

Dynamic Kosterlitz-Thouless transition in 2D Bose mixtures

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- **Dynamic Kosterlitz-Thouless transition**

with K. Günter, J. Dalibard, A. Polkovnikov

cond-mat/1112.1204

- Renormalization group dynamics, vortex unbinding, reverse Kibble-Zurek
- Experimental proposal for mixtures of hyperfine states



Quantum simulators

Feynman 1982

Spin chains

Greiner group 2011

Bose-Hubbard model

Greiner group 2010

Magnetic systems

Sengstock group 2011

Unitary Fermi gas

Zwierlein group 2011

Dirac equation

Blatt group 2010

Dynamic properties of 1D gases

Bloch group 2011

Open frontier in many-body physics: dynamic phase transitions, critical dynamics

→ We propose to apply the concept of quantum simulation to establish a theoretical framework for many-body dynamics

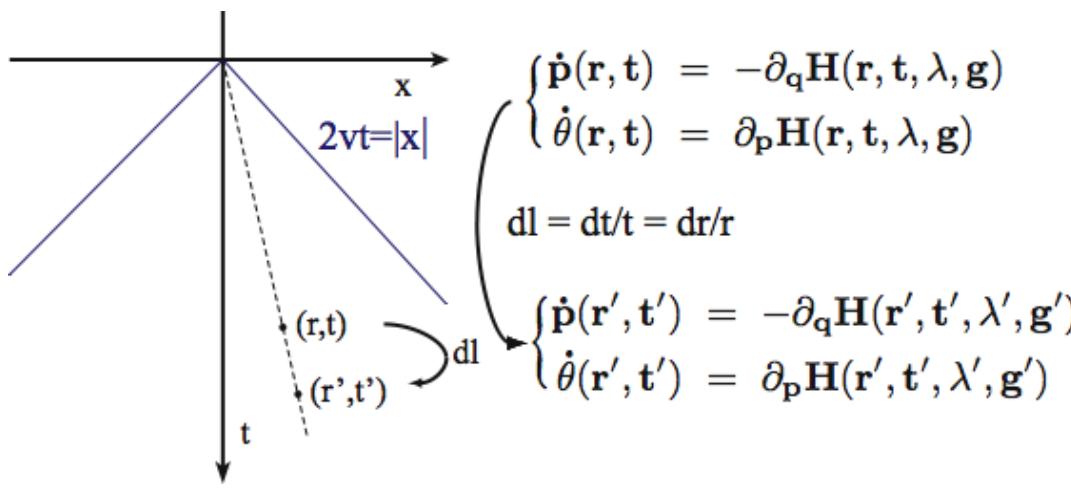
H. Weimer, et al., Nature Phys., 2010; M. Ortner, et al., New J. Phys., 2009; M. Lewenstein, et al., Adv. in Physics, 2007; H. P. Büchler, et al., Phys. Rev. Lett., 2005; N. Szpak, R. Schützhold, cond-mat/1109.2426; L. Mazza, et al., cond-mat/1105.0030; H. Weimer, cond-mat/1104.3081; Z. Lan, et al., cond-mat/1102.5283.



Renormalization group framework for critical dynamics

Phys. Rev.A 81, 033605 (2010)

We rescale real-time and real space, and correct for it up to 1-loop order



$$I: \frac{d\tau}{dl} = \alpha g^2$$

$$II: \frac{dg}{dl} = (2 - 2/\tau)g$$

$$t = t_0 e^l$$

This generates RG flow equations in real-time. To predict the dynamic behavior, we fix non-universal constants and time scales, and integrate the flow equations in time.



Bose gas in 2D: Kosterlitz-Thouless transition

Quasi-Order $G(x) = \langle \psi^+(x)\psi(0) \rangle \sim |x|^{-\frac{\tau}{4}}$ $\tau^{-1} \approx \frac{\pi\rho}{2mT} + C$

