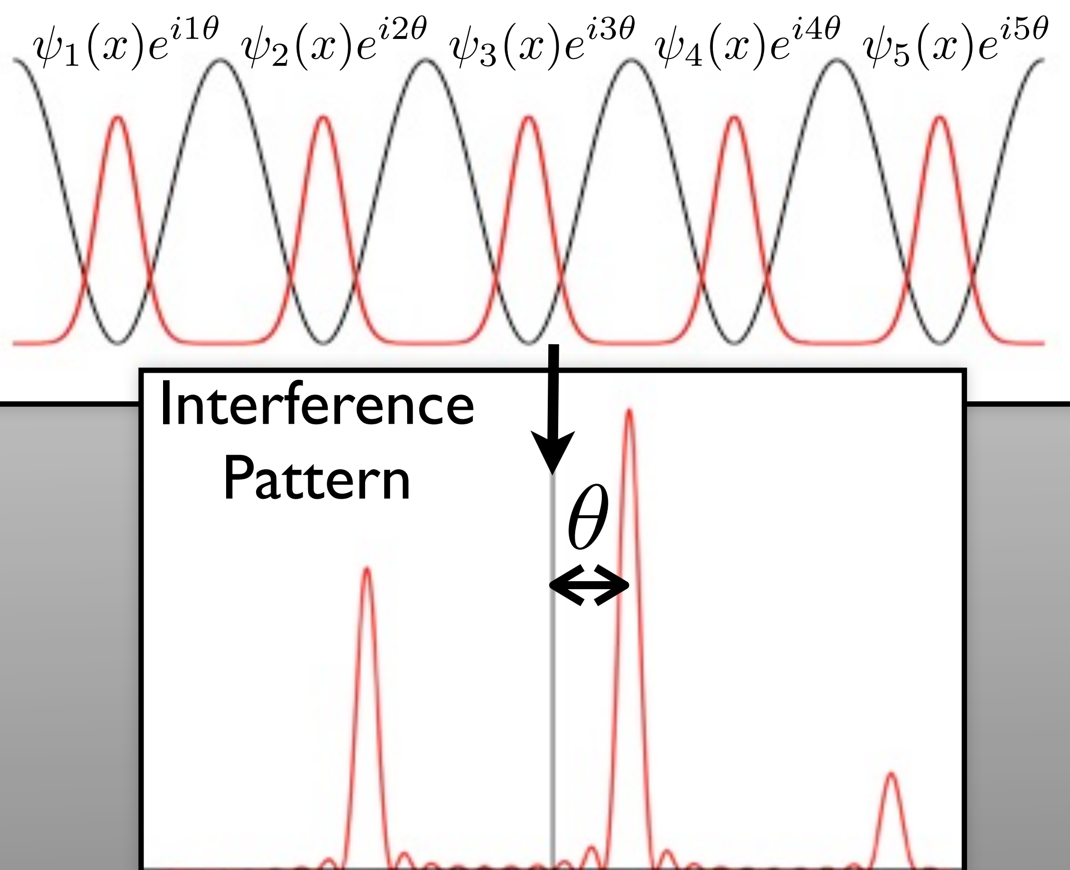
sub-shot noise preserved for

10 photons/atom

Multimode interferometry - BEC inside a lattice

Imprint :
Linear Bias

$$\hat{U}(\theta) = \exp\left(\theta \sum_{k=1}^{M} k \; \hat{a}_k^\dagger \hat{a}_k\right)$$



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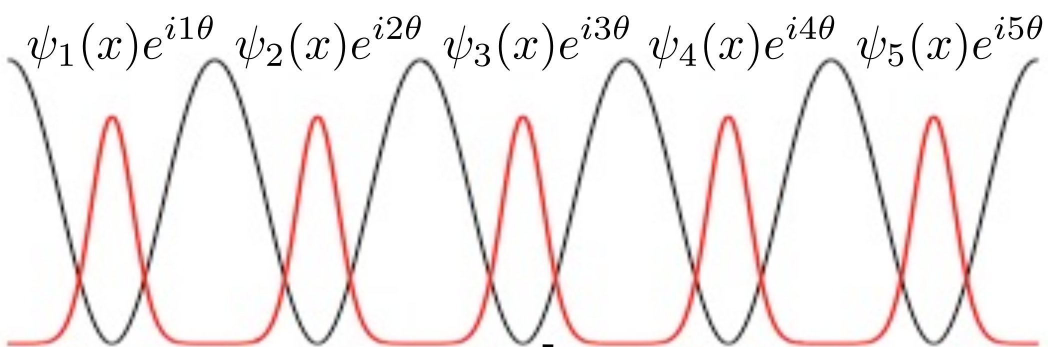

