

Theory of Quantum Gases and Quantum Coherence

Atom interferometry & quantum metrology

Kirchhoff Institut für Physik
University Heidelberg

The people



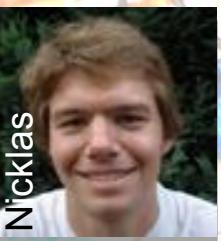
Steve



Groß



Zibold



Nicklas



Müssel



Strobel

Introduction

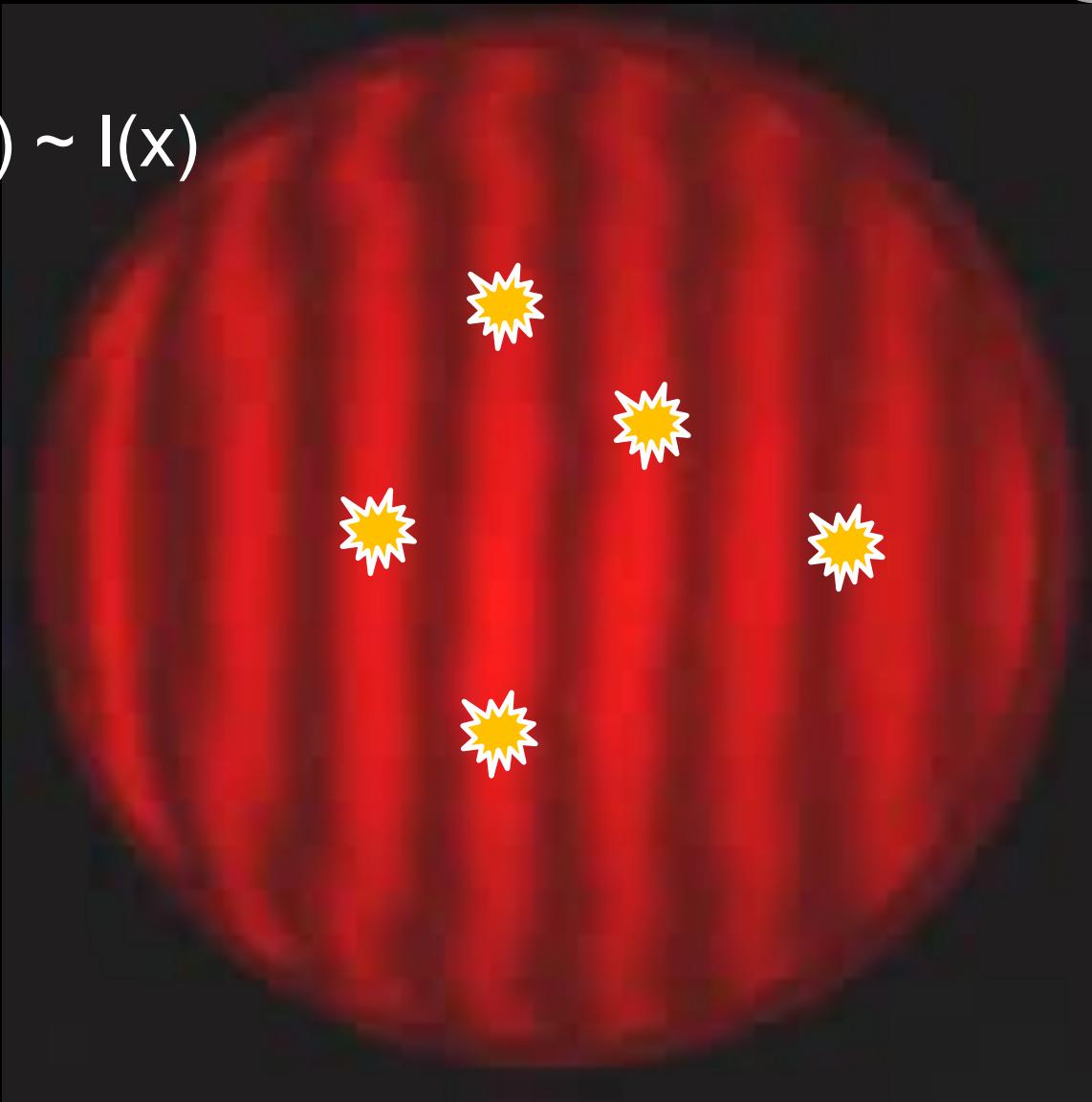
External degrees of freedom
non-classical many particle states

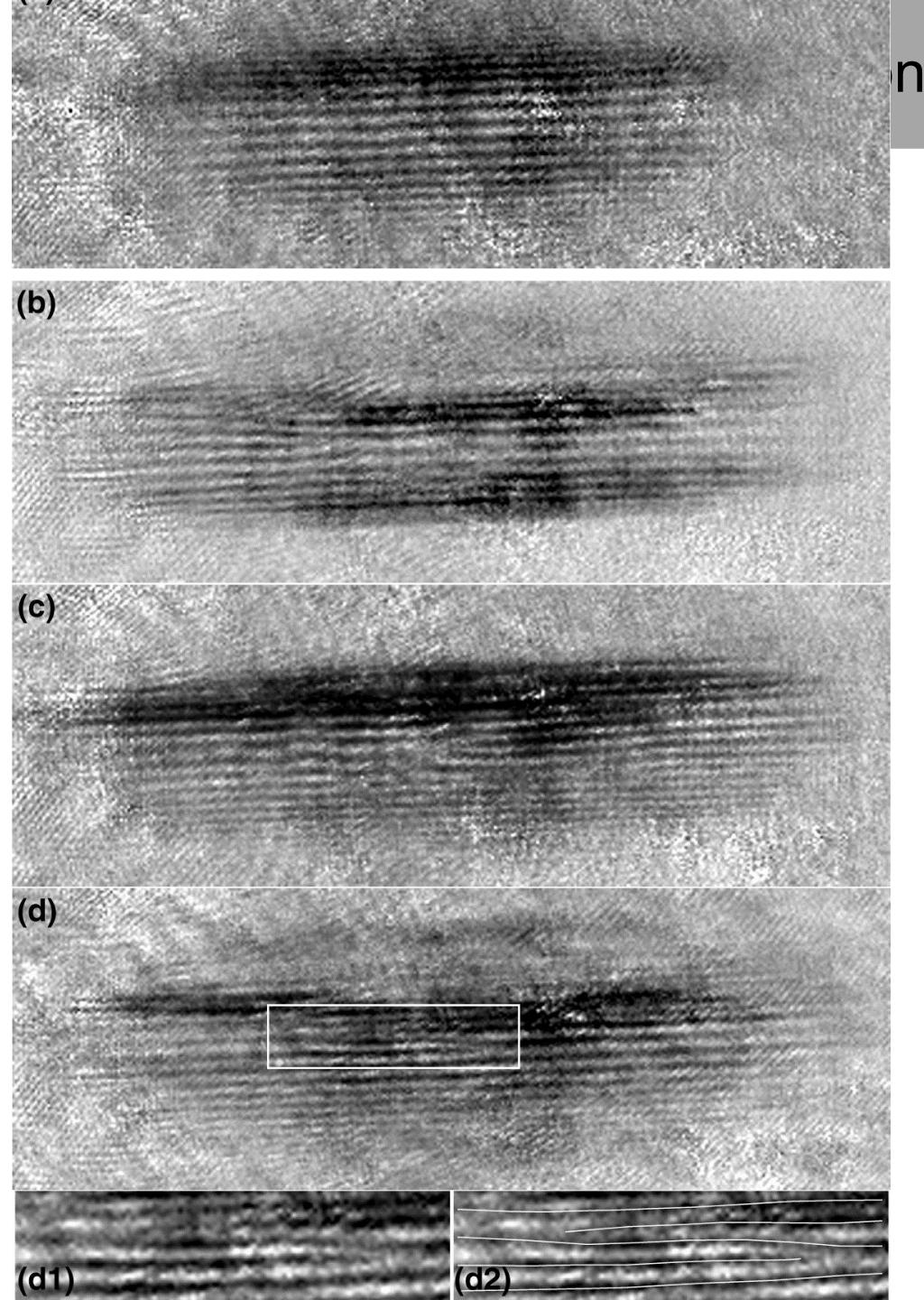
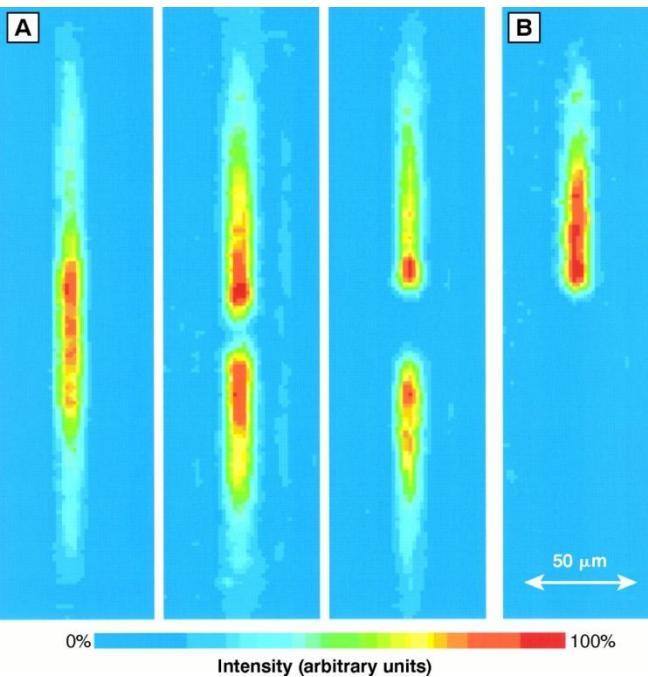
Internal degrees of freedom
non-linear matterwave interferometry

Generation of twin-atom fields
squeezing with spin changing collisions

introduction

$$p(x) \sim I(x)$$

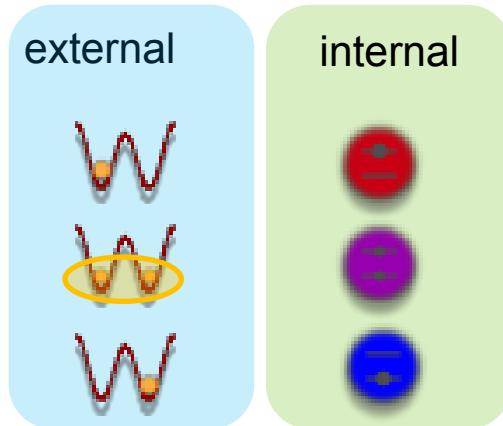
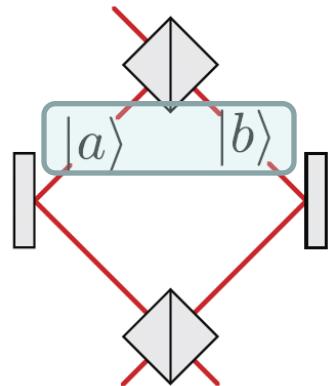




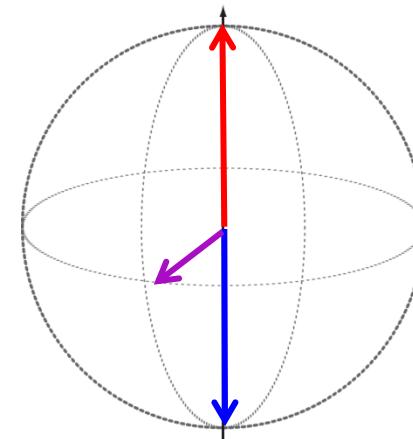
single versus many particles

introduction

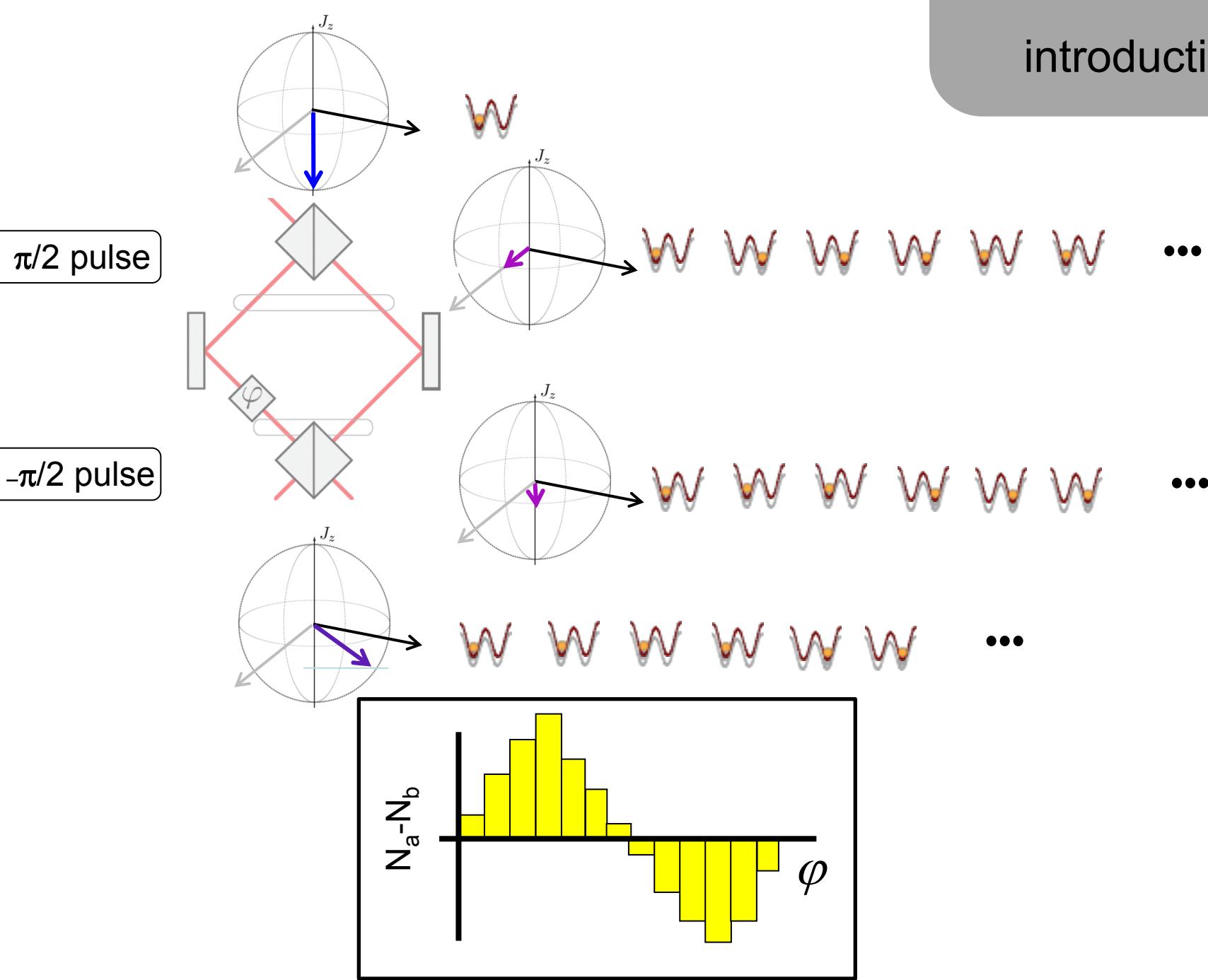
single particle interferometer



$$\begin{aligned} &|a\rangle \\ &|a\rangle + e^{i\varphi}|b\rangle \\ &|b\rangle \end{aligned}$$