Prokof'ev, Ruebenacker, Svistunov, '00, '01, '02

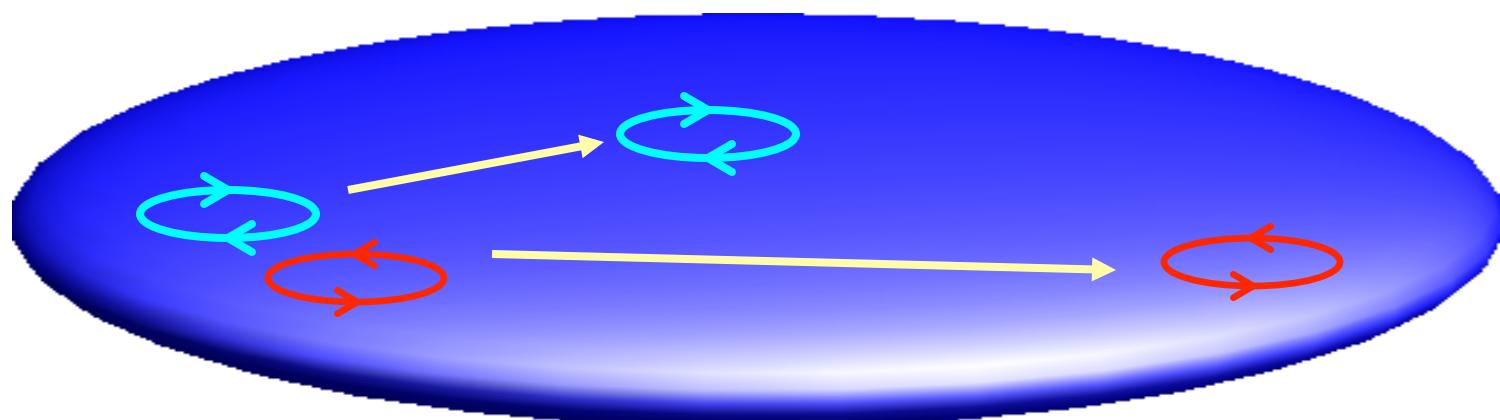
Kosterlitz-Thouless transition



driven by vortex unbinding

Disordered phase

$$G(x) = \langle \psi^+(x)\psi(0) \rangle \sim \exp(-|x|/x_0)$$

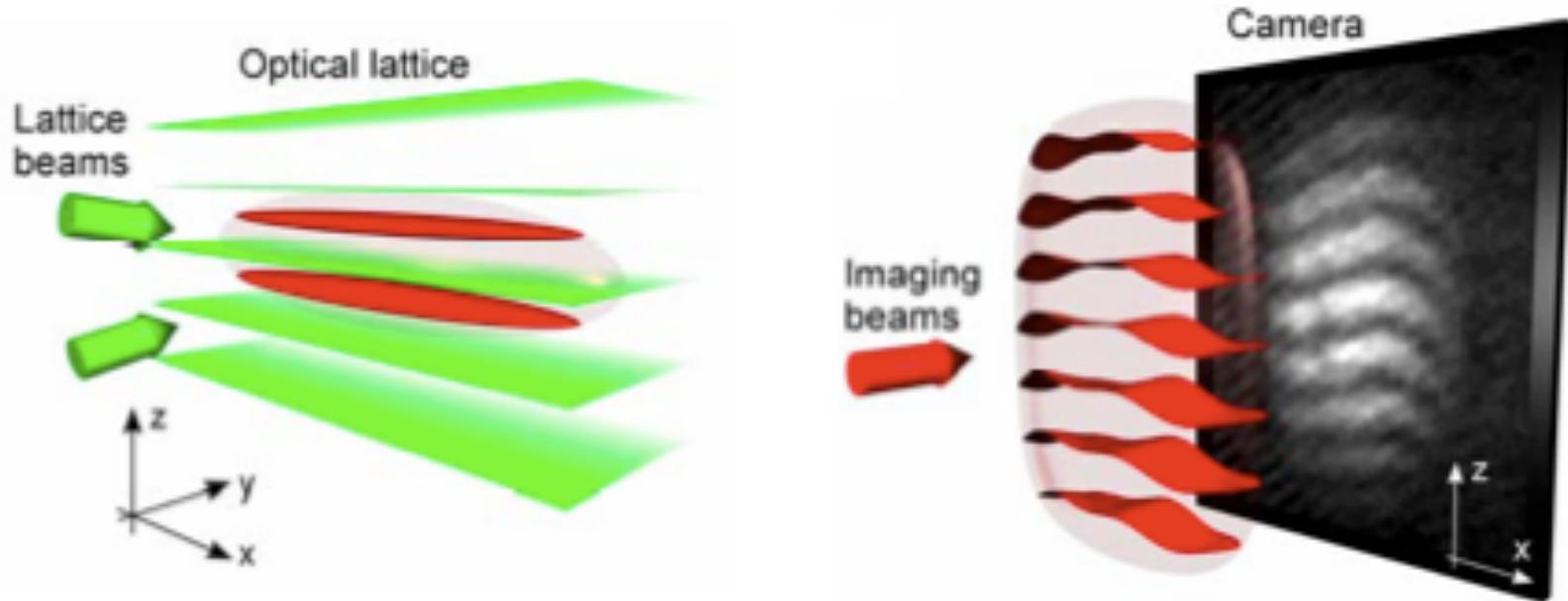




Interfering 2D Bose gases

- Realization of quasi-condensates and KT transition
- algebraic → exponential scaling. Vortices

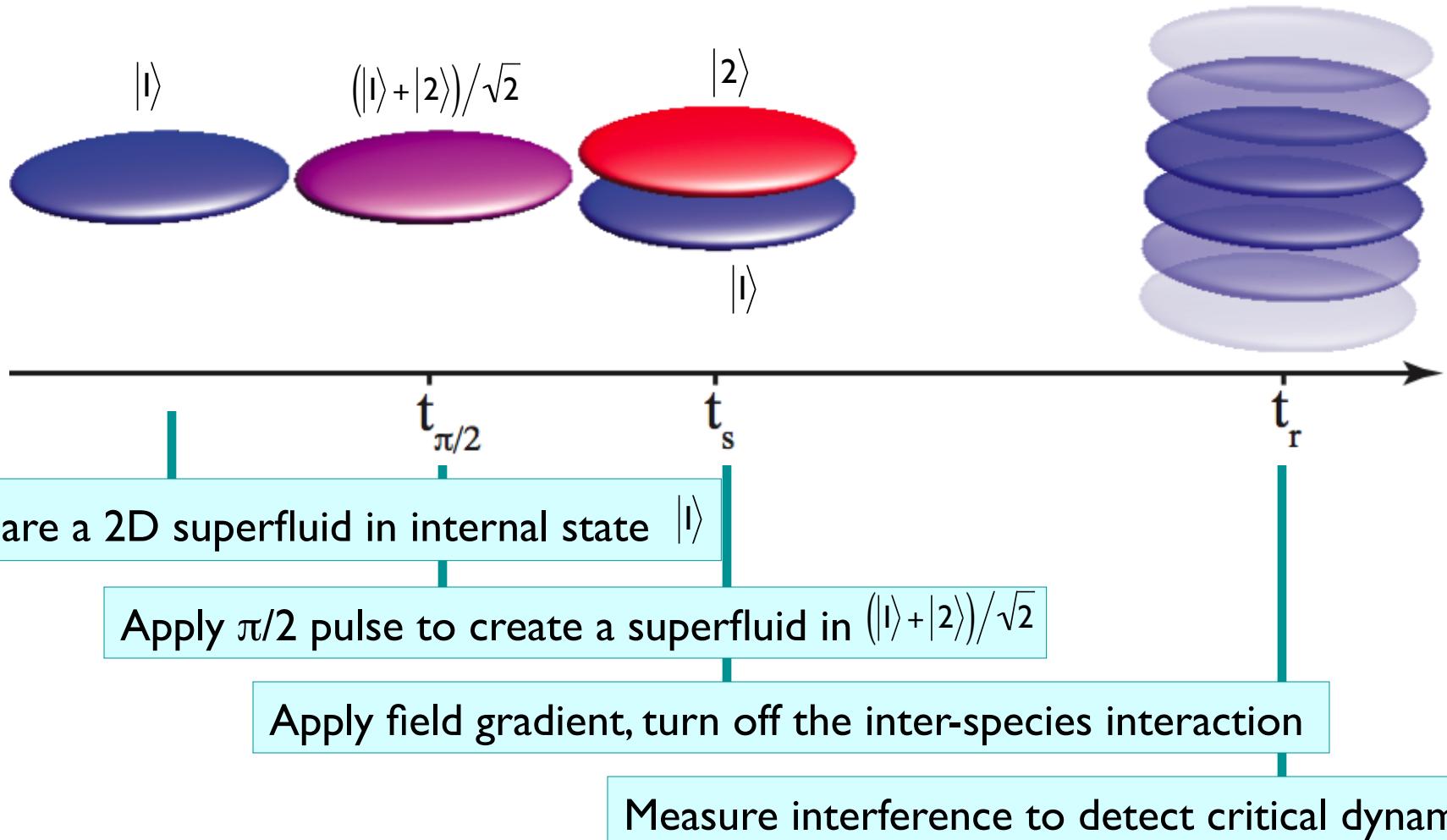
Dalibard group, Phillips/Helmerson group, Chin group, Jin group, ...



