Highly polarized limit of the quasi-2D Fermi gas

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Collaboration with
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Fermionic mixtures

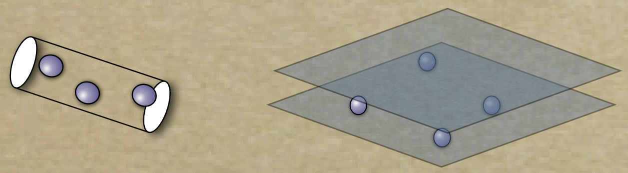
A gas of fermionic atoms

- *Either two hyperfine states of the same atom*
- *or two different atomic species (e.g. K & Li)*

Questions to address in these systems

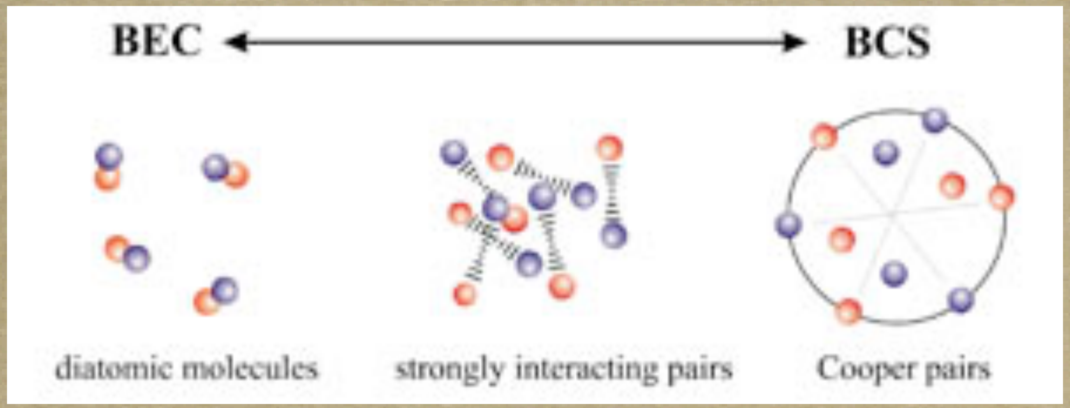
- *The role of interactions in determining many-body properties*

- *Dimensionality?*



- *Which phases can be obtained? (superfluid, phase separation, FFLO, ferromagnetic....)*

An equal mixture of the two components changes smoothly from a BCS to a BEC type superfluid



Outline

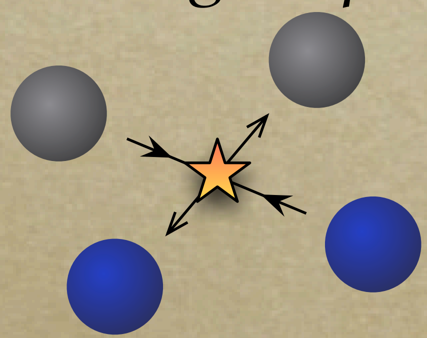
- Motivation: Quasi-2D vs 2D
- Highly polarized quasi-2D Fermi gas:
  - Polaron-molecule transition in quasi-2D
  - 2D-3D crossover
  - Upper branch

Part I: The quasi-2D Fermi gas

Atomic scattering

Consider two species with contact interactions:

s-wave scattering amplitude: $f(q) = -\frac{1}{a_s^{-1} + iq}$ $\quad a_s$ *=3D scattering length*



$$f(q) = \frac{2\pi}{\ln\left[1/(qa_{2\mathrm{D}})\right] + i\pi/2}$$

a_{2D} *=2D scattering length*

3D: Bound state when $a_s > 0$

2D scattering *always* allows a bound state

Strong interactions in the many-body system:

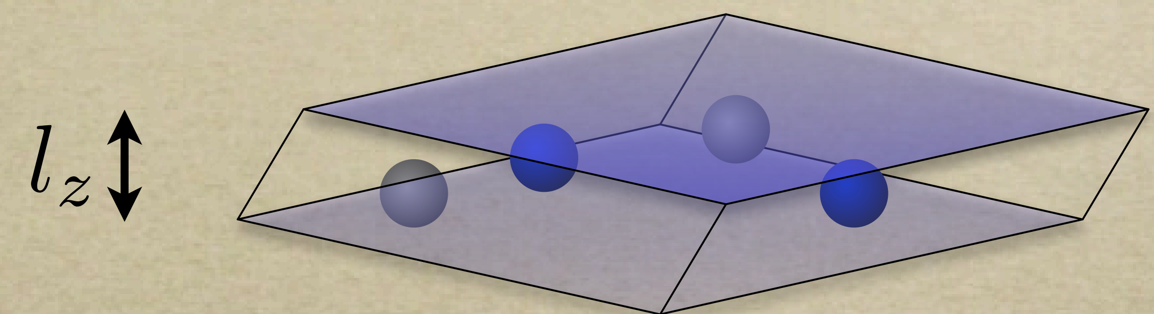
$|k_F a_s| \gg 1$ *(3D) or* $k_F a_{2D} \approx 1$ *(2D)*

- *Single interaction parameter in 2D:* ϵ_b/ϵ_F *or* $\log(k_F a_{2D})$

Quasi-2D

- Atoms confined to the plane by approximately harmonic confinement

$$V(z) = \frac{1}{2} m\omega_z^2 z^2$$



If $k_B T \ll \epsilon_F \ll \hbar\omega_z$ *the gas is quantum degenerate and collisions can be considered quasi-2D*
(transverse degrees of motion are frozen out)

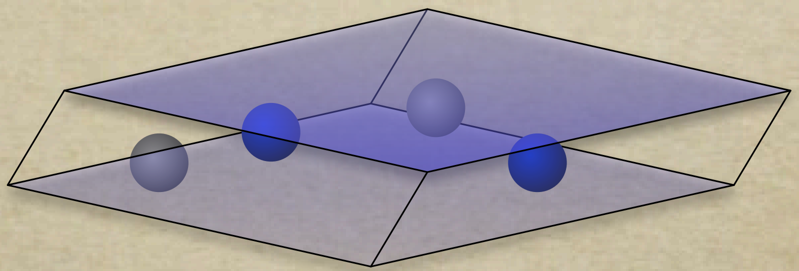
However, the binding energy of dimer is another energy scale

Quasi-2D

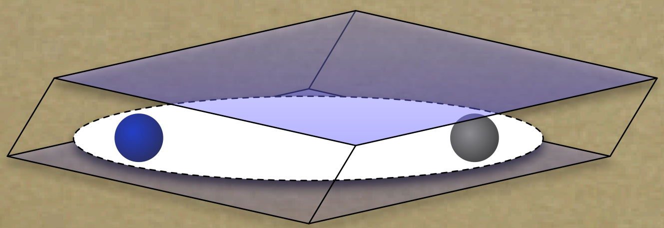
Scattering of two atoms in the H.O. ground state:

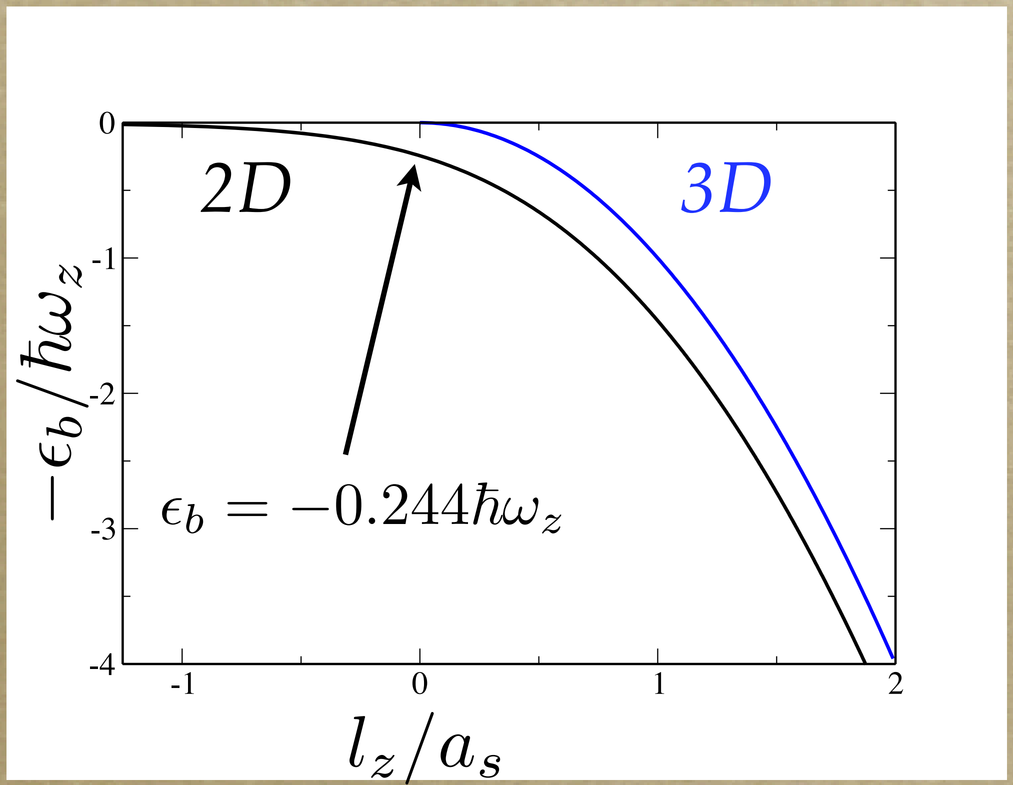
$$f_{00}(\epsilon) = \frac{2\sqrt{2\pi}}{l_z/a_s - \mathcal{F}(-\epsilon/\omega_z)}$$

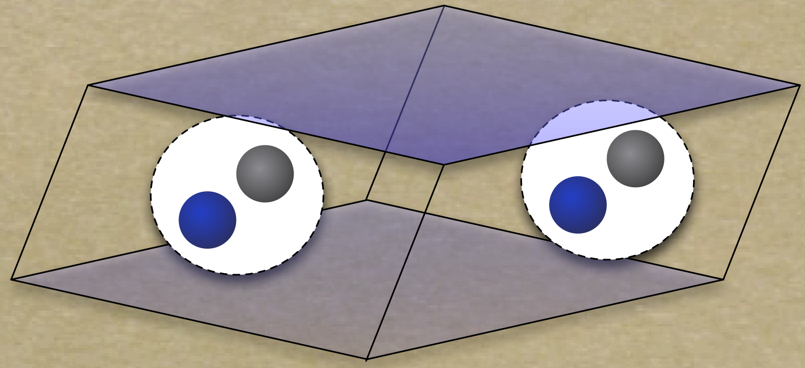
$$\mathcal{F}(x) = \int_0^\infty \frac{du}{\sqrt{4\pi u^3}} \left(1 - \frac{e^{-xu}}{\sqrt{[1-\exp(-2u)]/2u}}\right)$$



(Theory: Petrov & Shlyapnikov PRA 2001, Bloch, Dalibard, Zwerger RMP 2008)







Comparing theory and experiment

Often used to match theory and experiment:

$$f(q) = \frac{2\pi}{\ln\left[1/(q a_{2\mathrm{D}})\right] + i\pi/2}$$

$$\cancel{\epsilon_b = \frac{1}{m a_{2D}^2}}$$ *Only valid when* $\epsilon_b \ll \omega_z$

Instead use the quasi-2D scattering amplitude.
Low energy expansion:

$$\mathcal{F}(x) \approx \frac{1}{\sqrt{2\pi}} \ln\left(\pi x/B\right) + \frac{\ln 2}{\sqrt{2\pi}} x + \mathcal{O}(x^2)$$