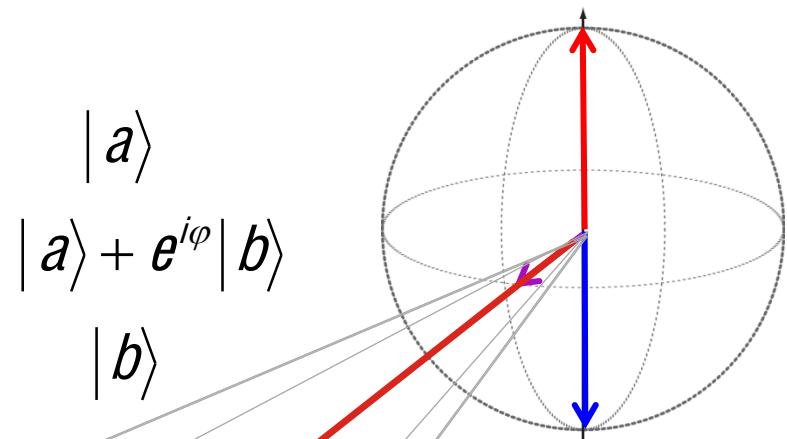
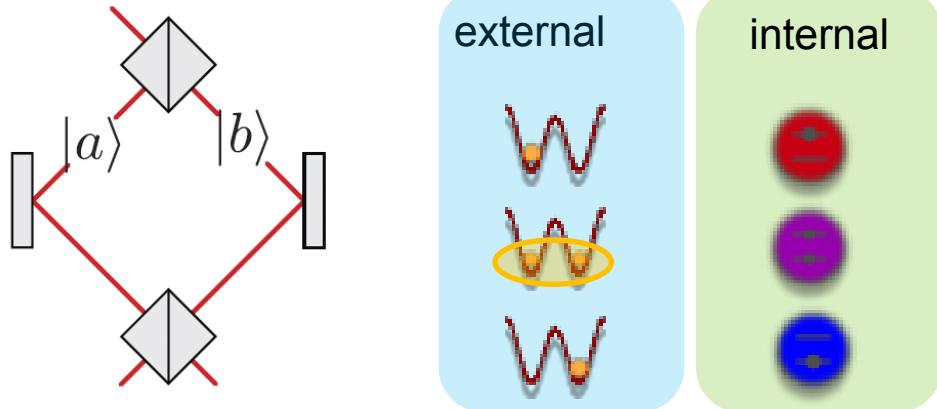
introduction



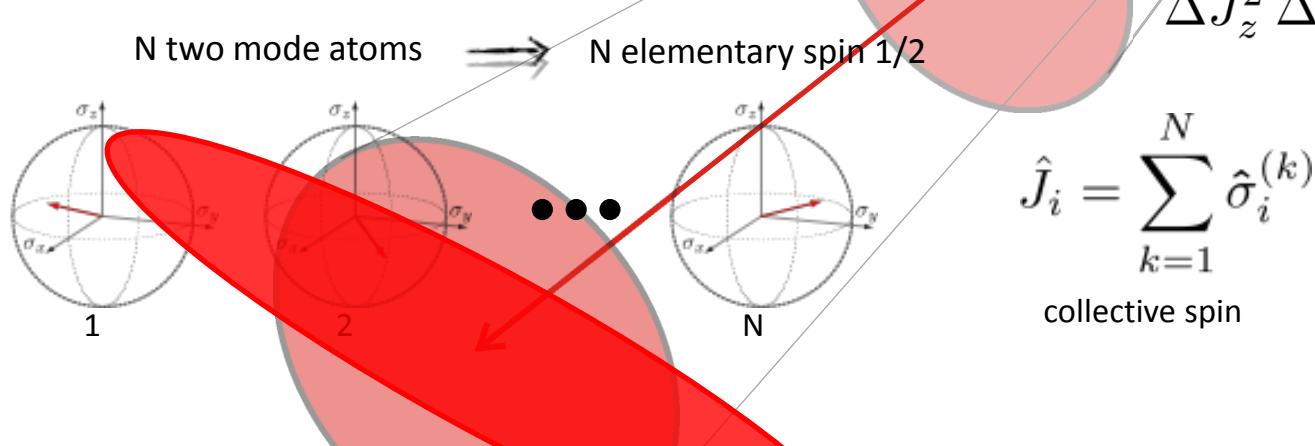
single versus many particles

introduction

single particle interferometer



many particle interferometer



$$\Delta J_z^2 \Delta J_y^2 \geq \langle J_x \rangle^2 / 4$$

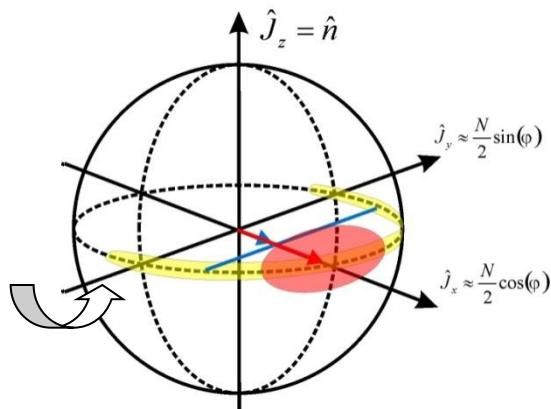
$$\hat{J}_i = \sum_{k=1}^N \hat{\sigma}_i^{(k)}$$

collective spin

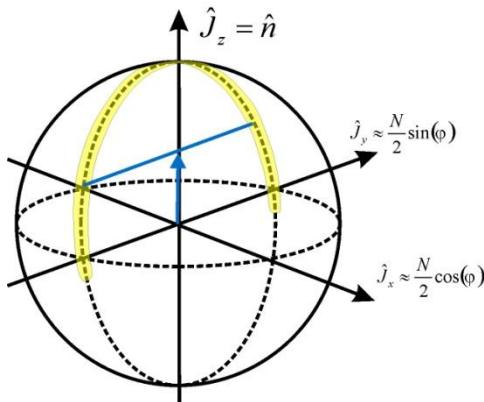
optimal squeezing

introduction

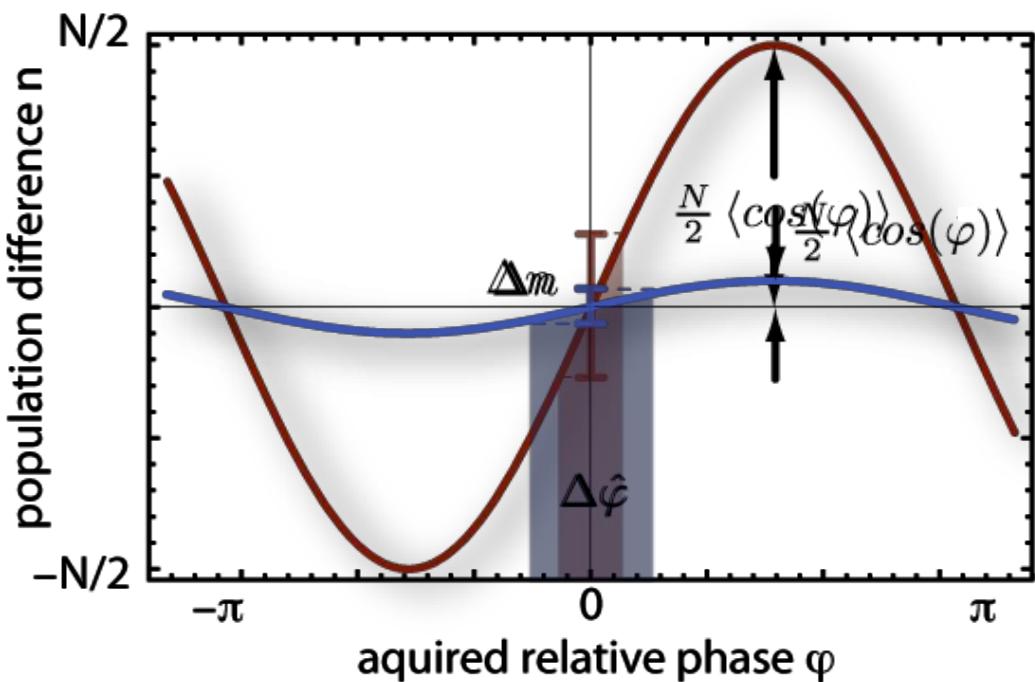
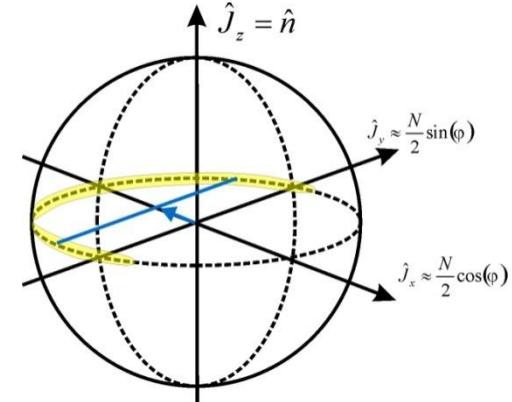
initial state



final state: $\phi=\pi/2$



final state: $\phi=\pi$



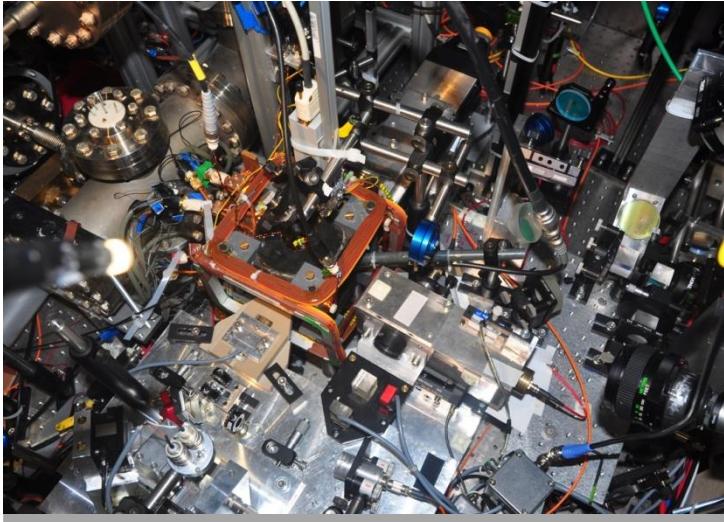
$$\xi_s^2 = \frac{4\Delta n^2}{N \langle \cos \varphi \rangle^2}$$

$$\xi_s^2 = \frac{N \text{var}(\hat{J}_z)}{\langle \hat{J}_x \rangle^2}$$

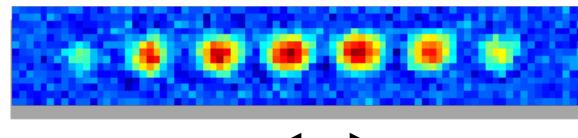
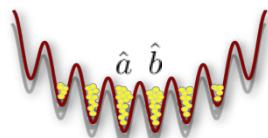
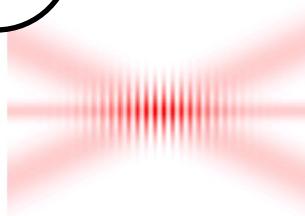
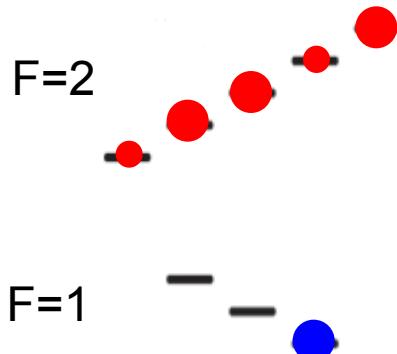
theory: Kitagawa & Ueda, PRA 47, 5138 (1993)
Wineland et al., PRA 50, 67 (1994)

experimental setup

introduction



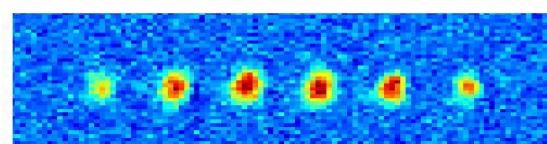
Rubidium BEC



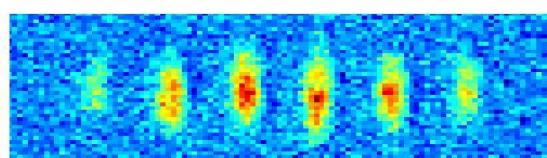
lattice period: $5.7 \mu m$

external

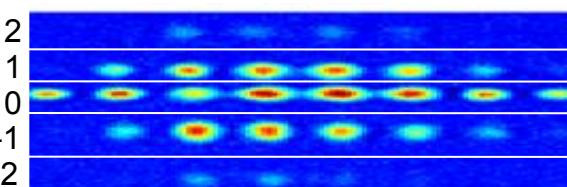
F=2



F=1

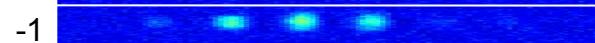


m_F



F=2

F=1

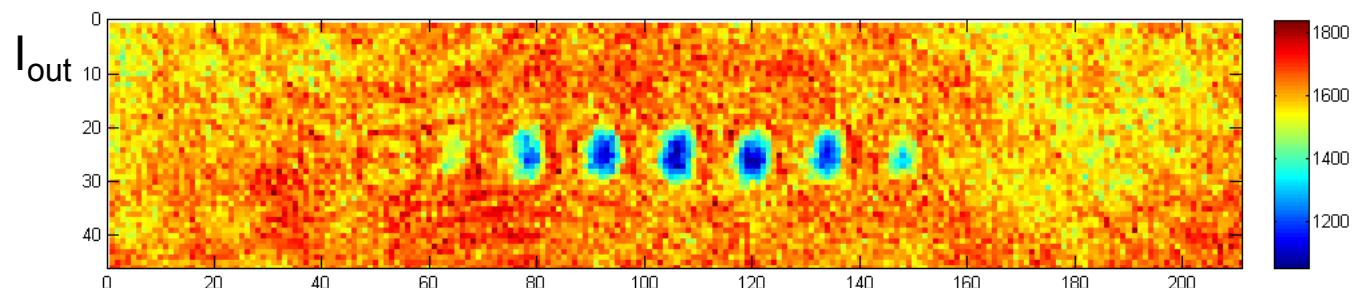
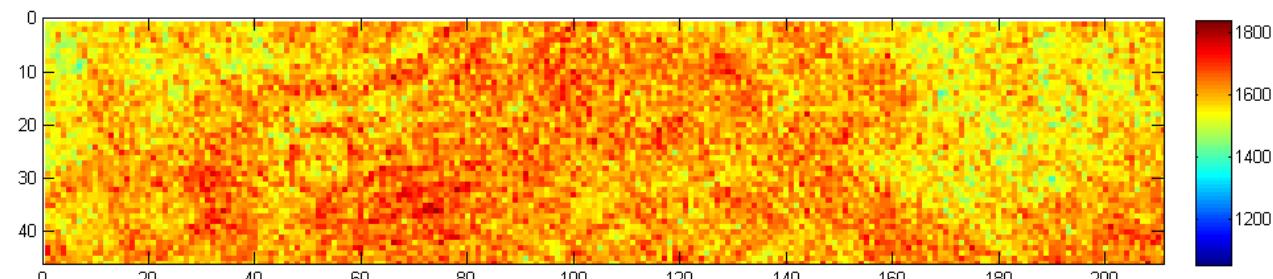
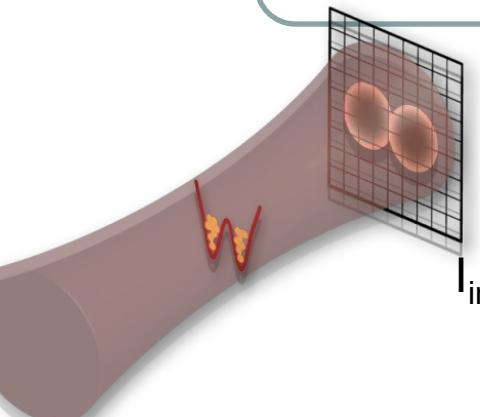


internal

state-of-the-art absorption imaging

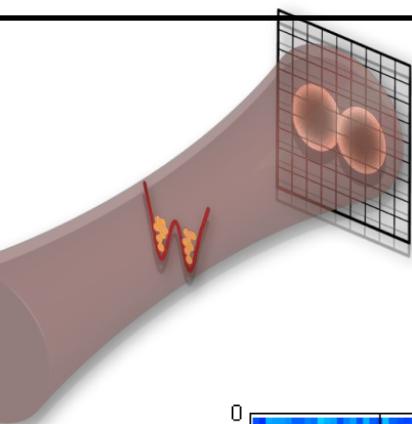
imaging

$$N(x_0, y_0) = \frac{A}{M^2} \frac{2I_{\text{sat}}}{\Gamma \hbar \omega_l} \left(\frac{I_{\text{in}}(x_0, y_0)}{I_{\text{sat}}} - \frac{I_{\text{out}}(x_0, y_0)}{I_{\text{sat}}} + \ln \frac{I_{\text{in}}(x_0, y_0)}{I_{\text{out}}(x_0, y_0)} \right)$$