Review: Z. Hadzibabic, J. Dalibard, Rivisto del Nuovo Cimento, 34, 389 (2011)



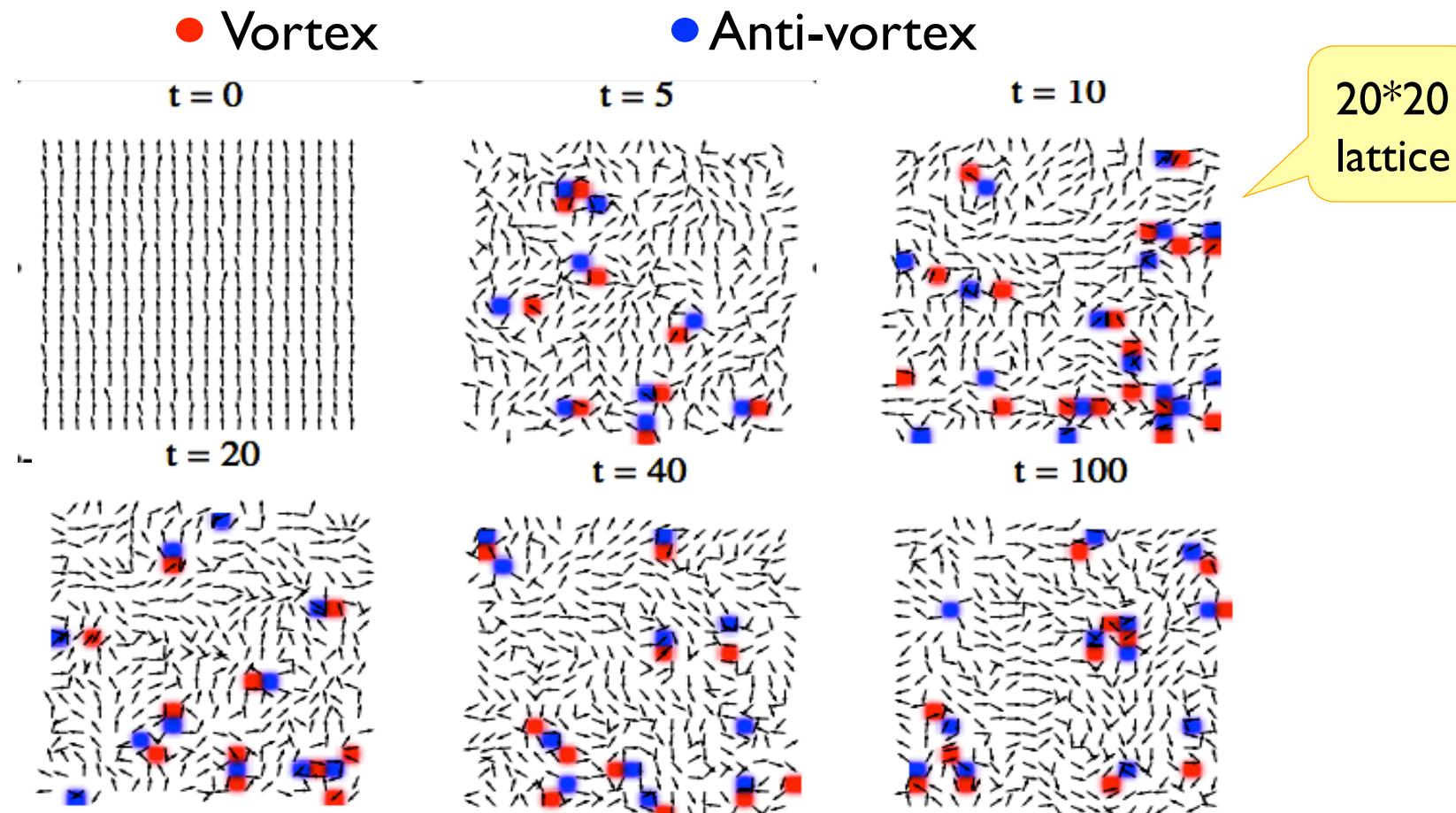
Preparation and measurement sequence





Numerical Approach: Truncated Wigner approximation

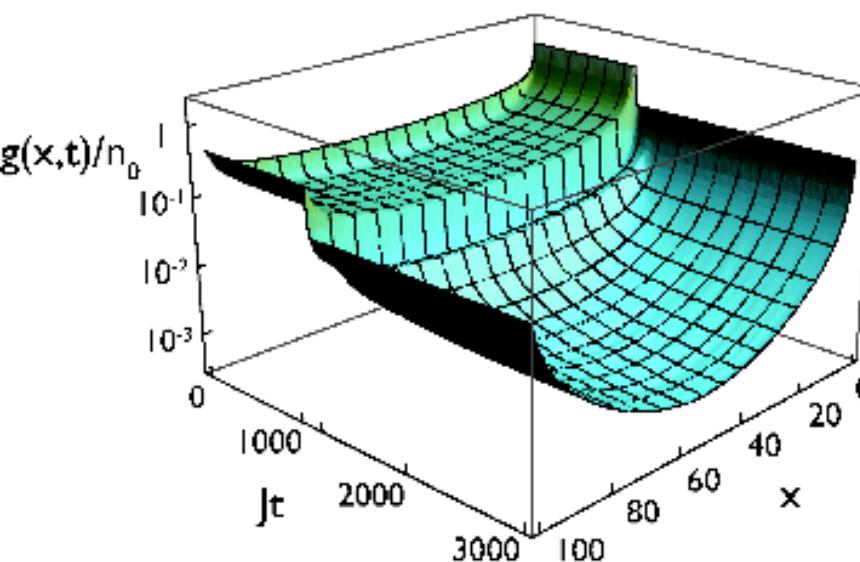
Sample over many Gross-Pitaevskii solutions according to the Wigner distribution of the initial state.
Includes quantum and thermal fluctuations.