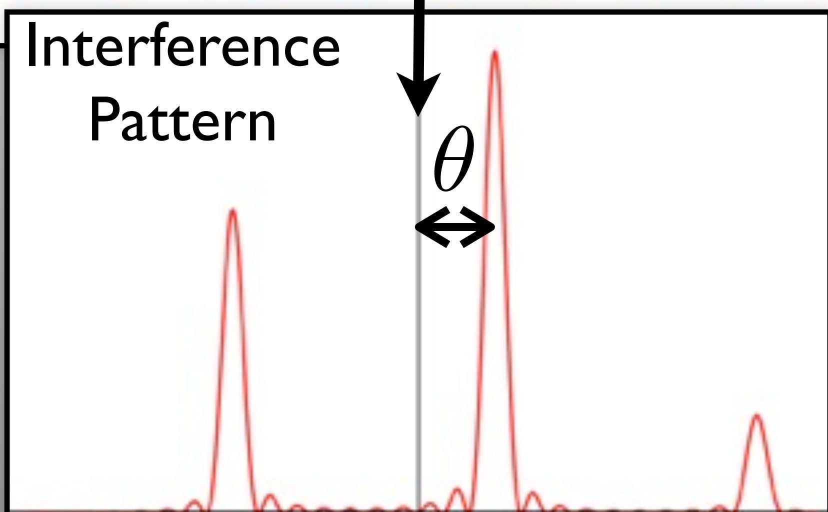

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Superfluid State

$$\Delta^2\theta_{\mathrm{ML}}^{(\mathrm{SF})} = \frac{1}{mN}\frac{3}{(M^2-1)}$$

"Heisenberg" sc. in number of sites

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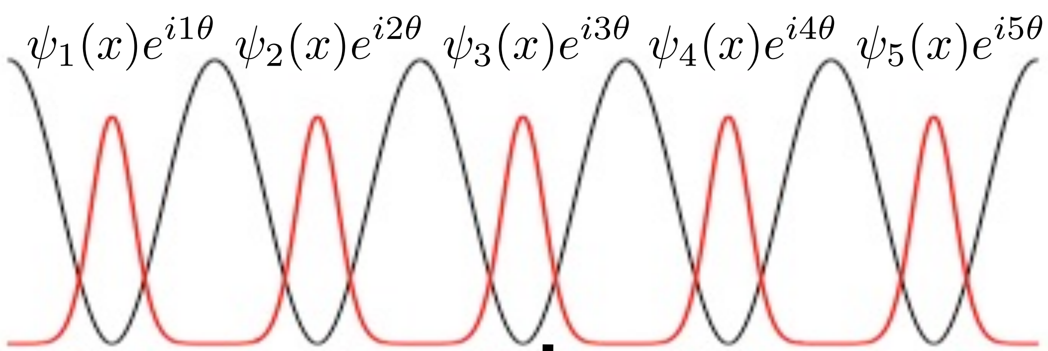