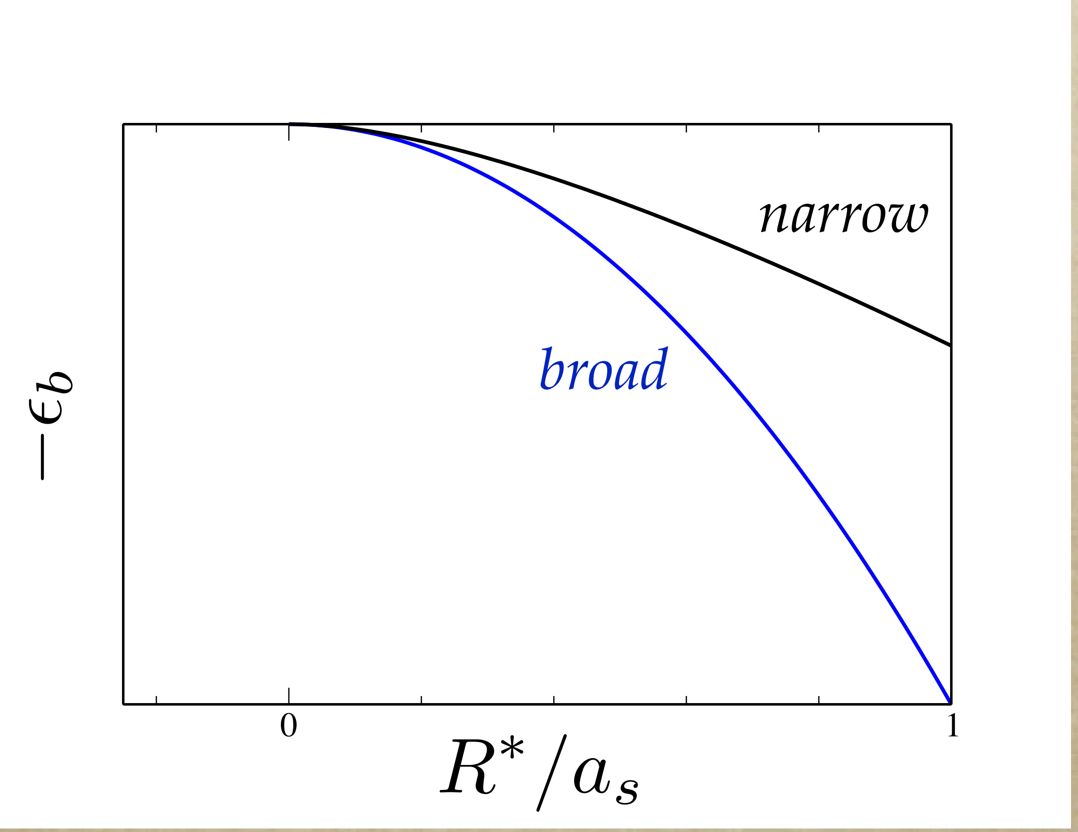
(Theory: Petrov & Shlyapnikov PRA 2001, Bloch, Dalibard, Zwerger RMP 2008)

Aside: 3D narrow Feshbach resonance

Narrow resonance characterized by narrow magnetic field width and by a large effective range R^*

$$\epsilon_b = \frac{1}{m a_s^2} \qquad \textit{broad}$$

$$\epsilon_b = \frac{(\sqrt{1+4R^*/a_s}-1)^2}{4mR^{*2}} \qquad \textit{narrow}$$



In quasi-2D the role of "effective range" is played by the confinement length

- *This analogy can be formalized by use of a two-channel model*
- *Captures few-body physics but misses many-body contributions*

2D vs Quasi-2D

Do theories of the 2D Fermi gas quantitatively describe current quasi-2D experiments?

Results from our studies of the highly polarized Fermi gas

Lower branch:

- *Quasi-2D nature important*

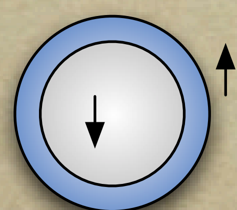
Upper branch:

- *May be described by 2D theory provided correct definition of* a_{2D} *is used*