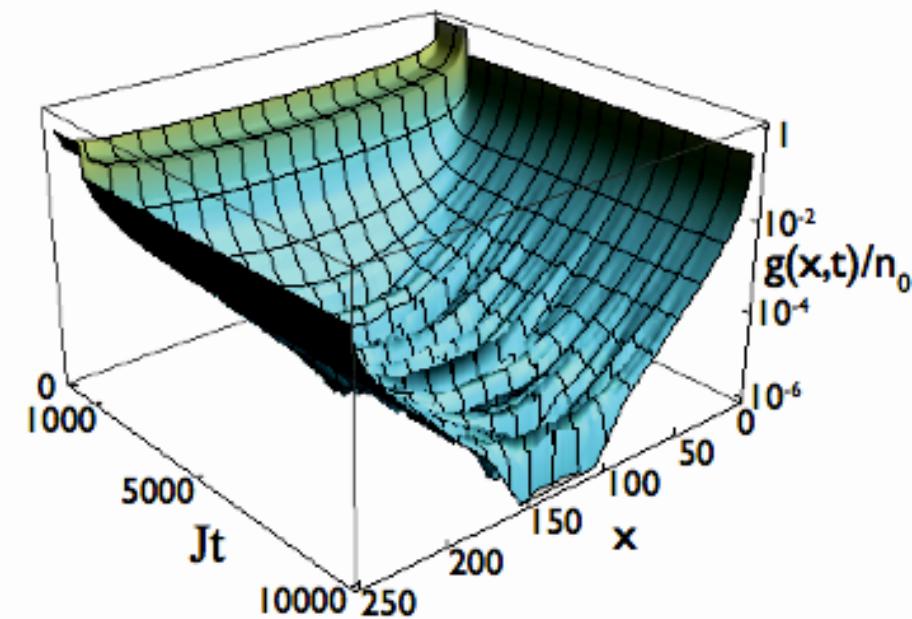
Phys. Rev.A 80, 041601(R) (2009); 81, 033605 (2010)
A. Polkovnikov, Annals of Phys. 325, 1790 (2010)



Dynamics of $G(x,t)$ on different time scales



On intermediate scales, a metastable supercritical state with algebraic scaling emerges.



On a very long time scale, the scaling changes from algebraic to exponential
→ dynamic KT transition!

$$\hbar/J = 0.3\text{ms} \quad \rho_0 = 50 / \mu\text{m}^2 \quad l = 0.3\mu\text{m} \quad {}^{87}\text{Rb}$$



Quantifying the change of functional form

We fit $g(x,t)$ with an algebraic and an exponential fitting function:

Algebraic: $\sim |x|^{-\tau/4}$

$$f_a(x) = C \left(|\sin(\pi x / L)| L / \pi \right)^{-\tau/4}$$

Exponential: $\sim \exp(-|x|/x_o)$

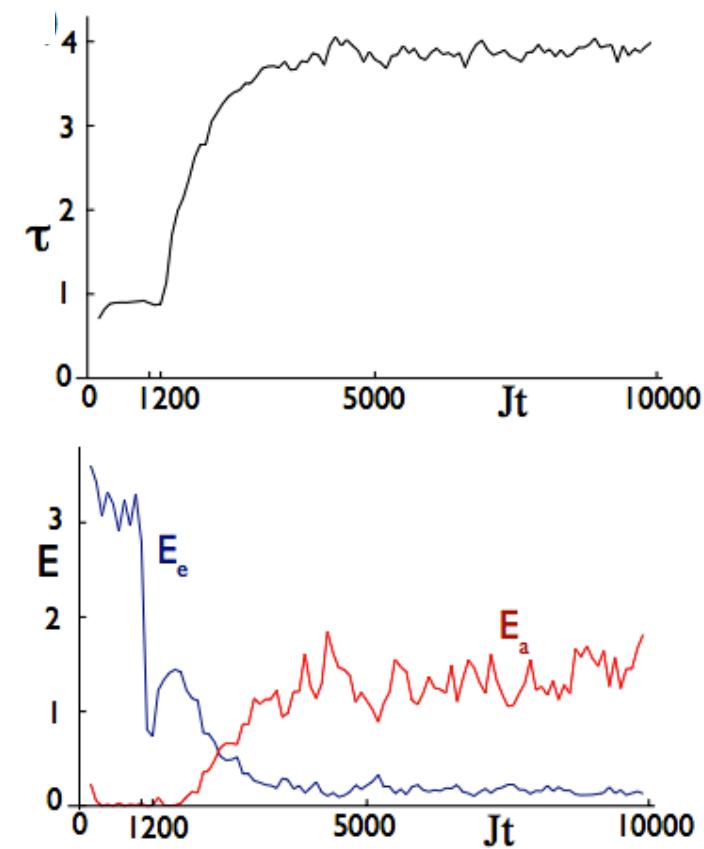
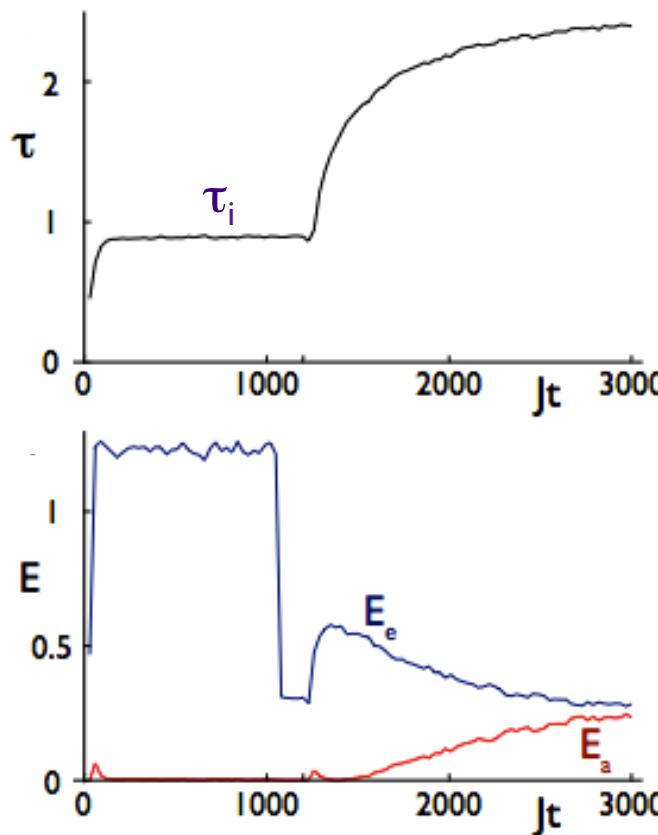
$$f_e(x) = C \exp(-|\sin(\pi x / L)| / x_o)$$