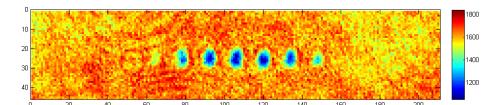
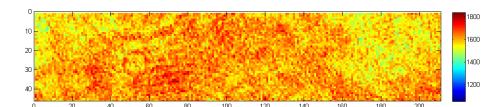


state-of-the-art absorption imaging

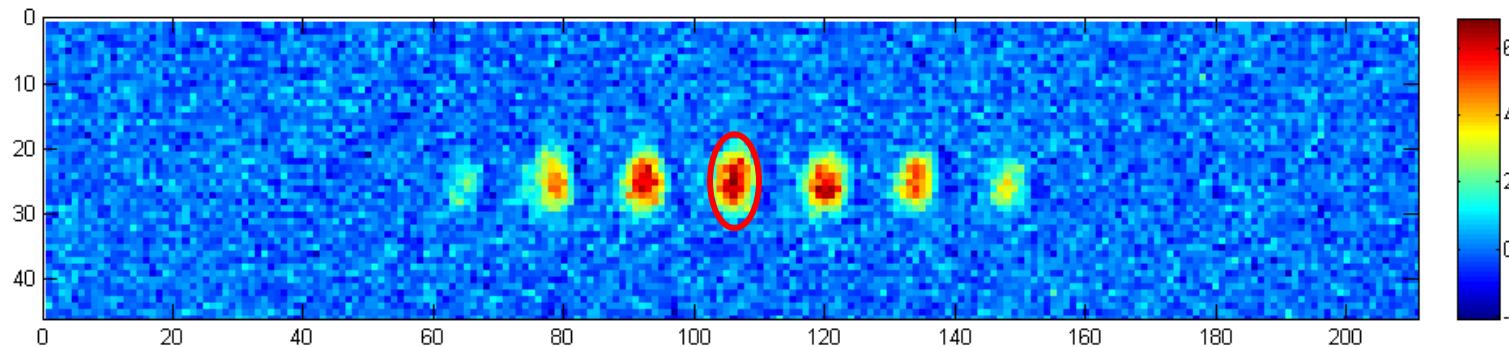
imaging



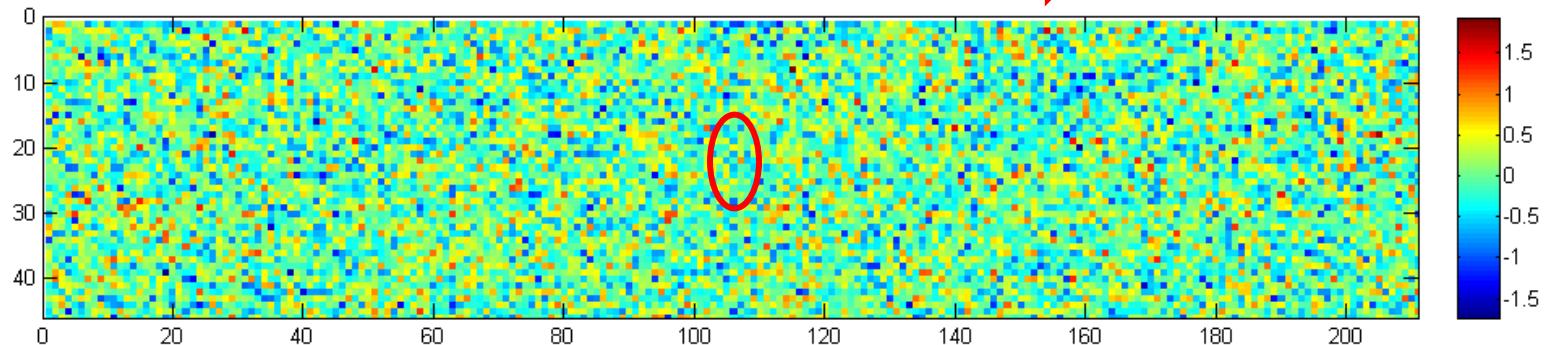
$$N(x_0, y_0) = \frac{A}{M^2} \frac{2I_{\text{sat}}}{\Gamma \hbar \omega_l} \left(\frac{I_{\text{in}}(x_0, y_0)}{I_{\text{sat}}} - \frac{I_{\text{out}}(x_0, y_0)}{I_{\text{sat}}} + \ln \frac{I_{\text{in}}(x_0, y_0)}{I_{\text{out}}(x_0, y_0)} \right)$$



230 atoms

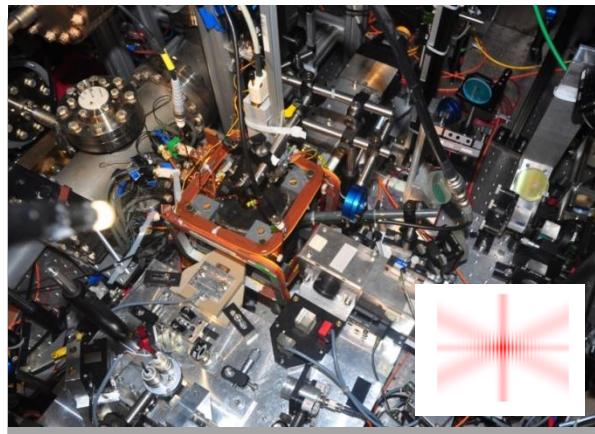
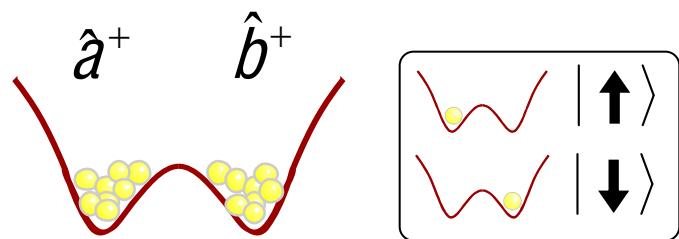
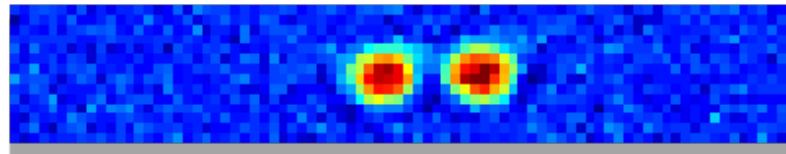


-6 atoms \rightarrow standard deviation 6 atoms



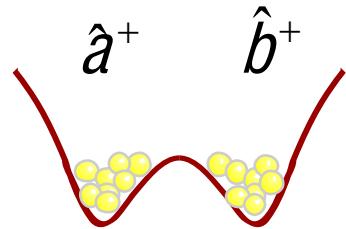
Lipkin-Meshkov-Glick Hamiltonian

double well



number difference fluctuations

ground state properties



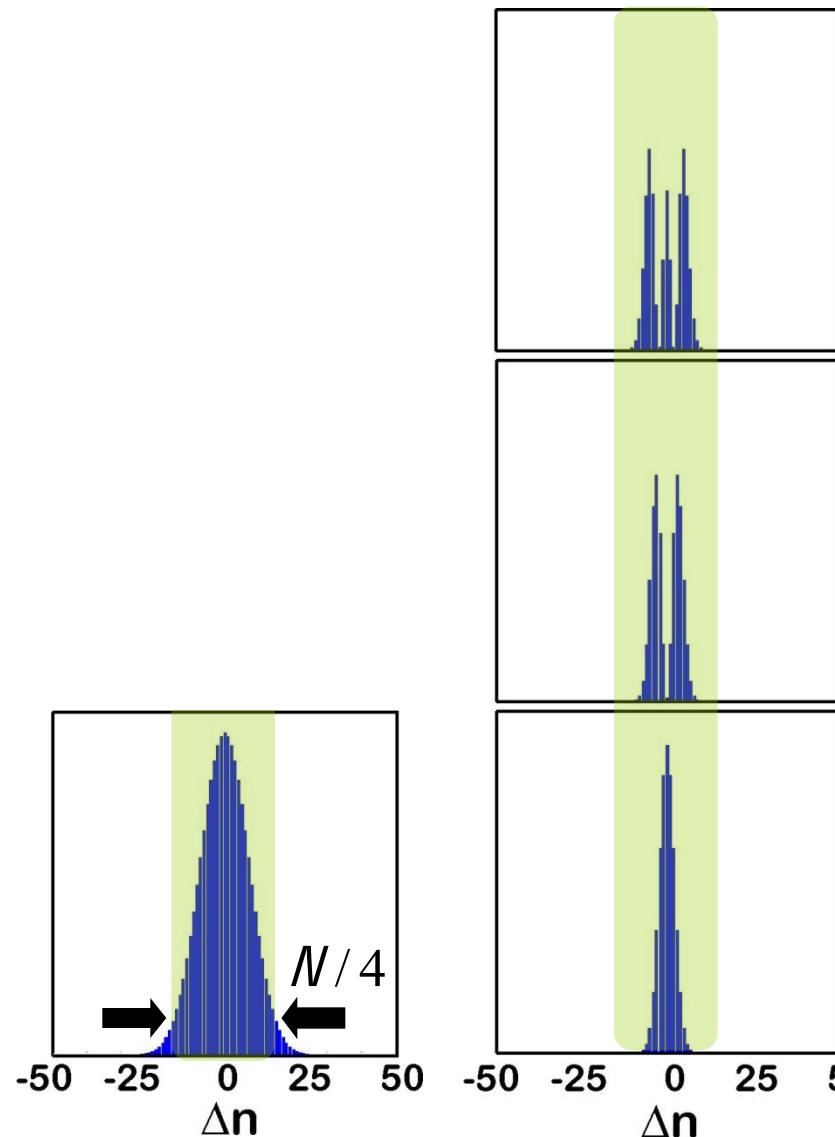
$$\hat{H} = \frac{E_c}{4} (\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})^2 - K(\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a})$$

interaction

tunneling

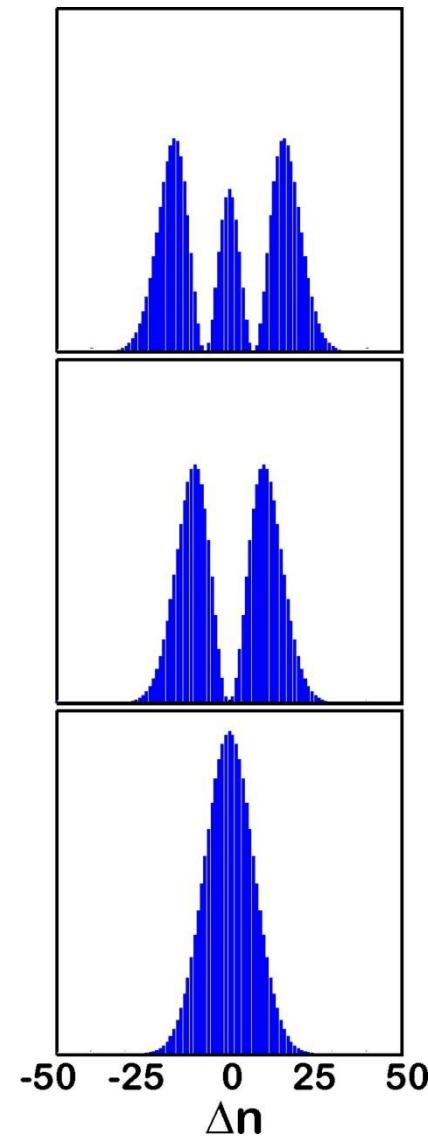
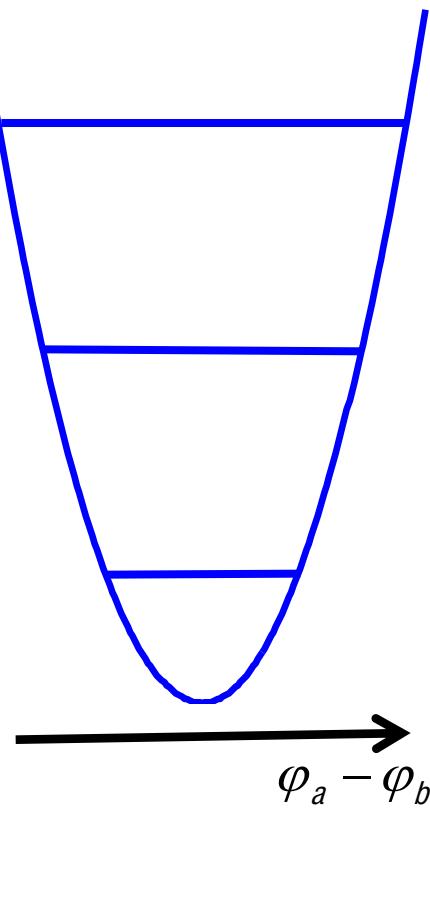
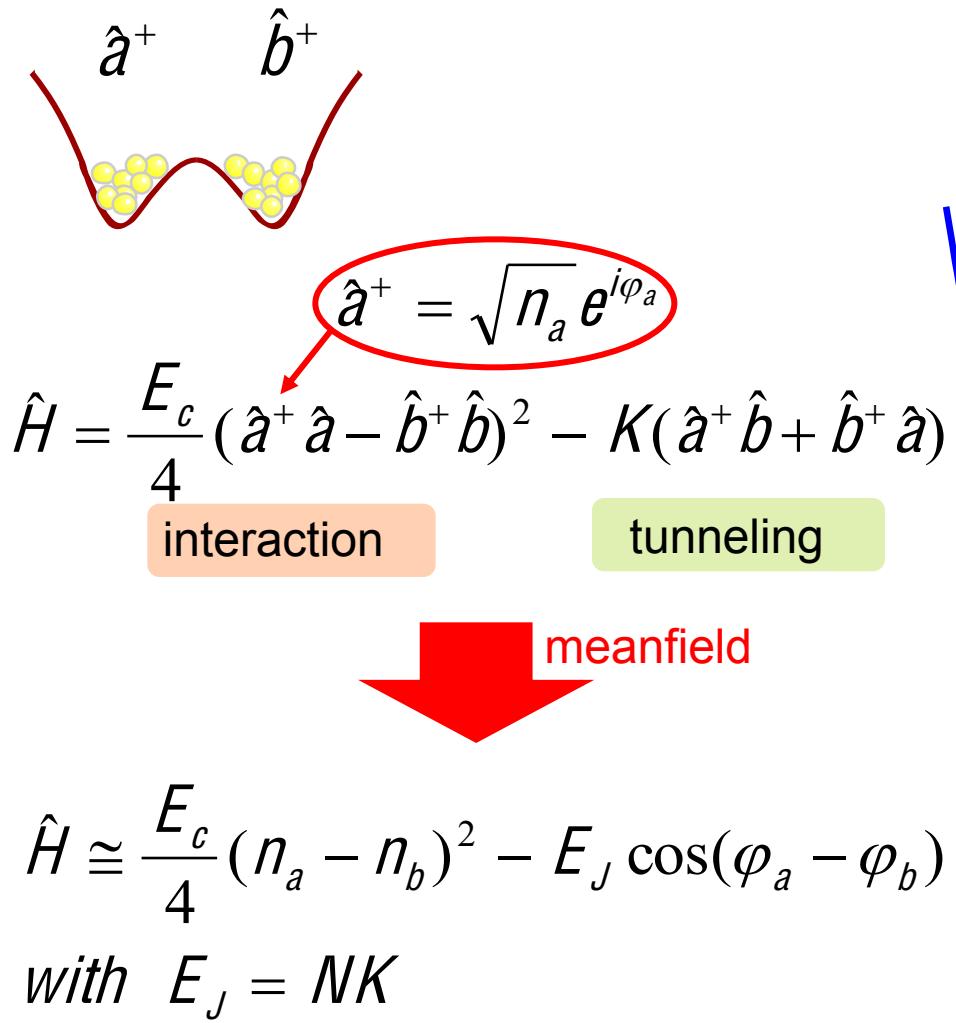
$$|\Psi\rangle \cong (\hat{a}^\dagger + \hat{b}^\dagger)^N |vac\rangle$$