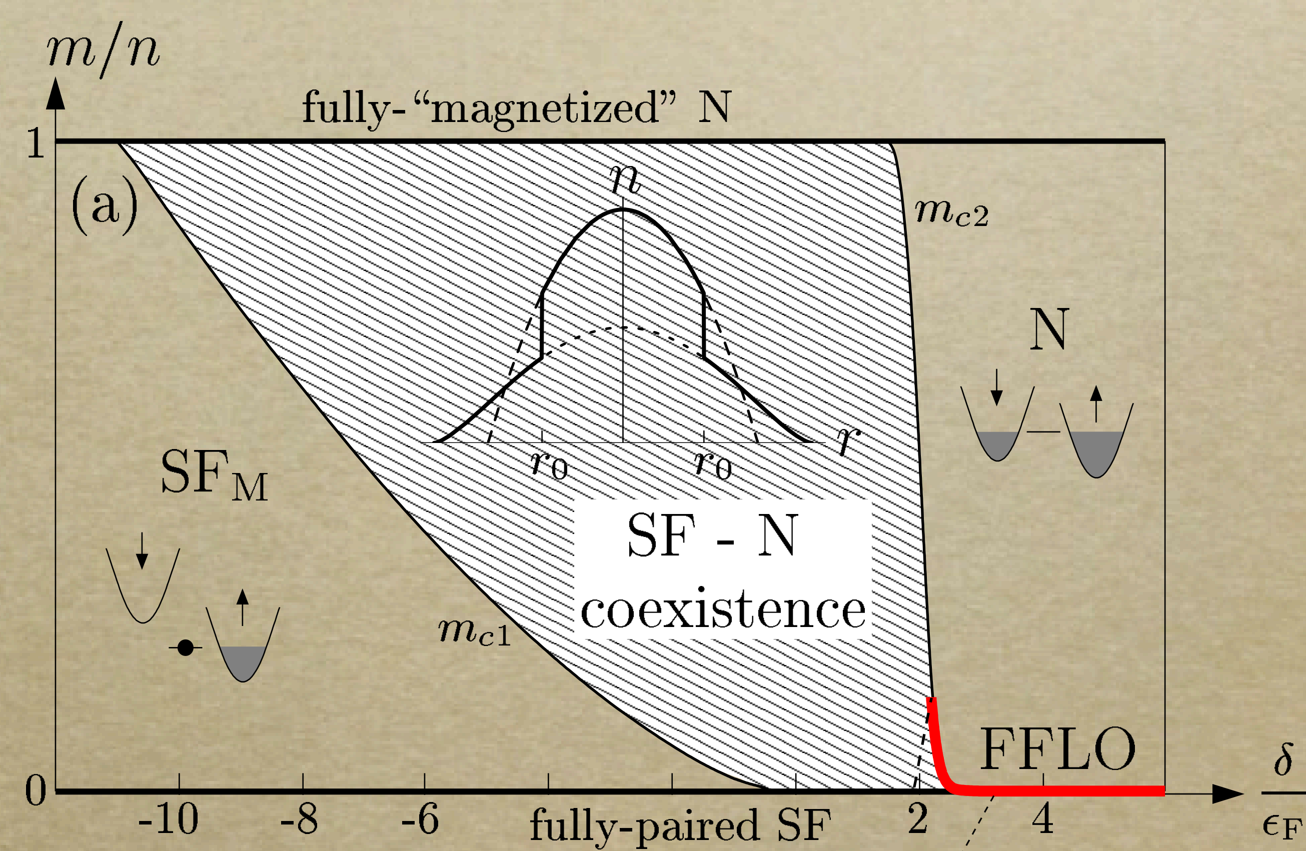
Part II: Highly polarized Fermi gas

Polarized Fermi gases

- *Introduce a density imbalance between the two components. What changes?*

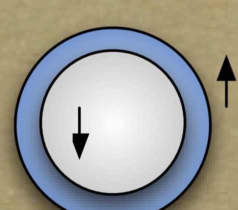
Frustration: 

Possibility of exotic pairing phenomena, such as FFLO



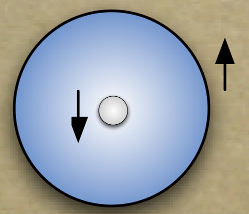
Sheehy & Radzihovsky, PRL 2006

Theoretical approach:



Single impurity in a Fermi gas

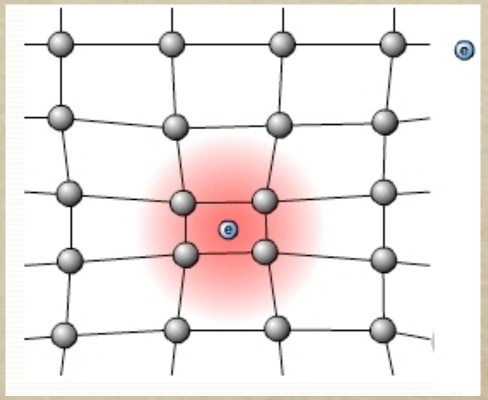
- Gives insight into the phase diagram of a polarized Fermi gas
- Tests our understanding of strongly correlated physics



Experiments:
Köhl (Cambridge): ^{40}K in 2D
Zwierlein (MIT): ^{6}Li, ^{40}K in 3D
Hulet (Rice): ^{6}Li (3D)
Grimm (Innsbruck): ^{40}K-^{6}Li mixture in 3D
Salomon (ENS): ^{6}Li (3D)
Thomas (North Carolina State): ^{6}Li (quasi-2D)
....

The Fermi polaron

A particle immersed in a quantum many-body system, such as an electron in the crystal lattice of a solid, will move in a cloud of excitations of its environment. The many-body system modifies the physical properties of the particle (mass, charge, etc.).

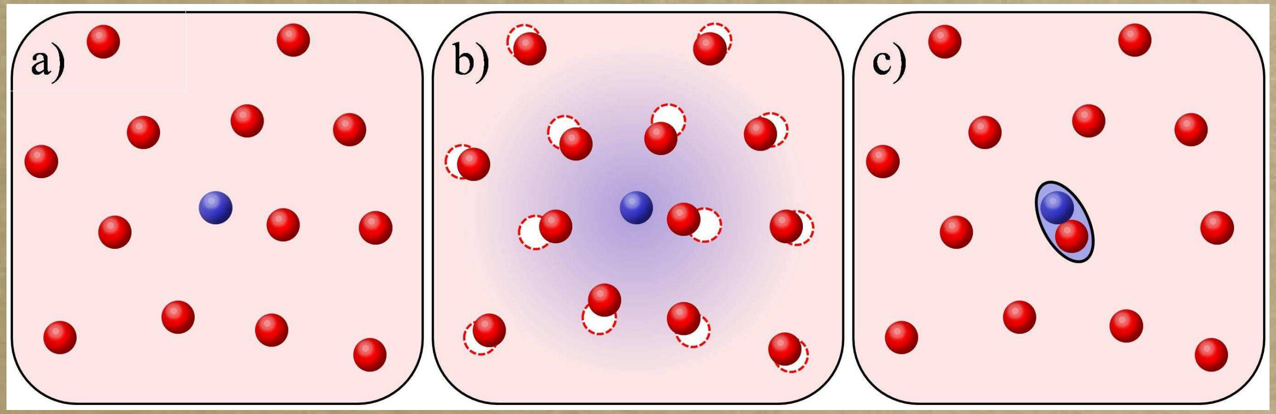


The standard picture of a Fermi polaron:

a) For weak interactions, the impurity moves freely through the Fermi sea

b) As the interactions are increased, the impurity attracts surrounding majority atoms

c) For strong attractive interaction the impurity will bind one majority atom to form a molecule



Variational description of an impurity in a Fermi gas

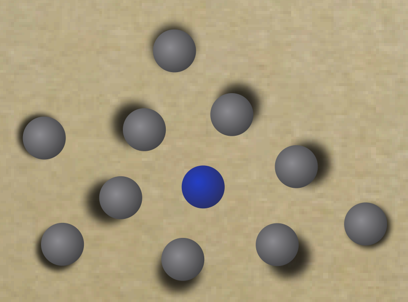
Dressing of impurity by one particle-hole excitation

Chevy, Lobo, Recati, Combescot...
Svistunov & Prokof'ev
MIT, ENS, Innsbruck, Cambridge

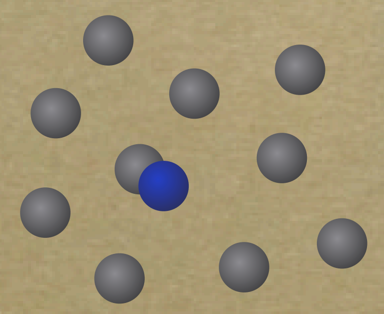
- Has been shown to give good estimates in 3D
- Approximate cancellation of contributions from terms with 2 or more particle hole pairs