We define the two fitting errors:

$$E_{e,a}(t) = \sum (g(x,t) - f_{e,a}(x))^2$$



Fitting results

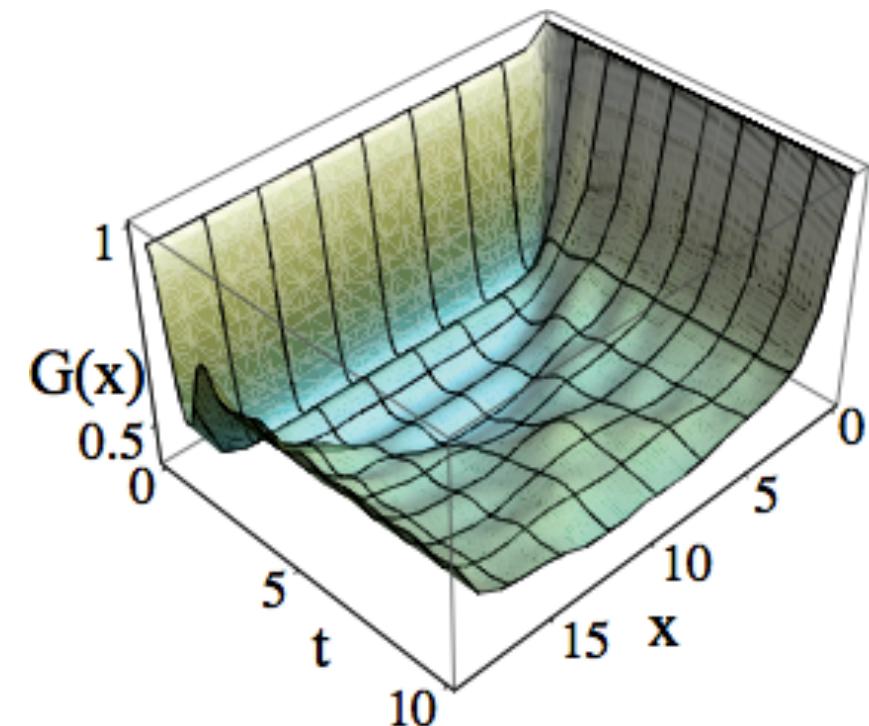
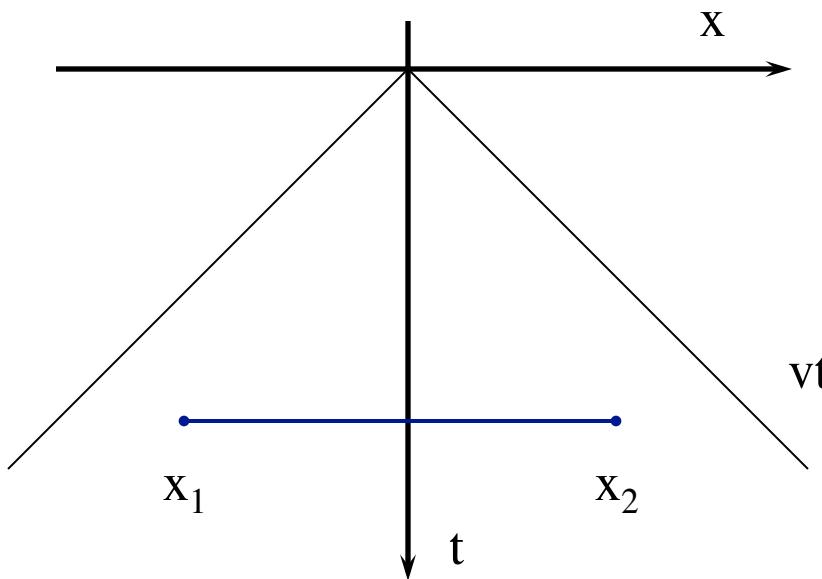


- The system relaxes to a steady state with exponent τ_i
- A supercritical superfluid state is observed
- The relaxation to the disordered groundstate is slowed down critically



Light cone dynamics

Dynamics separates $G(x,t)$ into connected and disconnected part.

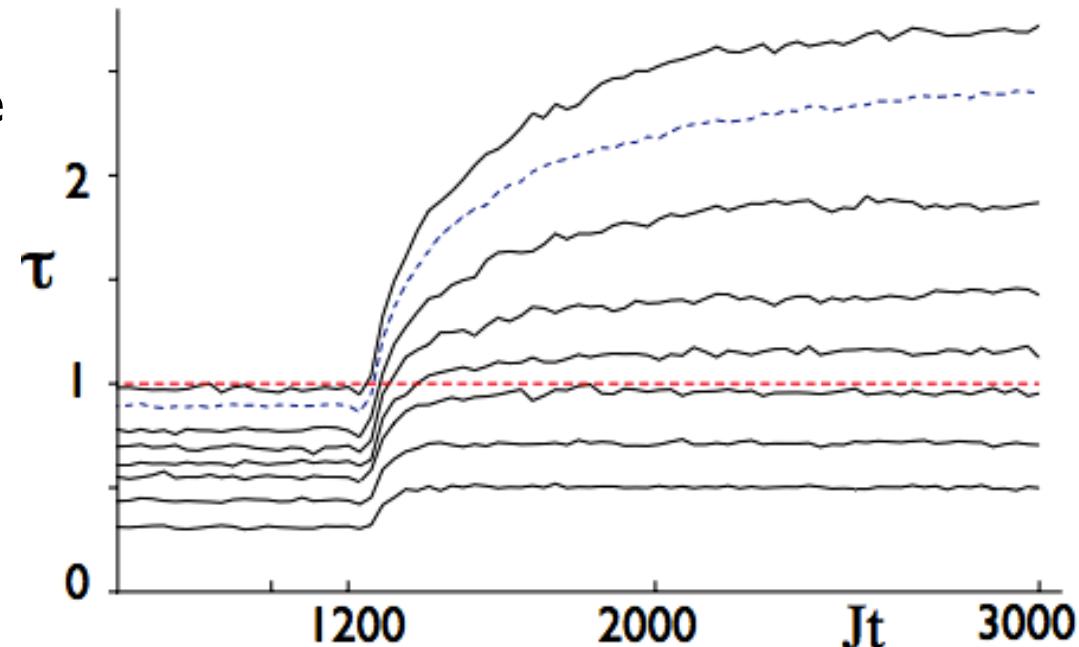


After the quench, $G(x,t)$ is only piece-wise algebraic, thus the fitting error spikes up.