no interaction

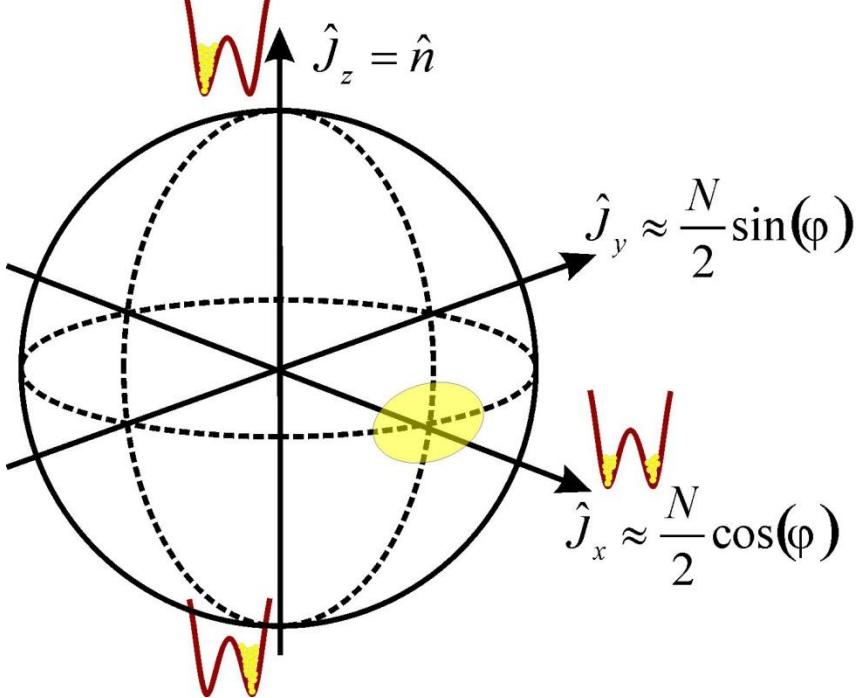


semi-classical picture

properties of many particle state



What can be measured ?



$\hat{J}_z = \frac{1}{2} (\hat{a}^+ \hat{a} - \hat{b}^+ \hat{b})$

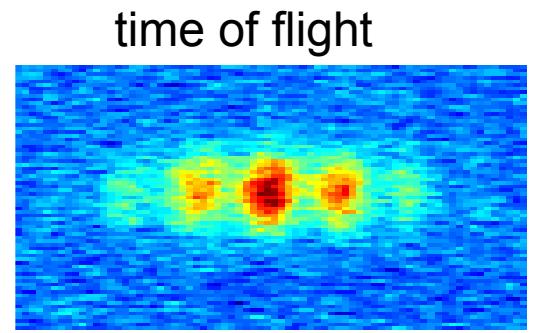
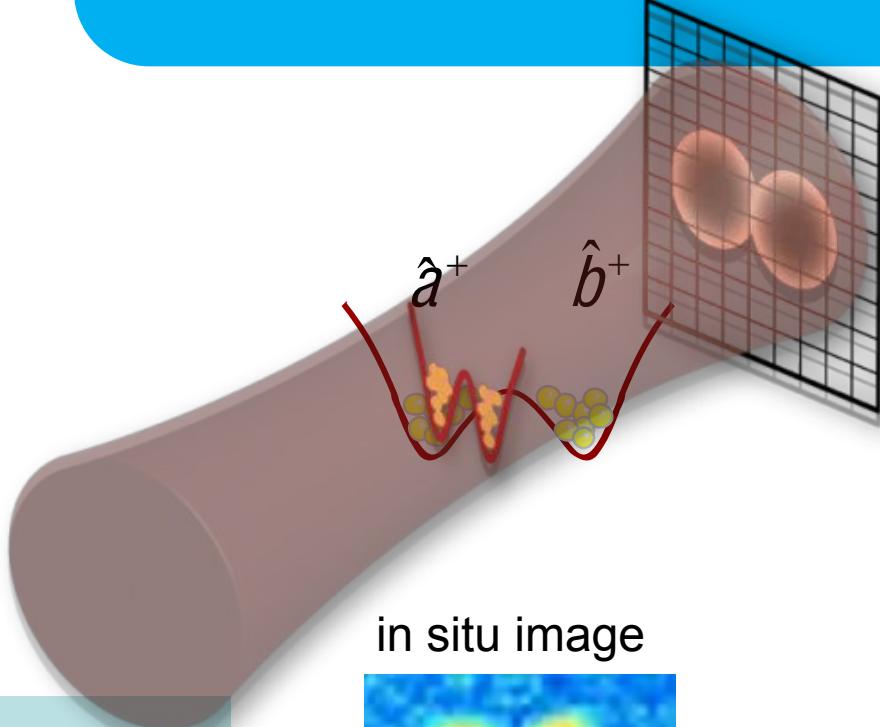
$\hat{J}_x = \frac{1}{2} (\hat{a}^+ \hat{b} + \hat{b}^+ \hat{a})$

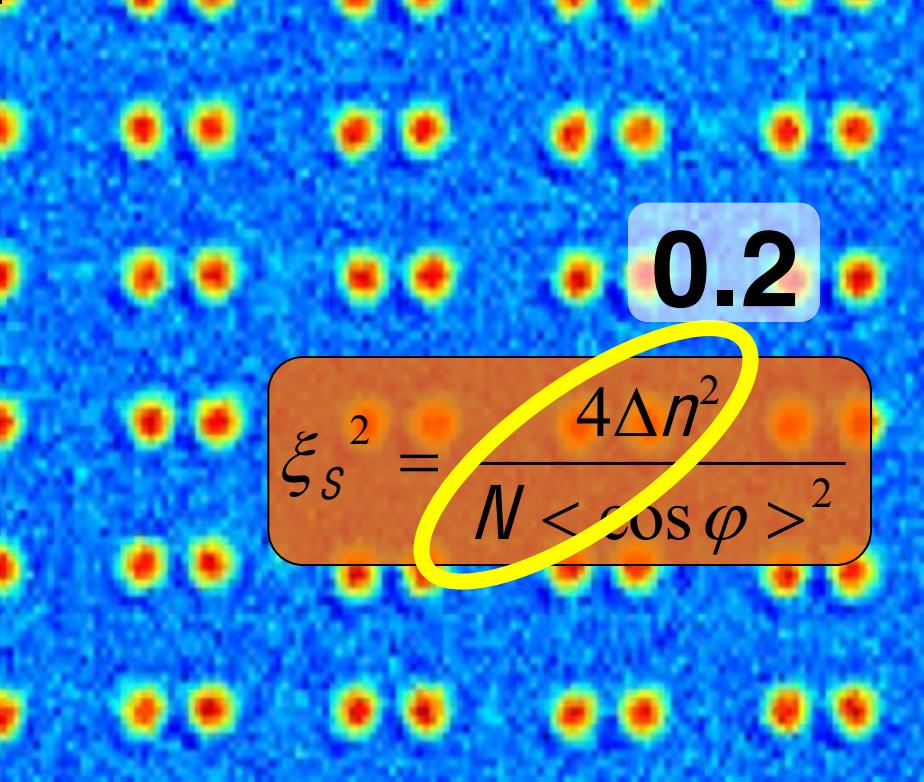
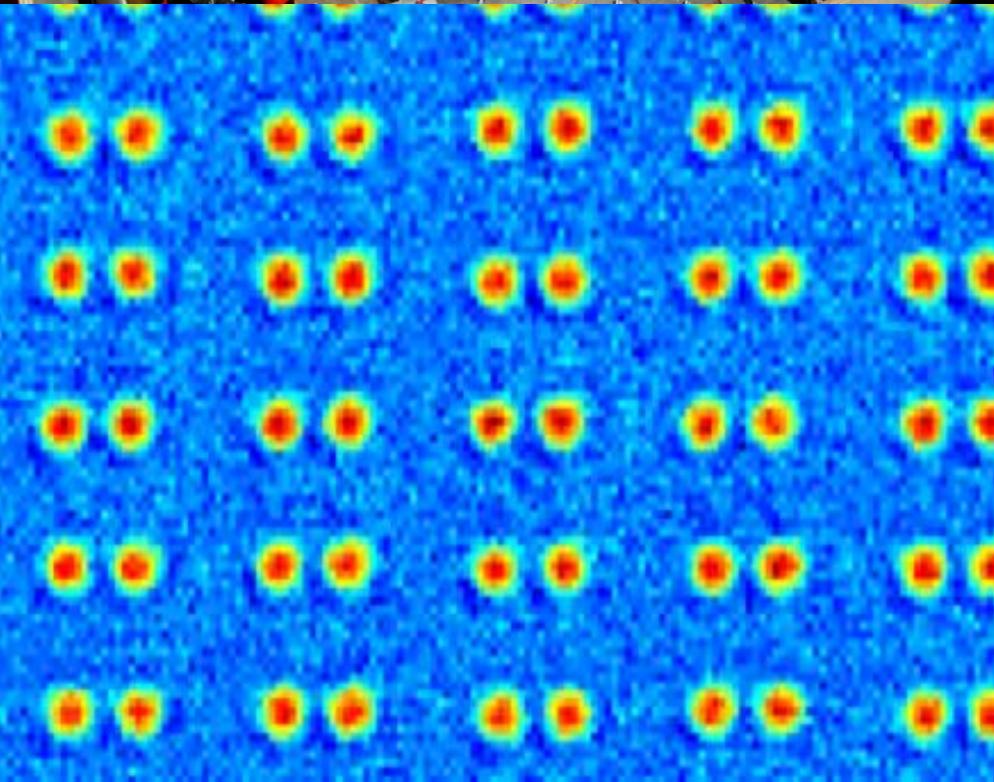
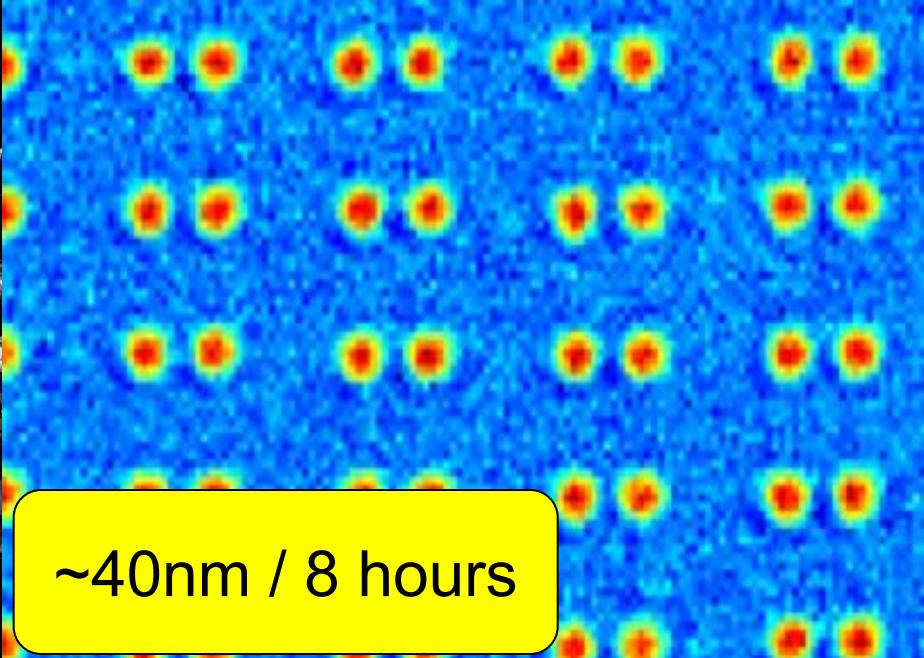
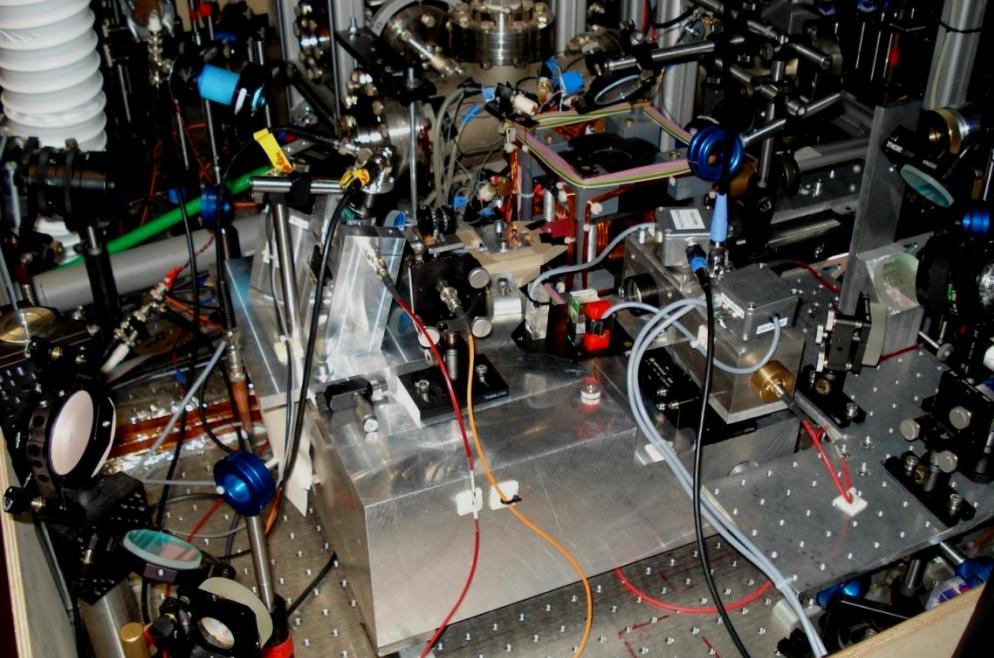
$\hat{J}_y = \frac{1}{2i} (\hat{a}^+ \hat{b} - \hat{b}^+ \hat{a})$

$\hat{a}^+ = \sqrt{n_a} e^{i\varphi_a}$

'meanfield'