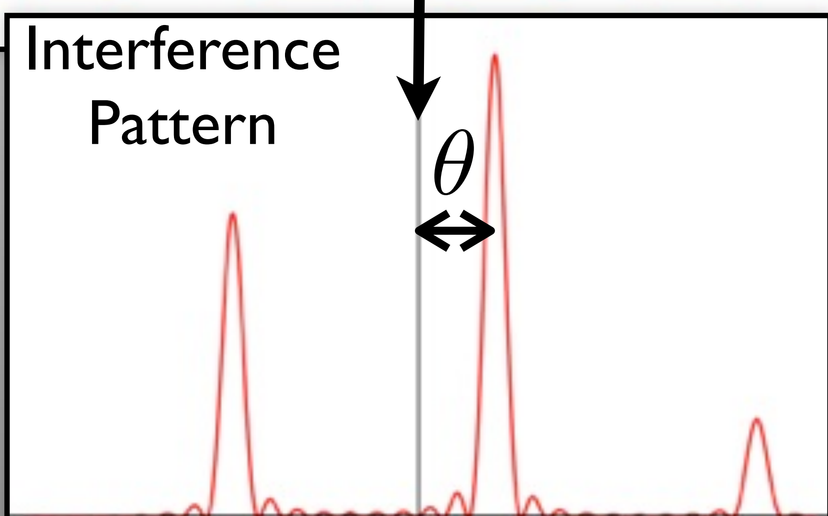

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Improves for Phase Squeezed

Creation of number-squeezing *C. Gross, et al., PRA 84 (2011)*

Humboldt
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Tuesday, June 12, 12

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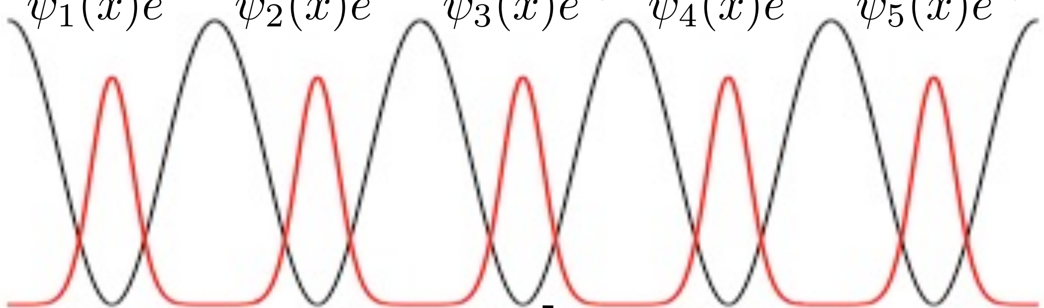
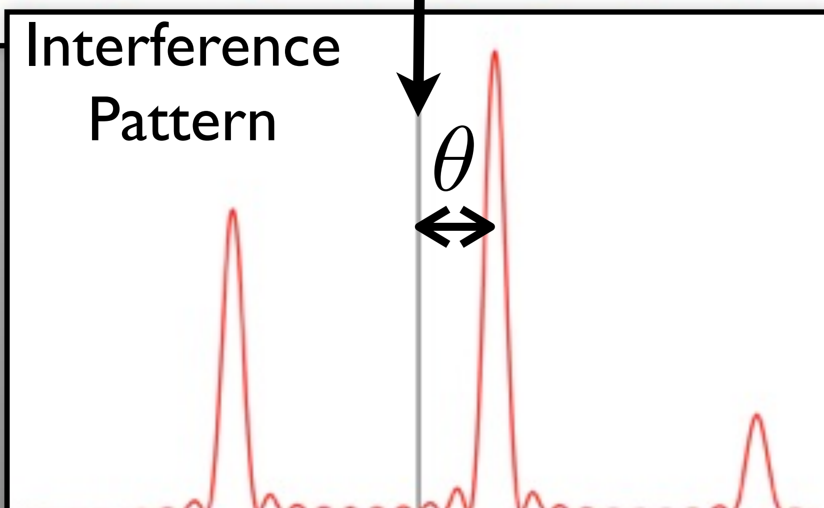
$\psi_1(x)e^{i1\theta}$ $\psi_2(x)e^{i2\theta}$ $\psi_3(x)e^{i3\theta}$ $\psi_4(x)e^{i4\theta}$ $\psi_5(x)e^{i5\theta}$





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Particular **SU(M)** interferometer

Implementation of Beam Splitter, is not obvious !
A Vourdas, et al., PRA 71(2005)

Interference Pattern does it "automatically" !

Summary & Outlook

1. General expression for the sensitivity of parameter estimation from one-body density
    - Two/Multi-mode, any interferometer, bosons/fermions
2. Measurement on interfering BEC
    - SSN sensitivity in double-wells fitting the interference density
    - "Heisenberg" scaling with number of lattice sites

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Next...

1. SU(M) interferometers: general sensitivity bounds
   - Two/Multi-mode, any interferometer, bosons/fermions
   - Useful entangled states in optical lattice (see *C. Gross, et al. PRA 84(2011)*)
2. Temperature estimation (Poster *J. Chwedenczuk*)
3. Look for useful correlation in other systems (low dimensions, fermions,...)