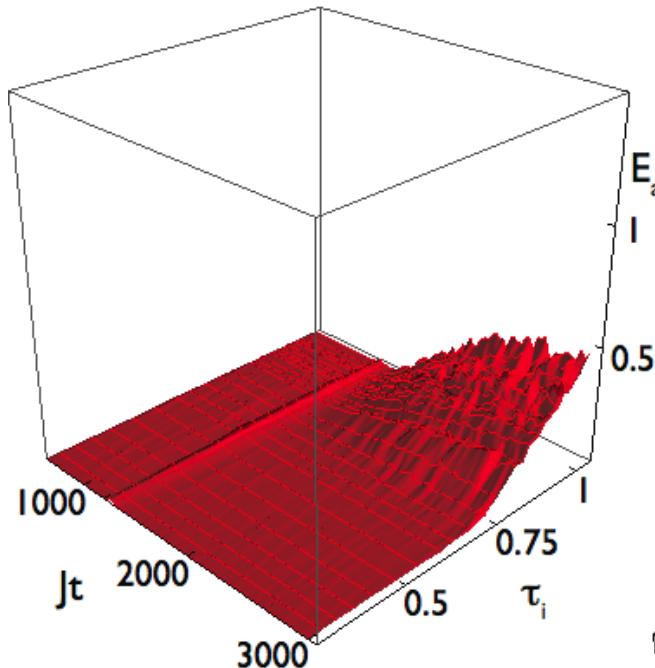
Mapping out the dynamical phase transition

- $\tau(t)$, with τ_i as initial value
- For small τ_i , the system equilibrates at some final $\tau_f > \tau_i$
- For larger τ_i , the system continues to increase, and eventually relaxes to thermal equilibrium via vortex unbinding



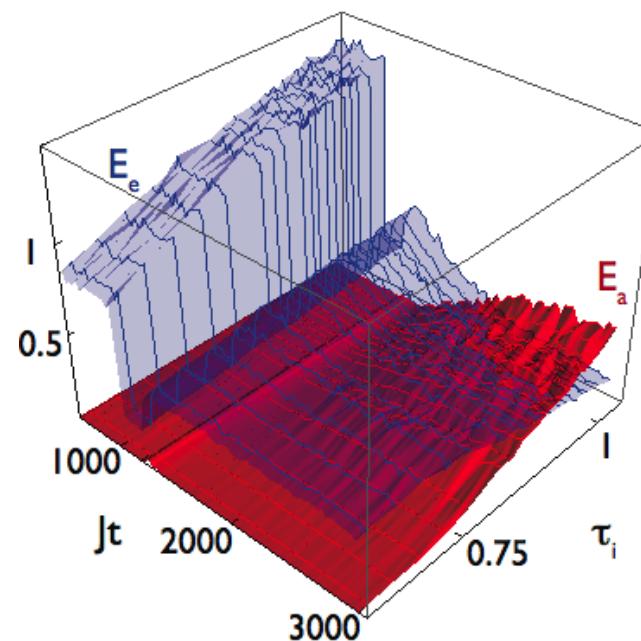


Errors indicate change of functional form

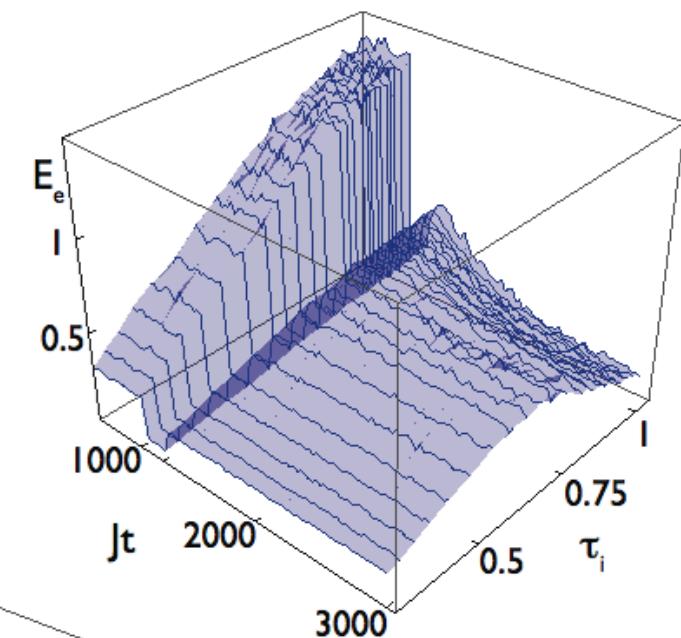


Algebraic error

Dynamic
KT transition!