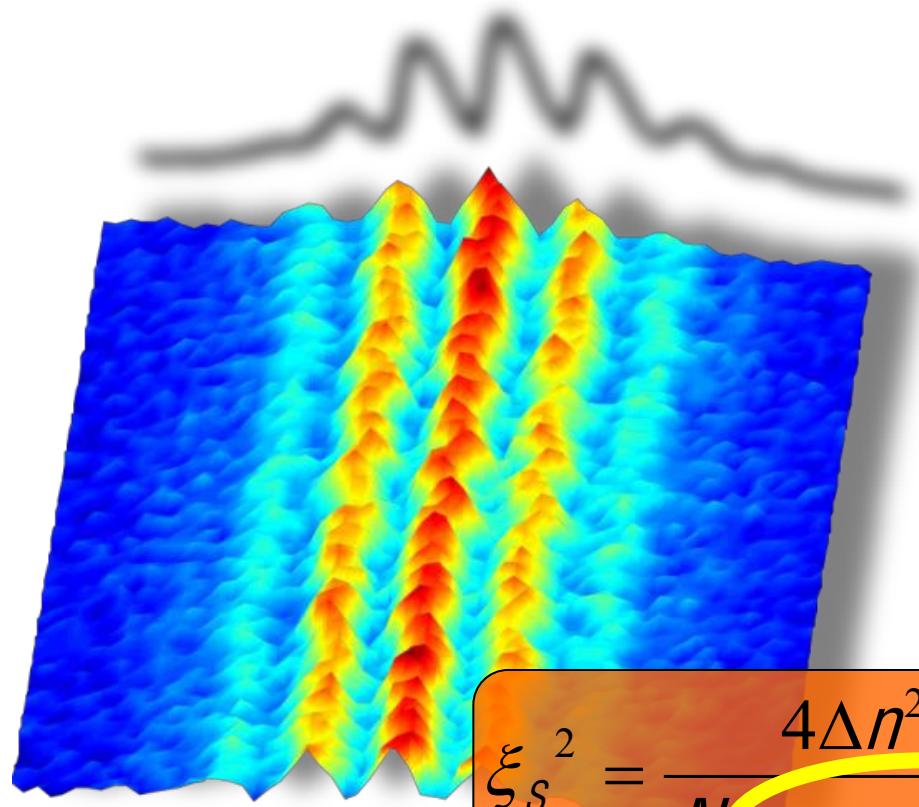
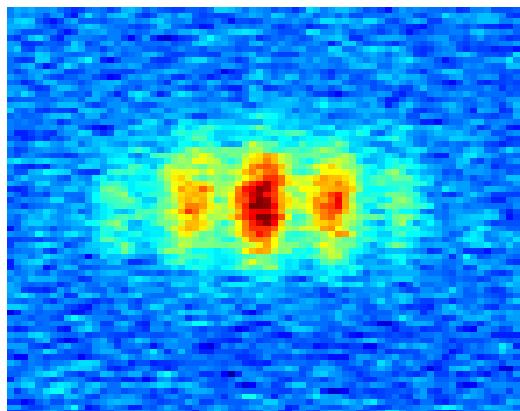
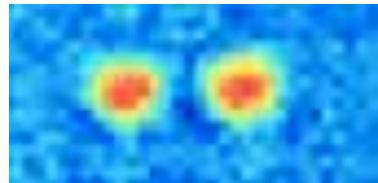
$$\langle \hat{J}_z \rangle \approx \frac{\langle n_a - n_b \rangle}{2}$$





conjugate variable – ,phase‘

coherence



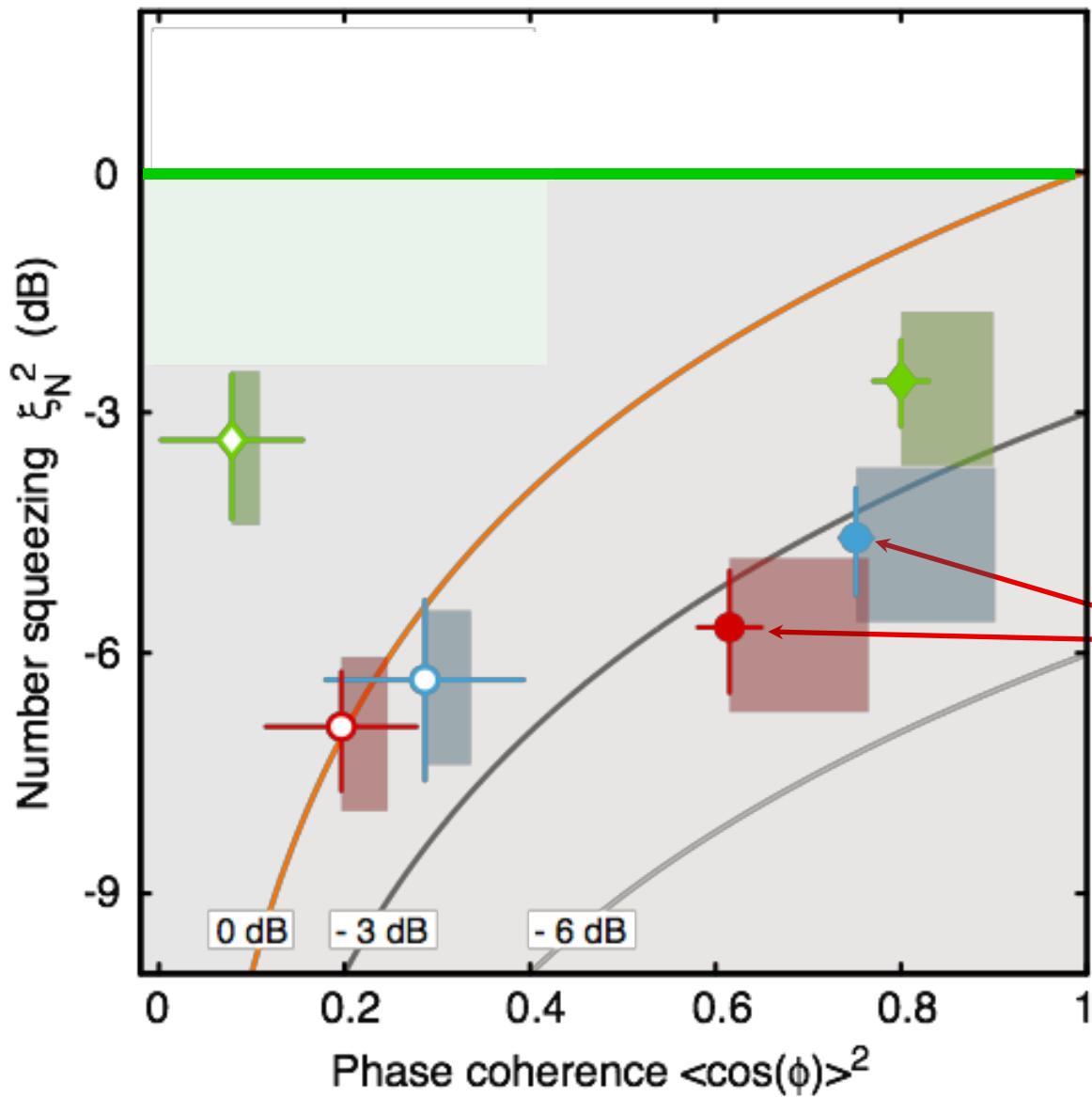
$$\xi_s^2 = \frac{4\Delta n^2}{N \langle \cos \varphi \rangle^2}$$

0.8

atomic coherent spin states

Nature 455, 1216 (2008)

external



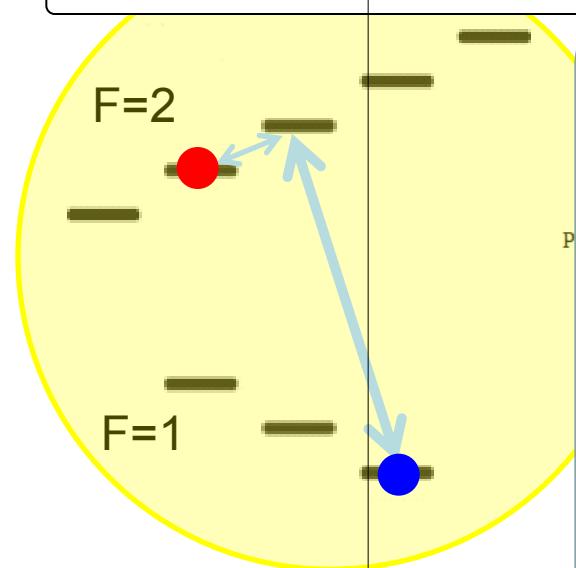
$$\xi_S^2 = \frac{4\Delta n^2}{N \langle \cos \varphi \rangle^2}$$

Over 1000 measurements
 $\xi_S^2 = -3.8^{+0.3}_{-0.4}$ dB

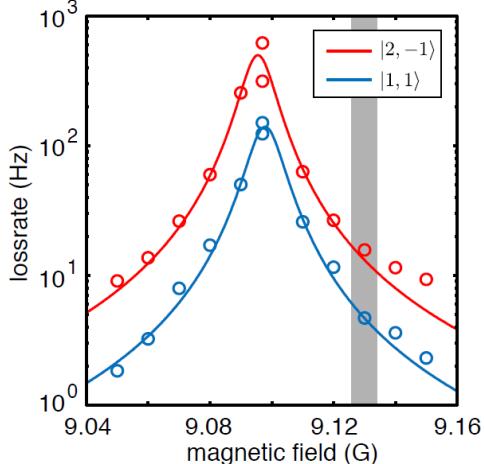


Josephson physics

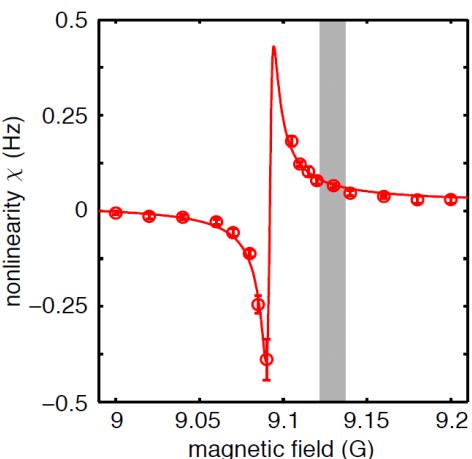
internal system



Feshbach resonance -loss



Feshbach resonance E_c



IL 1998

$$\Delta n = \frac{N_I - N_r}{2}$$

$$\varphi = \varphi_r - \varphi_I$$

$$H = \frac{E_c}{2} \Delta n^2 - E_j \sqrt{1 - \frac{4\Delta n^2}{N^2} \cos \varphi}$$

Charging energy: E_c

$$E_c \propto (a_{11} + a_{22} - 2a_{12}) \int |\Phi|^4 dr$$

Josephson energy: E_j

$$E_j \approx M\Omega$$

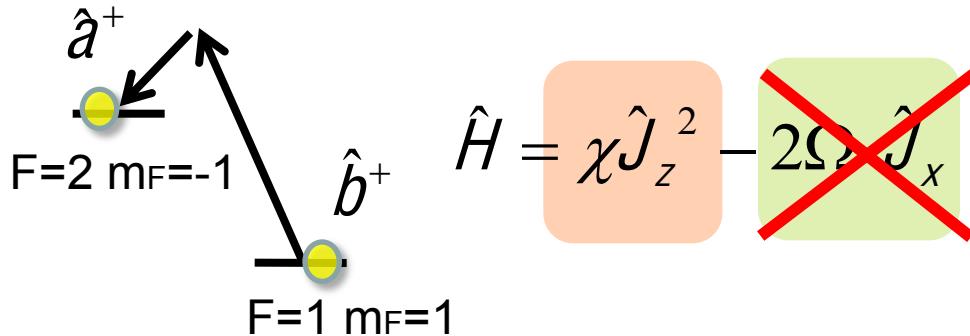
Rubidium

$$a_{11} : a_{22} : a_{12} = 100.44 a_B : 95.47 a_B : 97.7 a_B \Rightarrow E_c \sim 0$$

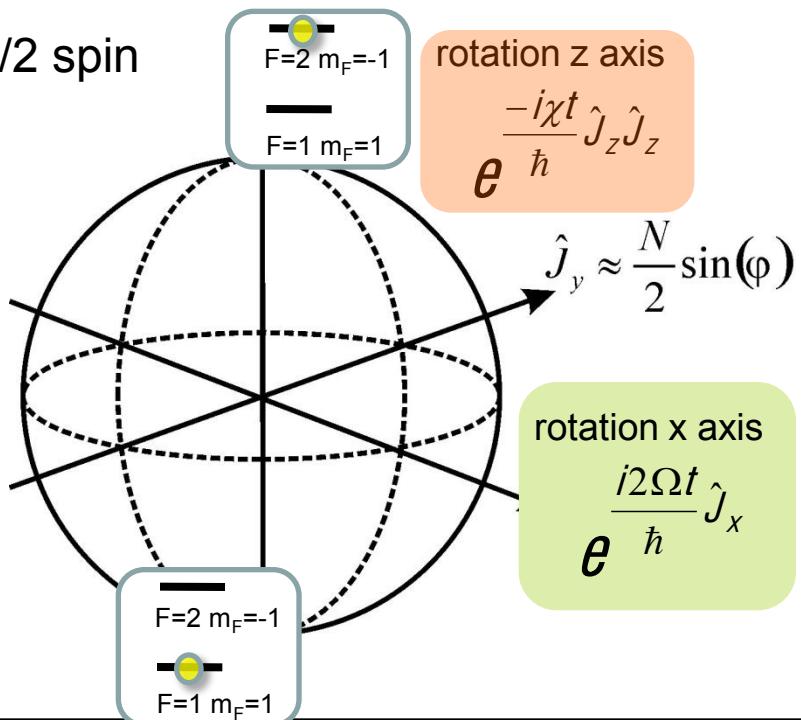
one axis twisting

M. Kitagawa, M. Ueda PRA 47, 5138 (1993)

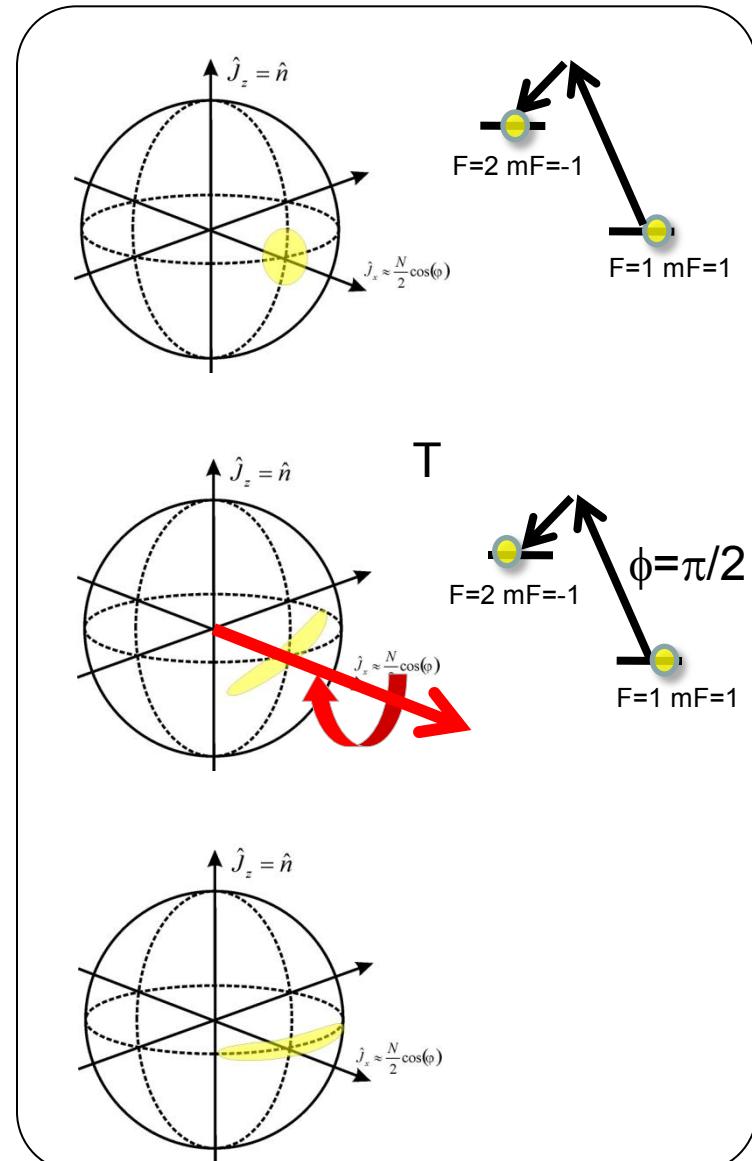
Sorensen, et al. Nature 409, 63 (2001)