Combescot, Giraud PRL 2008

Polaron:

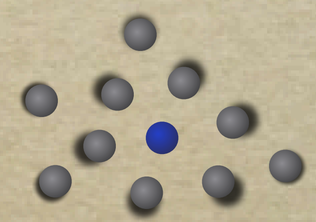


Molecule:

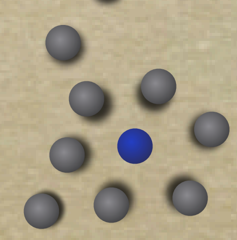


Two solutions:

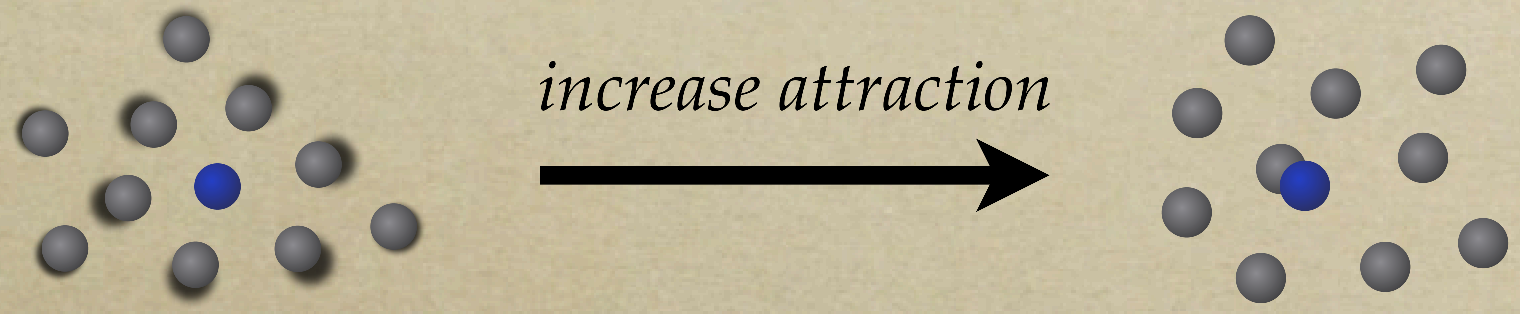
- *Attractive polaron*



- *Repulsive polaron*



Polaron-molecule transition



- 2D prediction: $\epsilon_b/\epsilon_F \approx 10$ *Parish PRA(R) 2011*

This transition has recently been observed in the group of M. Köhl *Koschorreck et al, arXiv:1203.1009*

- Parameters of experiment:

$$\epsilon_F = h \times 10\text{kHz}$$

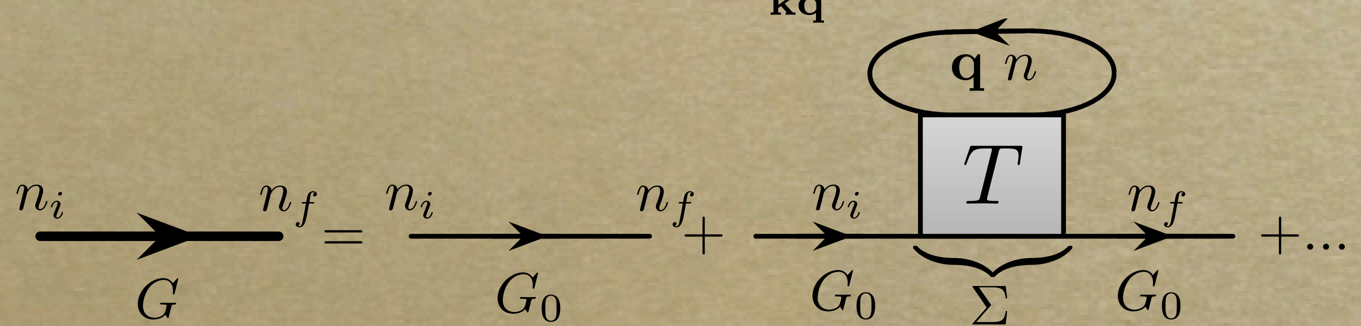
$$\omega_z = 2\pi \times 80\text{kHz}$$

Is the 2D approximation valid for these parameters?