Exponential
error





Scaling exponent of the metastable state

- The spectrum separates into total and relative density+phase fluctuations

$$\psi_{1,2} \sim (n_0/2 + \delta n_{1,2})^{1/2} \exp(i\phi_{1,2})$$

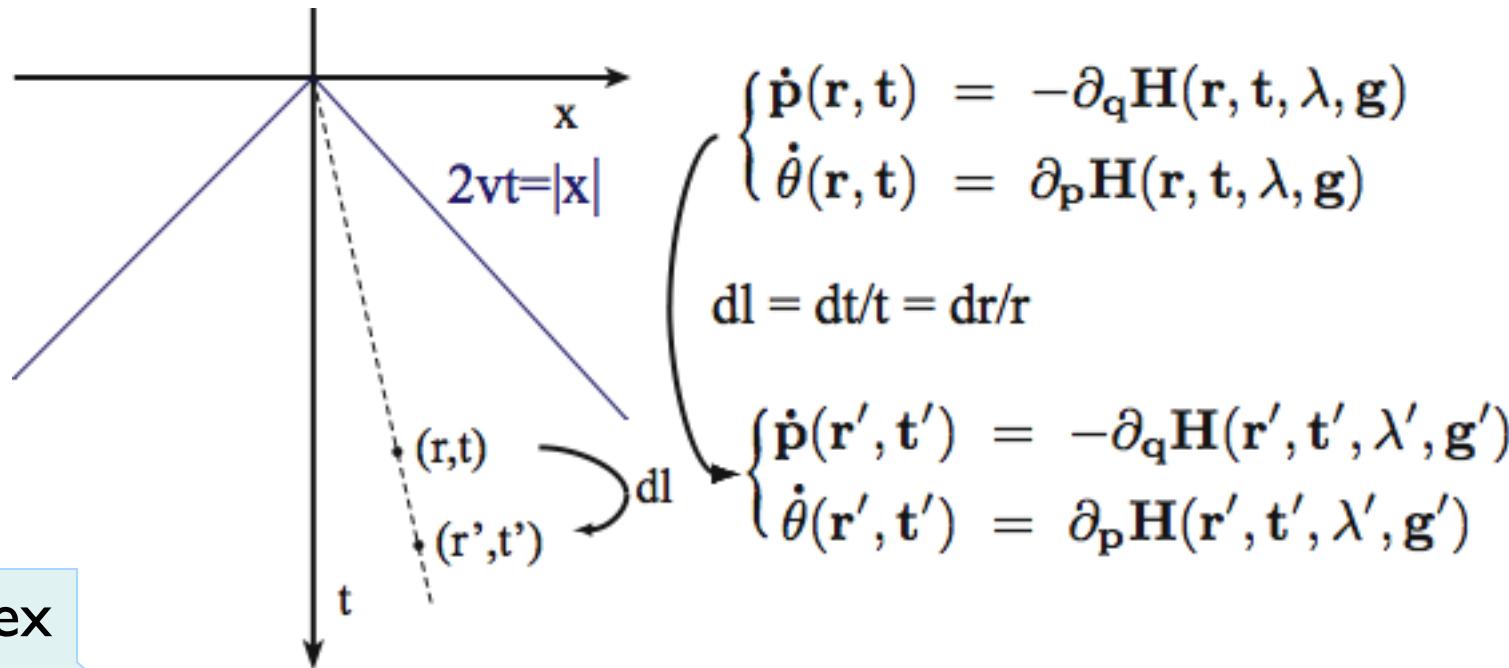
- Symmetric and anti-symmetric sector equilibrate on a time scale shorter than the vortex unbinding scale.
- For $T \gg U n_0/2$, the total energy scales as T^2
- So $T_f = T_i / \sqrt{2}$
- But $\tau^{-1} \approx \frac{\pi\rho}{2mT} + C$, so

$$\tau_f = \left(\frac{1}{\sqrt{2}\tau_i} + D \right)^{-1}$$

For 1D gases, sym and anti-sym sector stay out-of-equilibrium for a much longer time,
see T. Kitagawa, et al., NJP 2011