$J=N/2$ spin

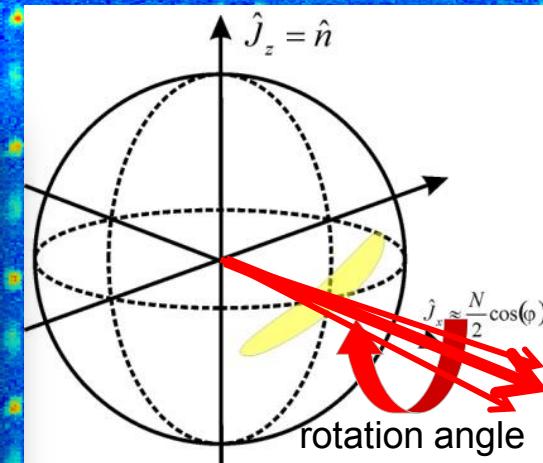
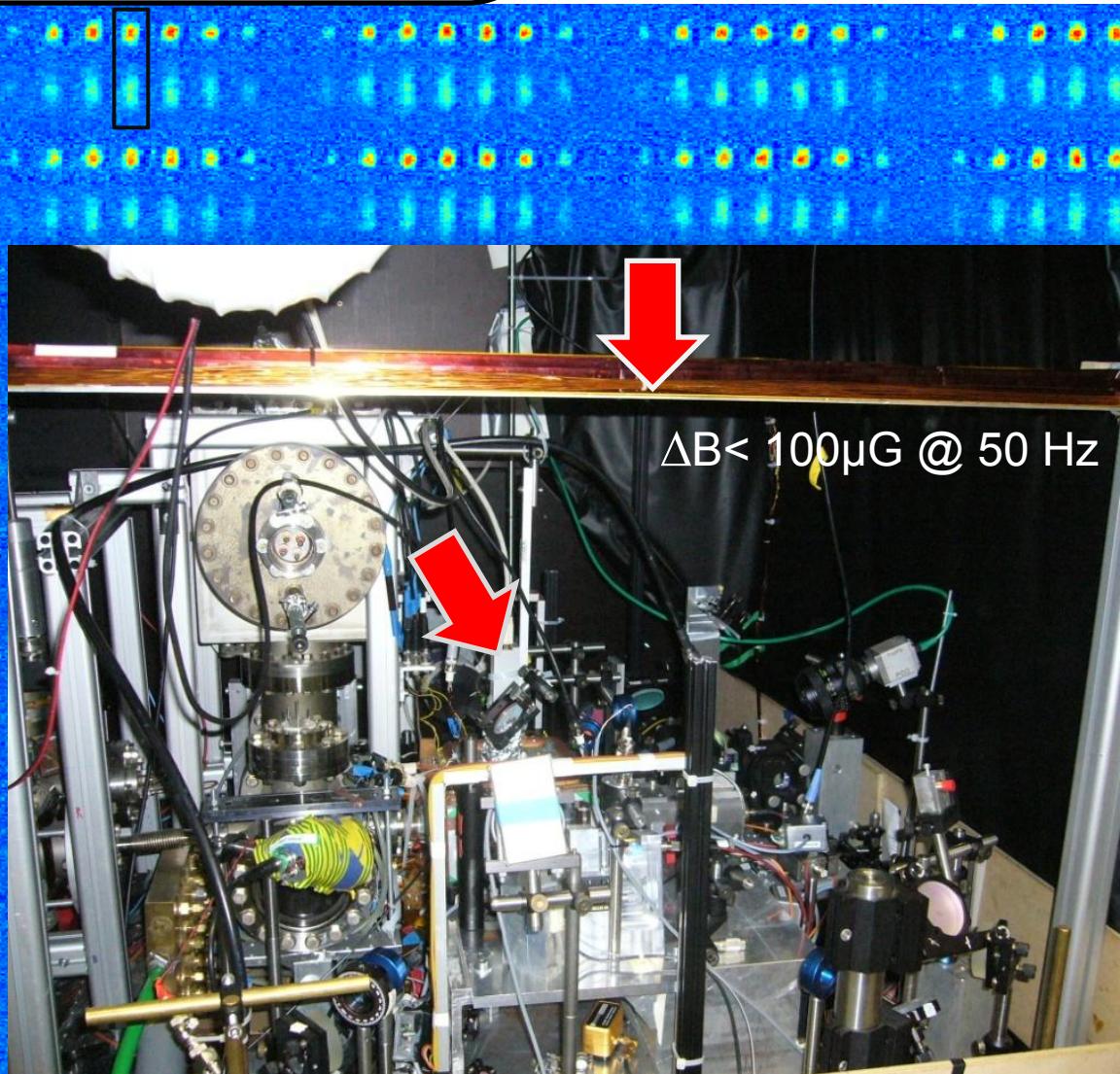


internal



one axis twisting

internal

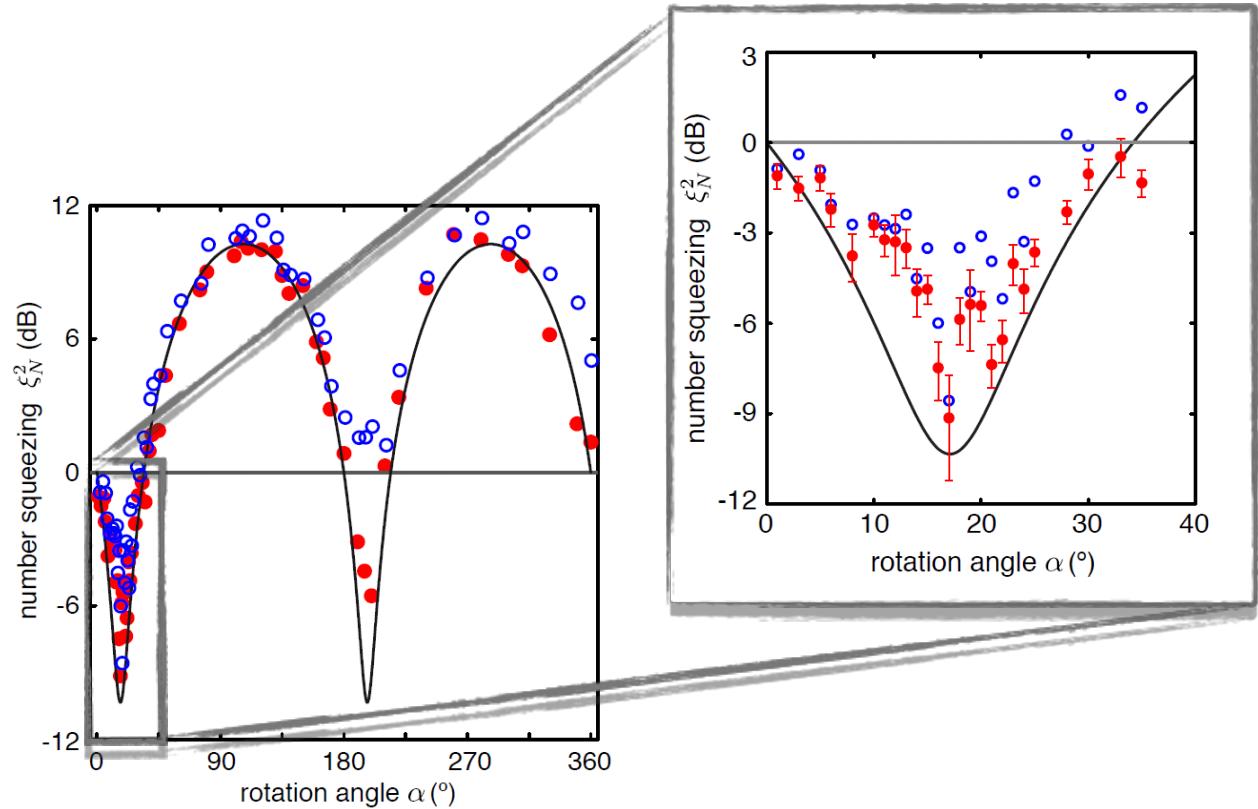
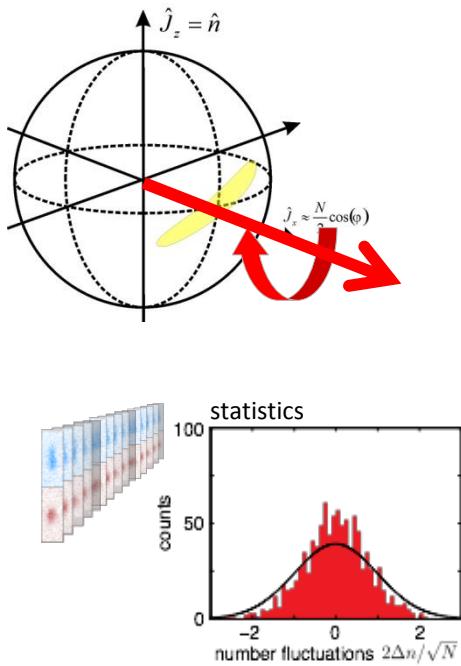


one axis twisting

Nature 464, 1165 (2010)

See also BEC on atomchip: Munich/Basel Nature 464, 1170 (2010)

internal



► inferred spin squeezing:

$$\xi_S^2 = \frac{N \Delta J_z^2}{\langle J_x \rangle^2} = -8.2^{+0.9}_{-1.2} \text{ dB}$$

$$\xi_N^2 = \frac{4 \Delta J_z}{N} = -8.2 \text{ dB}$$

vapor cells: Hald et al., PRL 83, 1319 (1999)

Kuzmich et al., PRL 85, 1594 (2000) Cold thermal atoms + cavity: Leroux et al., PRL 104 073602 (2010)

cold atoms: Appel et al., PNAS 106, 10960 (2009) BEC on atomchip: Munich, Philipp Treutlein group Nature 464, 1170 (2010)

entanglement

Nature 464, 1165 (2010)

internal

- Many body entanglement:

$$\rho \neq \sum_k P_k \rho_k^1 \otimes \rho_k^2 \otimes \dots \otimes \rho_k^m \otimes \dots \otimes \rho_k^N$$

non-separable!

- Possible entanglement witness:

$$\xi_S^2 < 1 \quad \text{implies entanglement}$$

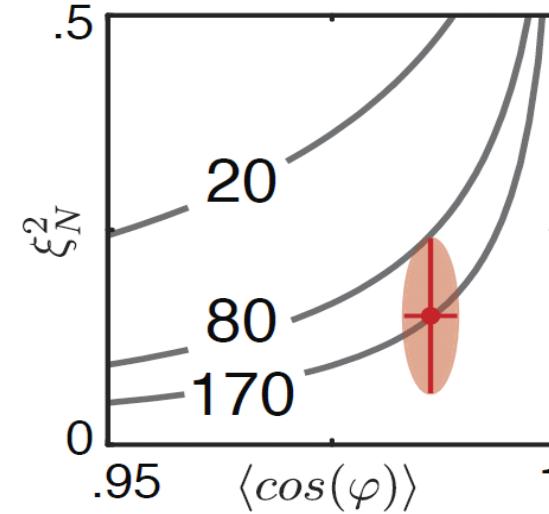
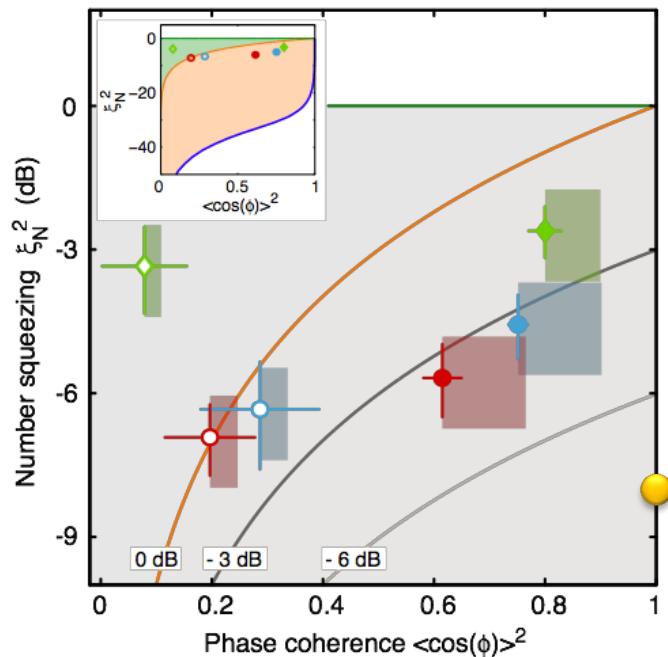
Sørensen et al., Nature 409, 63 (2001)

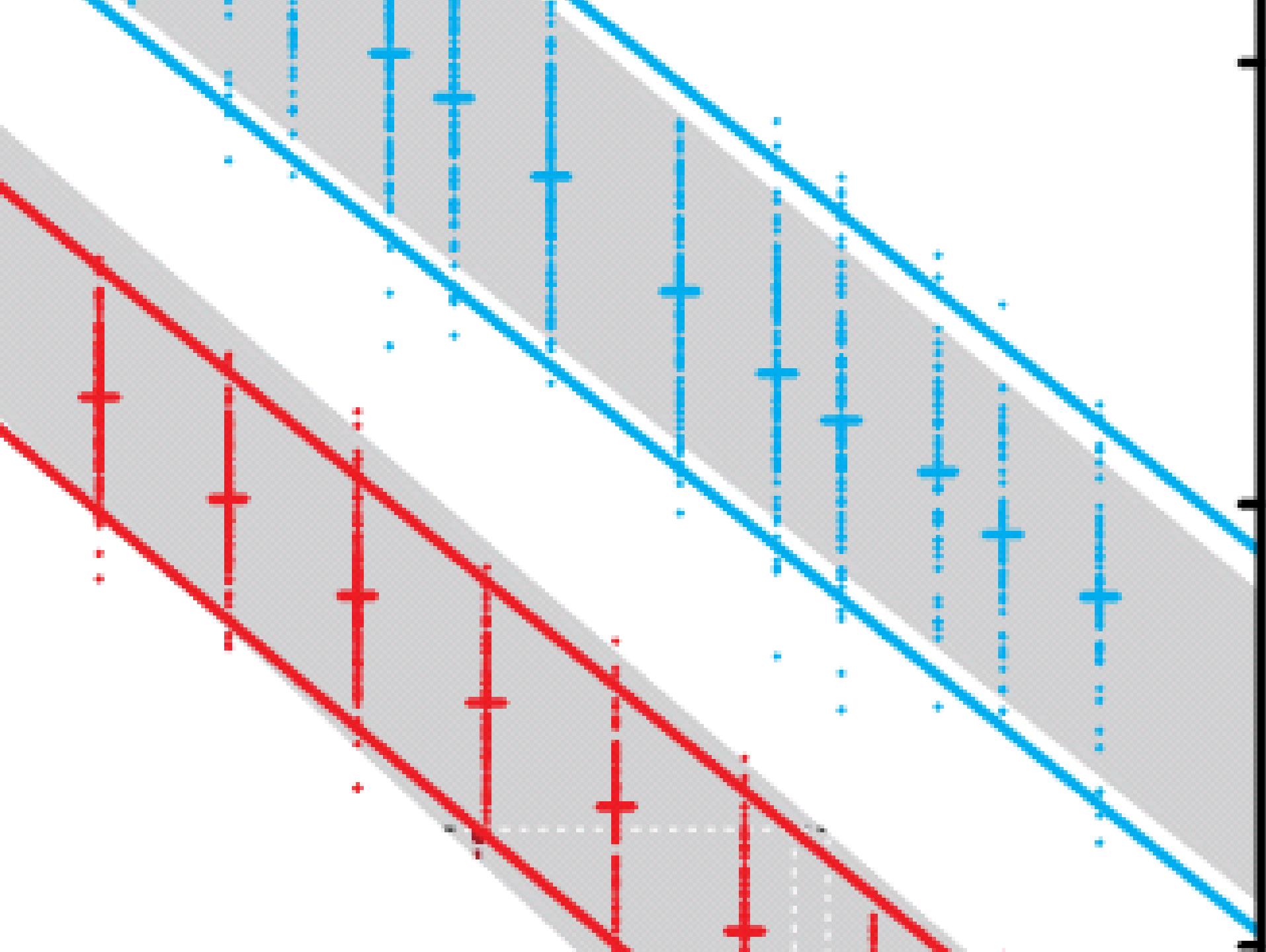
- Depth of entanglement:

$$\rho = \sum_k P_k \rho_k^{1..m} \otimes \rho_k^{m+1..2m} \otimes \dots \otimes \rho_k^{N-m+1..N}$$