Dynamics as a renormalization group process



g : vortex
fugacity

$$I: \frac{d\tau}{dl} = \alpha g^2$$

$$II: \frac{dg}{dl} = (2 - 2/\tau)g$$

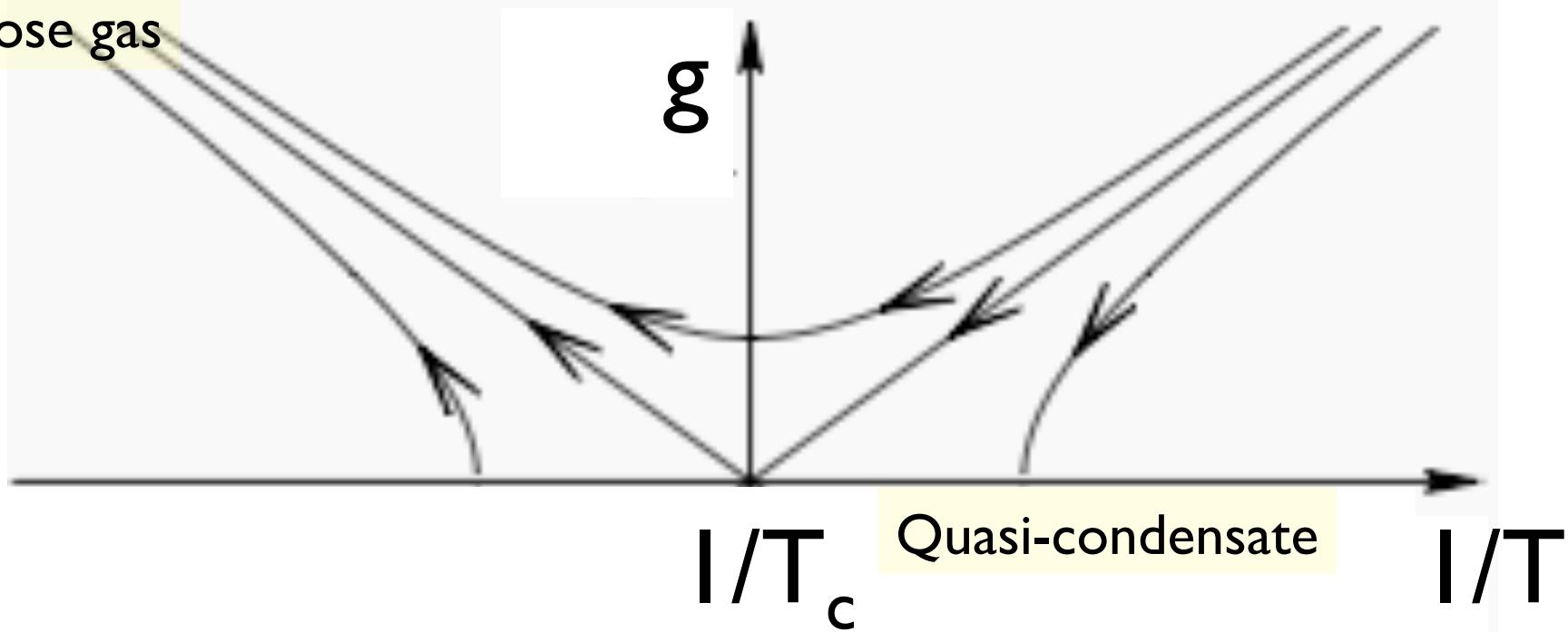
$$t = t_0 e^l$$

→ **Dynamic KT transition**



Renormalization group flow vs. dynamics

Thermal
Bose gas

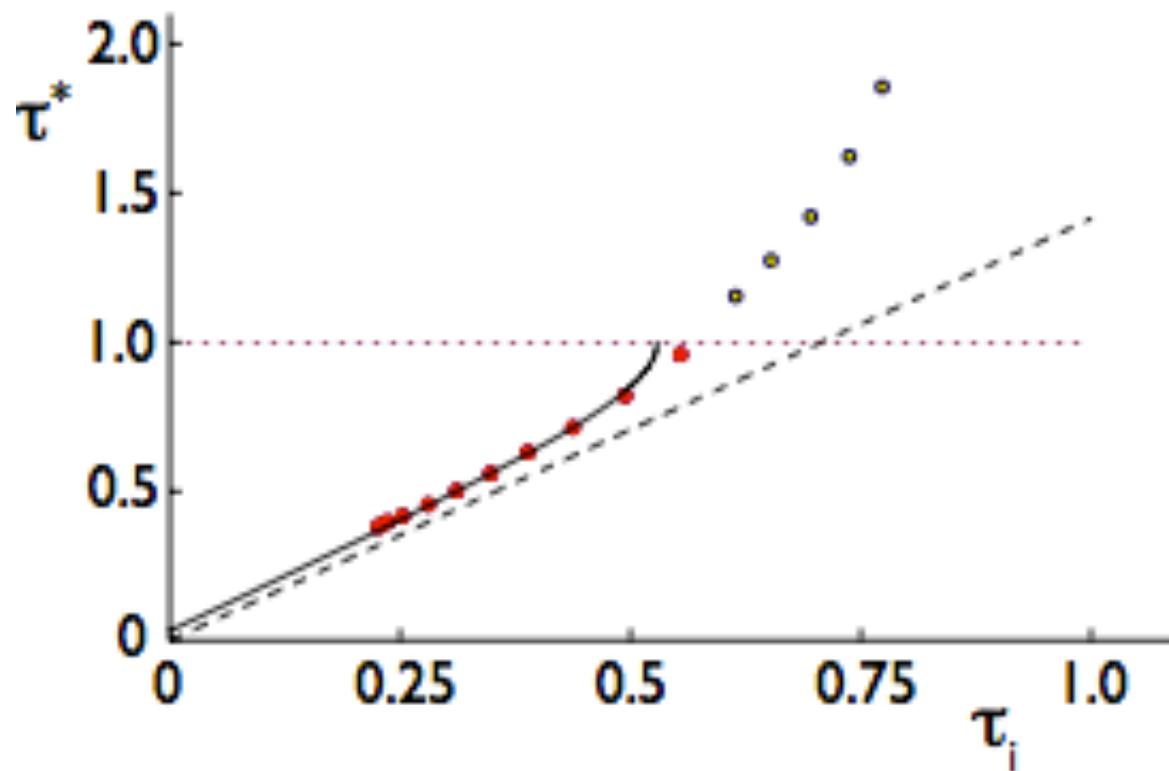


Dynamics resemble the RG flow of the equilibrium system



Subcritical regime, fixing $\alpha g^2(0)$

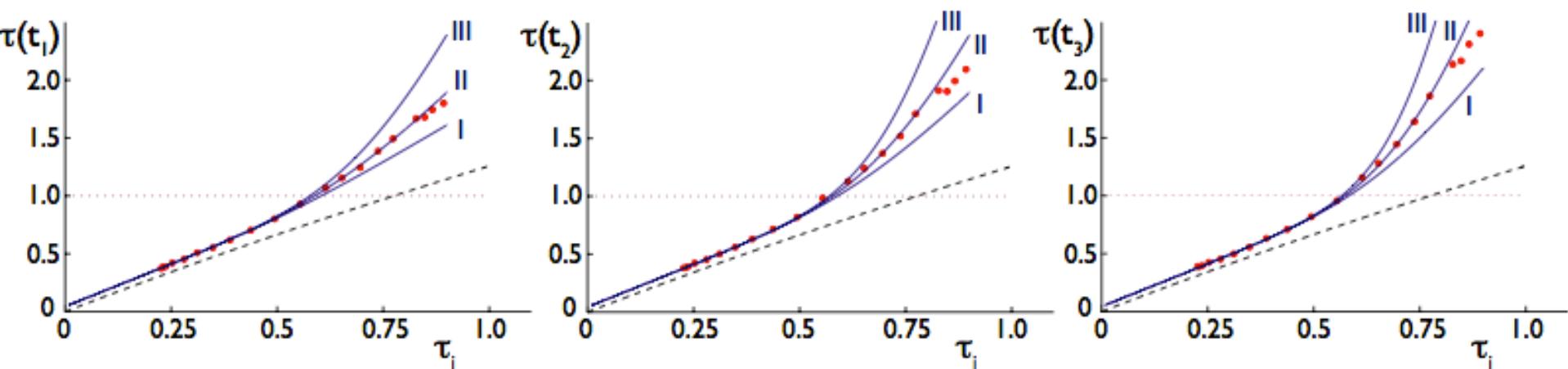
- We write $\frac{d(\alpha g^2)}{dl} = (4 - 4/\tau)(\alpha g^2)$
- By using the asymptotic form of τ , and fitting it to the subcritical data, one can extract $\alpha g^2(0)$ and C .





Predicting critical dynamics via real-time RG

- We integrate the flow equations and find agreement with the numerics.



Red: τ data at times $t_1 = 300, t_2 = 1200, t_3 = 1800$.

Blue: RG prediction; 'II' is the correct prediction, 'I' and 'III' are two near-by solutions for visible comparison.

Analytical description of critical dynamics!



Time scales of vortex unbinding

From RG, we derive the time scale

Near criticality:

$$t^* \sim \exp\left(\frac{\exp(-E_c / 2T)}{\sqrt{1 - T / T_c}}\right)$$

Away from criticality:

$$t^* \sim \exp\left(\frac{E_c}{T - T_c}\right)$$

→ Exponential increase at criticality

The energy range of suppression is given by the vortex core energy E_c



Conclusion

- Critical dynamics can be described using a novel RG approach
- Realistic, experimental proposal to create a dynamic Kosterlitz-Thouless transition of 2D superfluids
- Metastable, supercritical state emerges
- Dynamical vortex unbinding (Reverse Kibble-Zurek effect)