▷ block size of the largest non-separable part: m

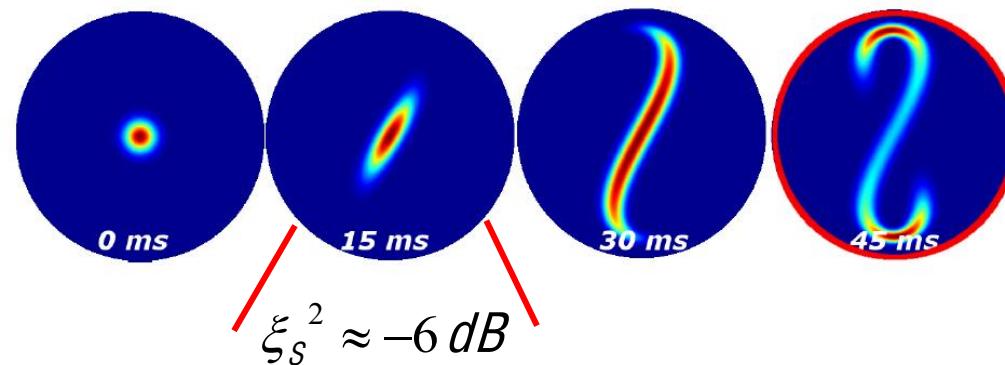
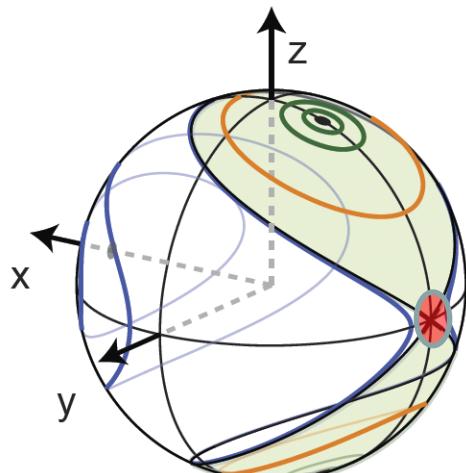
Sørensen & Mølmer, PRL 86, 4431 (2001)



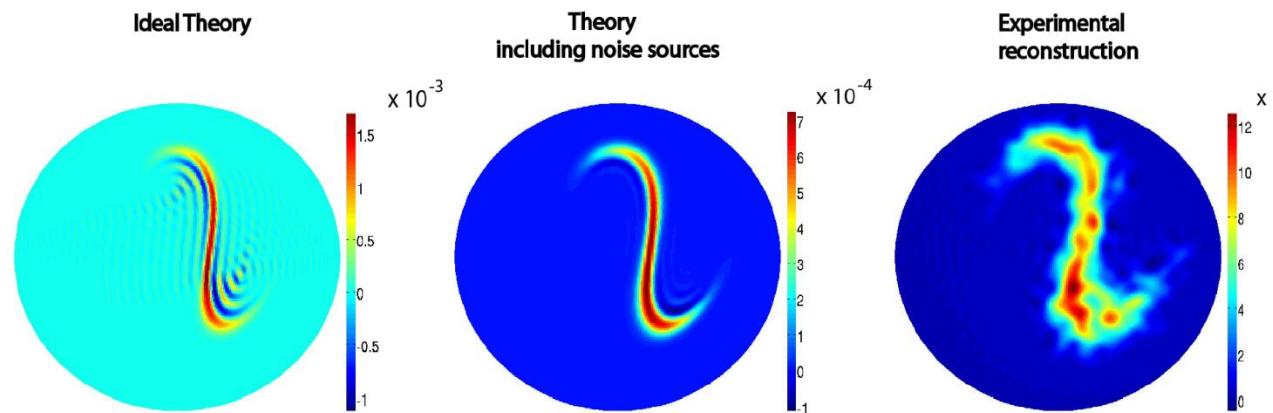


Latest squeezing - use unstable fixpoint

Not yet published



A.I. Lvovsky, J. Opt. B: Quantum Semiclass. Opt. 6 S556 (2004)



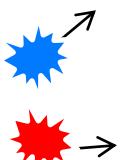
postselected: 340 ± 10 atoms

Quantum optics paradigm system

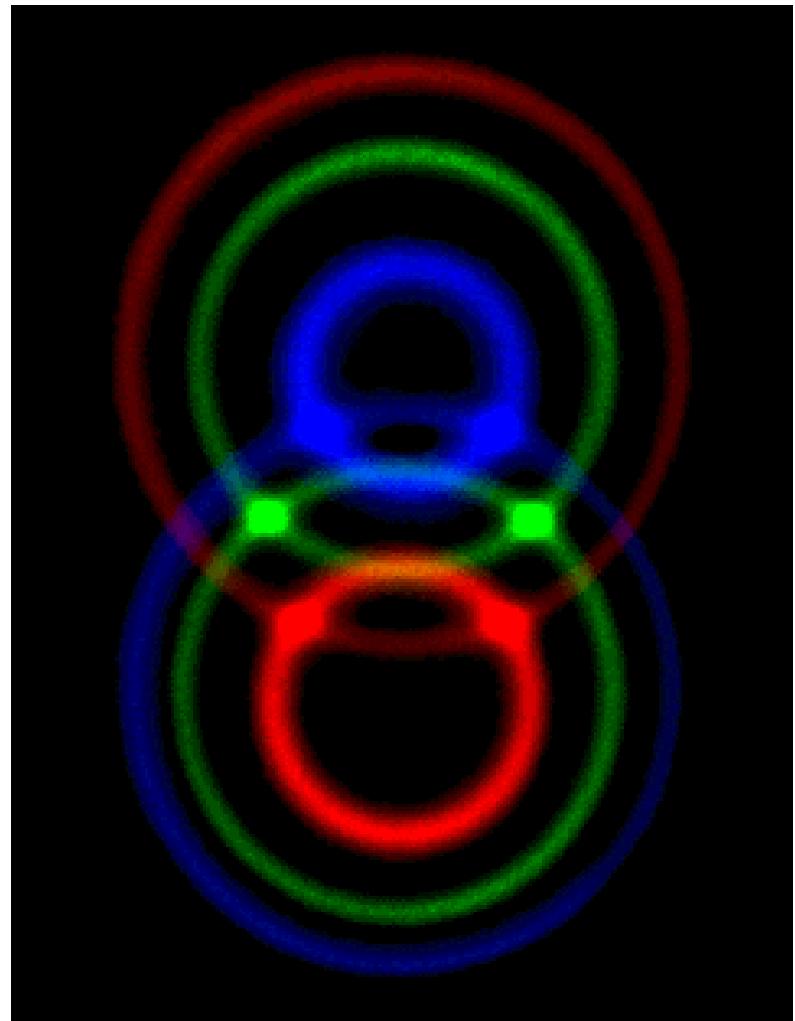
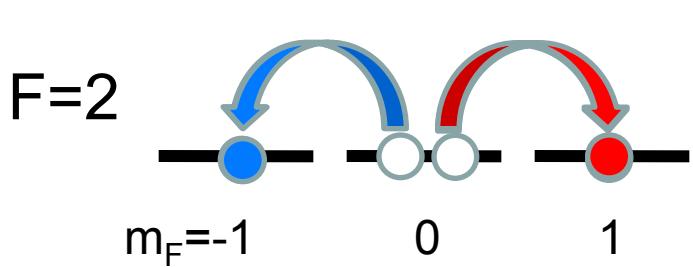
latest



$$P \sim \chi_2 E^2$$

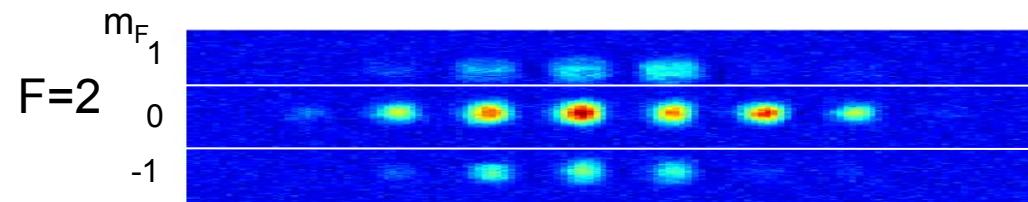
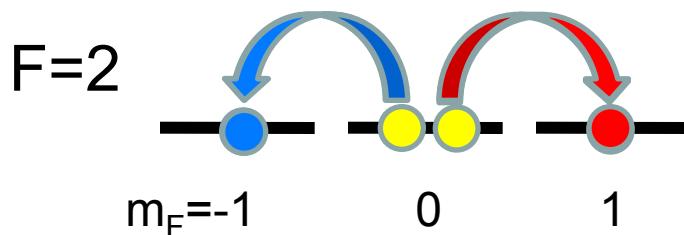


$$P \sim \chi_2 E^2$$



Spin changing collisions

latest

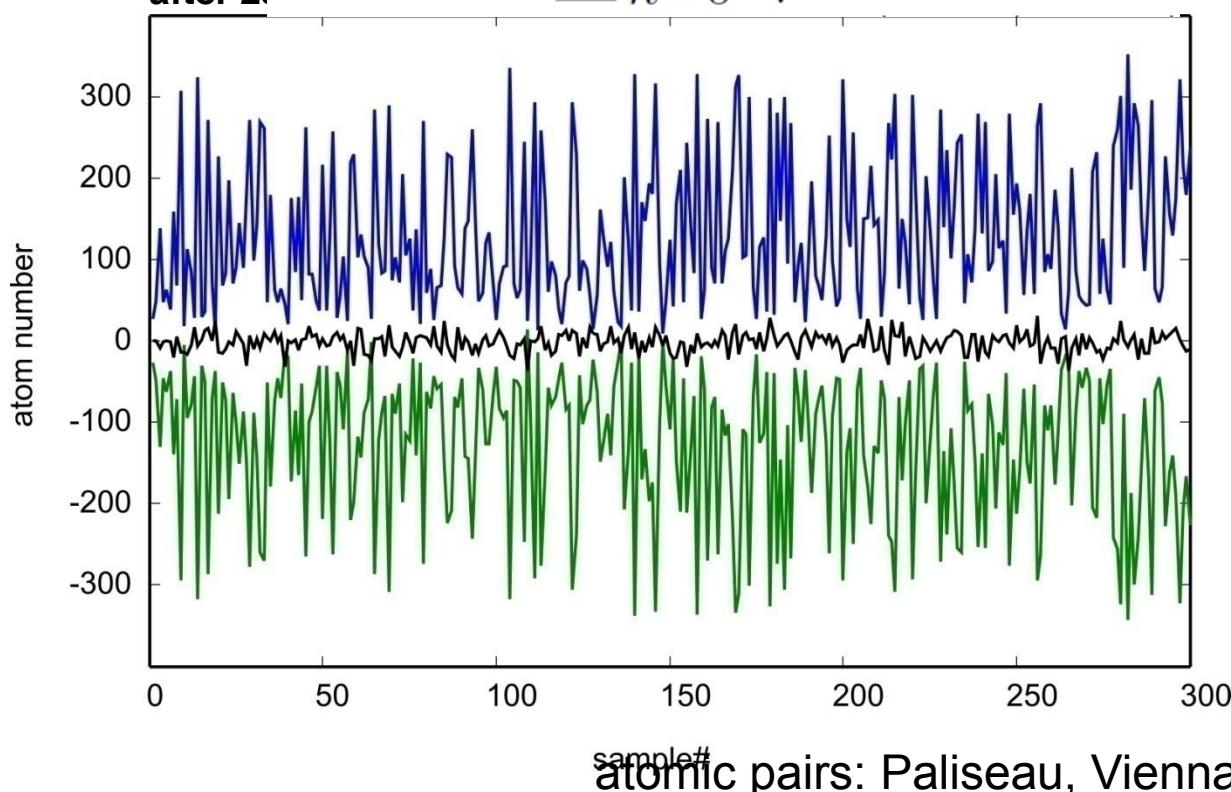


two mode squeezed vacuum

after 2! $|\psi_{\bar{n}}\rangle = \sum_{n=0}^{\infty} \sqrt{p_n(\bar{n})} |n, n\rangle$

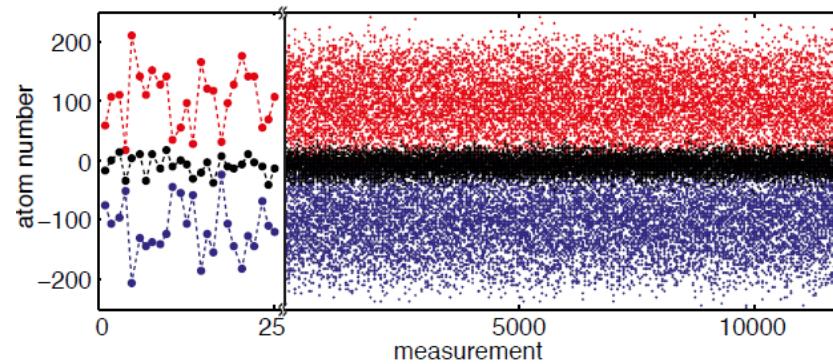
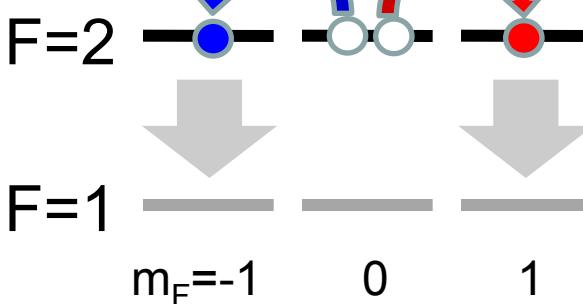
$$p_n(\bar{n}) = (1 - t_{\bar{n}}) t_{\bar{n}}^n$$

$$t_{\bar{n}} = 1 / (1 + 2/\bar{n})$$

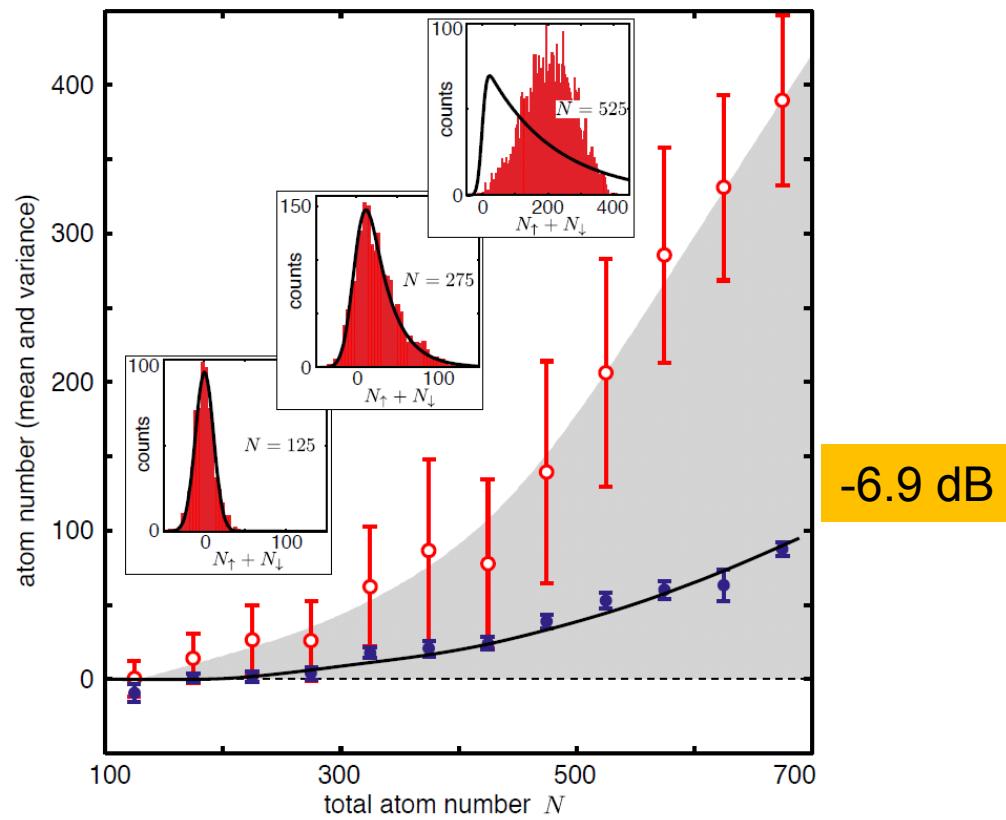


atomic pairs: Paliseau, Vienna, Hannover, Georgia Tech, ...

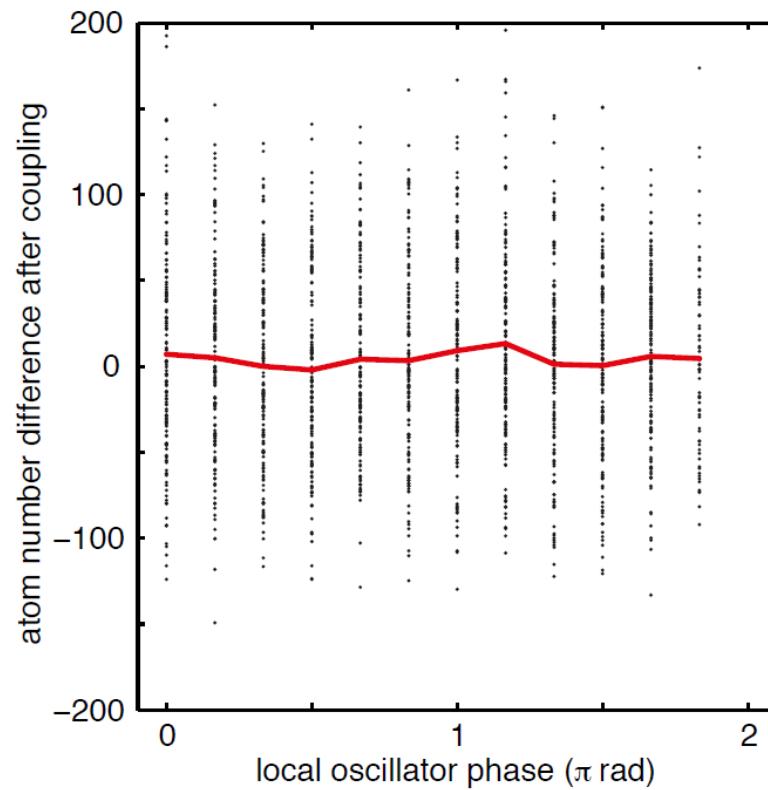
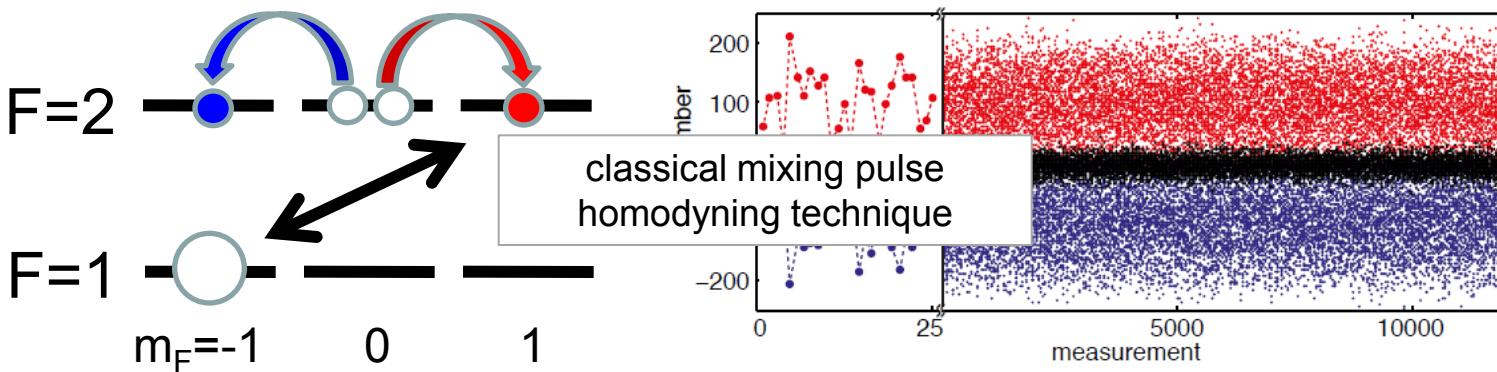
latest



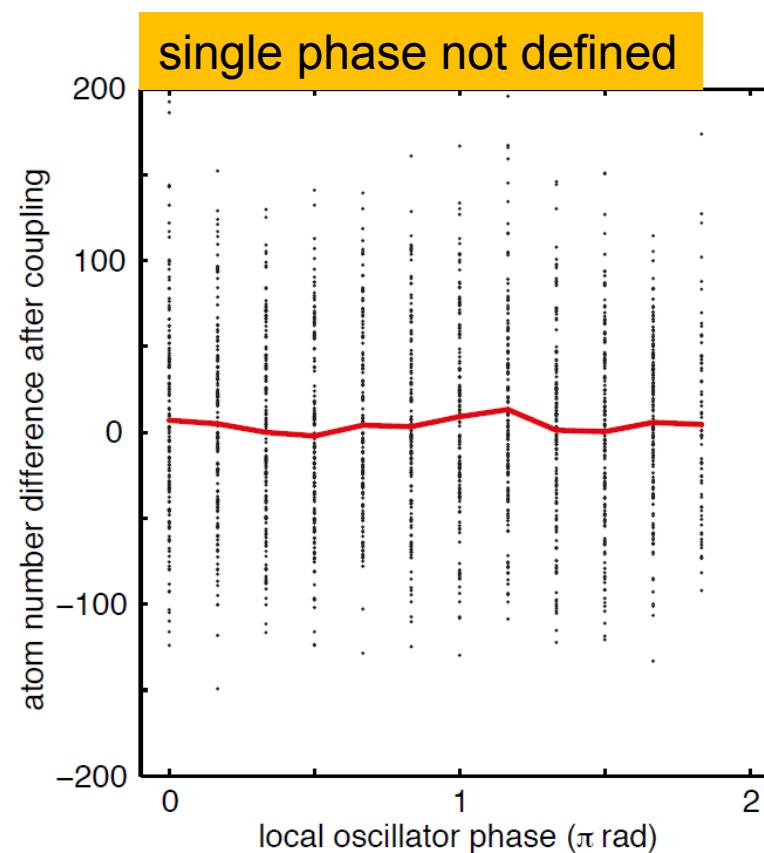
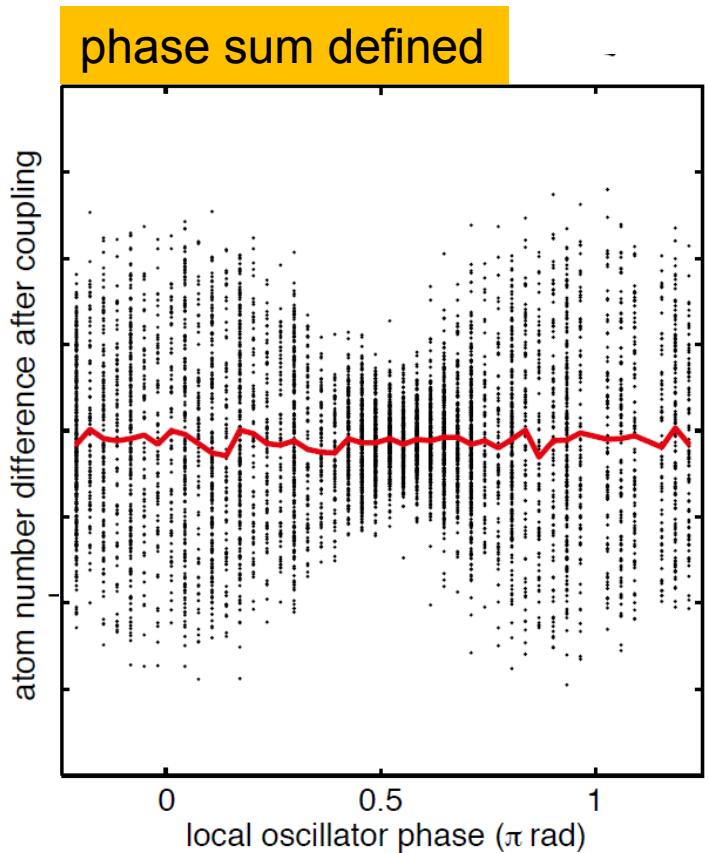
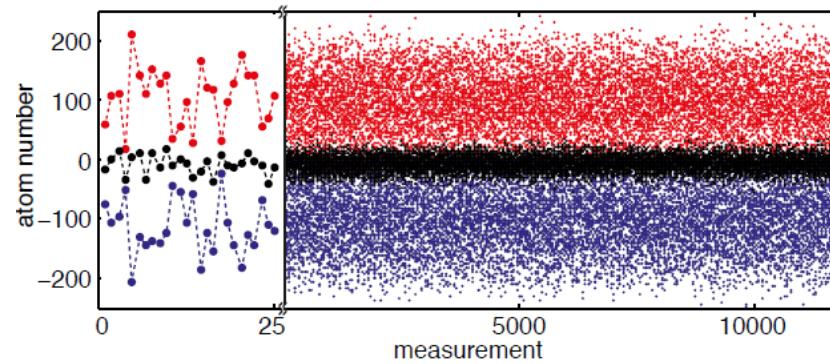
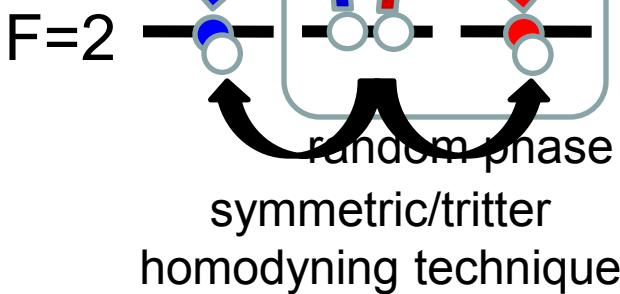
$$|\psi_{\bar{n}}\rangle = \sum_{n=0}^{\infty} \sqrt{p_n(\bar{n})} |n, n\rangle \quad p_n(\bar{n}) = (1 - t_{\bar{n}}) t_{\bar{n}}^n \quad t_{\bar{n}} = 1 / (1 + 2/\bar{n})$$



latest

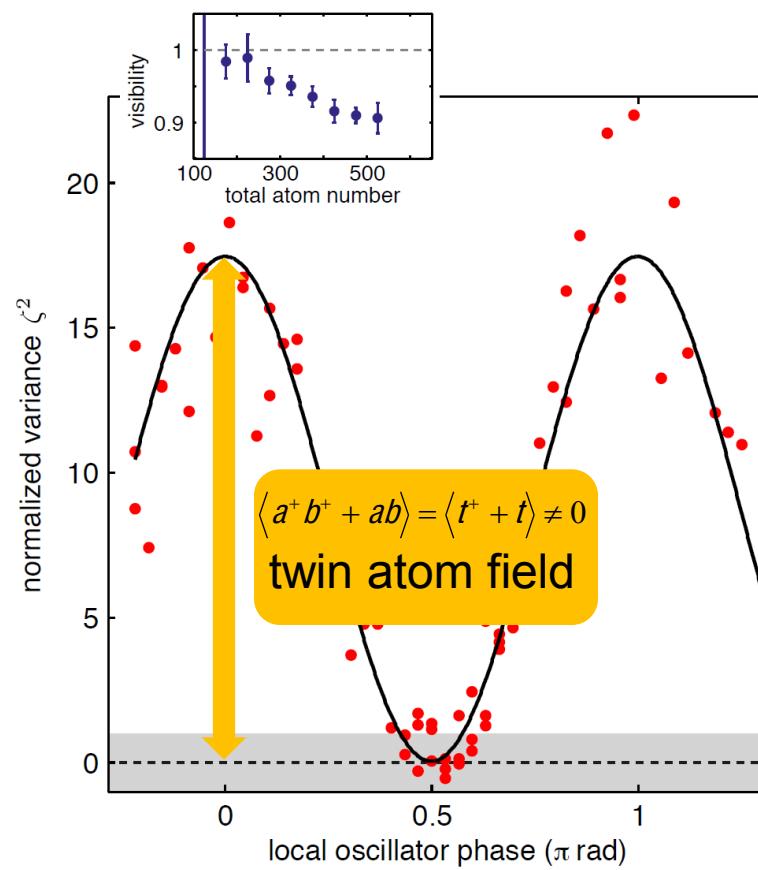
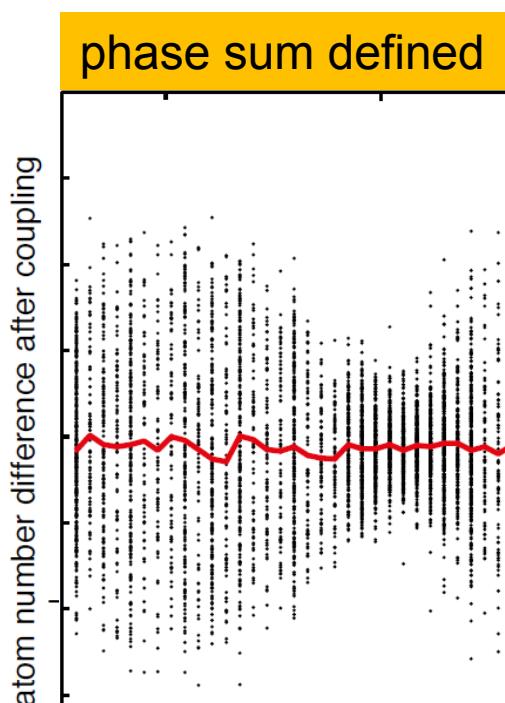
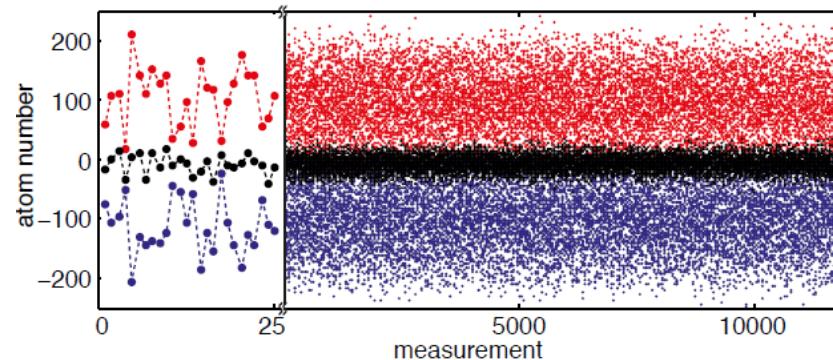
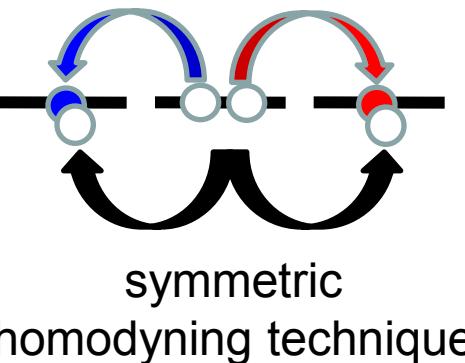


latest



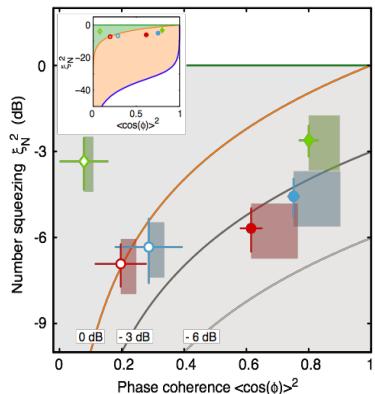
latest

F=2

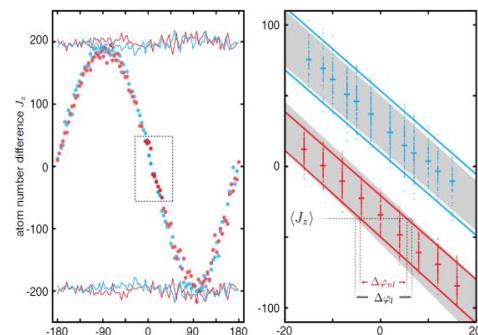


conclusion

External squeezing



Nonlinear interferometry



Two mode squeezing