Polaron in quasi-2D

- The presence of the Fermi sea couples center of mass and relative motion
  - *Harmonic oscillator quantum number of the impurity can be changed by interactions with the Fermi sea*
  - *Variational wavefunction for the polaron:*

$$|P\rangle = \sum_{n} \phi_n c^\dagger_{\downarrow \mathbf{0} n} |\mathrm{FS}, N_\uparrow\rangle + \sum_{\substack{nn'm \\ \mathbf{kq}}} \phi^{nn'm}_{\mathbf{kq}} c^\dagger_{\downarrow \mathbf{q}-\mathbf{k} n} c^\dagger_{\uparrow n' \mathbf{k}} c_{\uparrow m \mathbf{q}} |\mathrm{FS}, N_\uparrow\rangle$$



Pietilä, Pekker, Nishida, Demler, PRA 2012

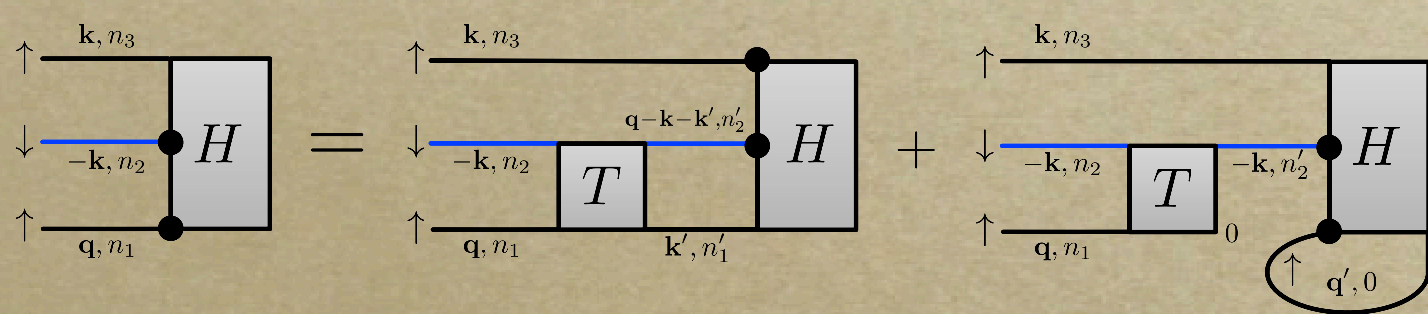
Matrix equation: $G(\mathbf{p},\epsilon) = \left[G_0^{-1}(\mathbf{p},\epsilon) - \Sigma(\mathbf{p},\epsilon)\right]^{-1}$

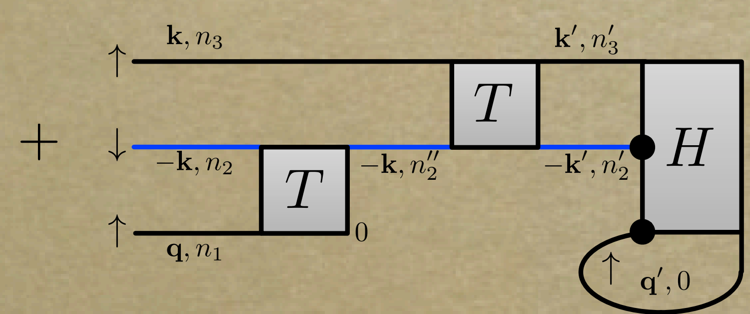
$$\Sigma_{n_1 n_2}(\mathbf{p},\epsilon) = \sum_{\mathbf{q},n} T^{n_1 n}_{n_2 n}(\mathbf{p}+\mathbf{q}, \epsilon + \epsilon_{\mathbf{q}n})\, n_{F\uparrow}(\mathbf{q},n)$$

Molecule in quasi-2D

- Variational wavefunction for the molecule:

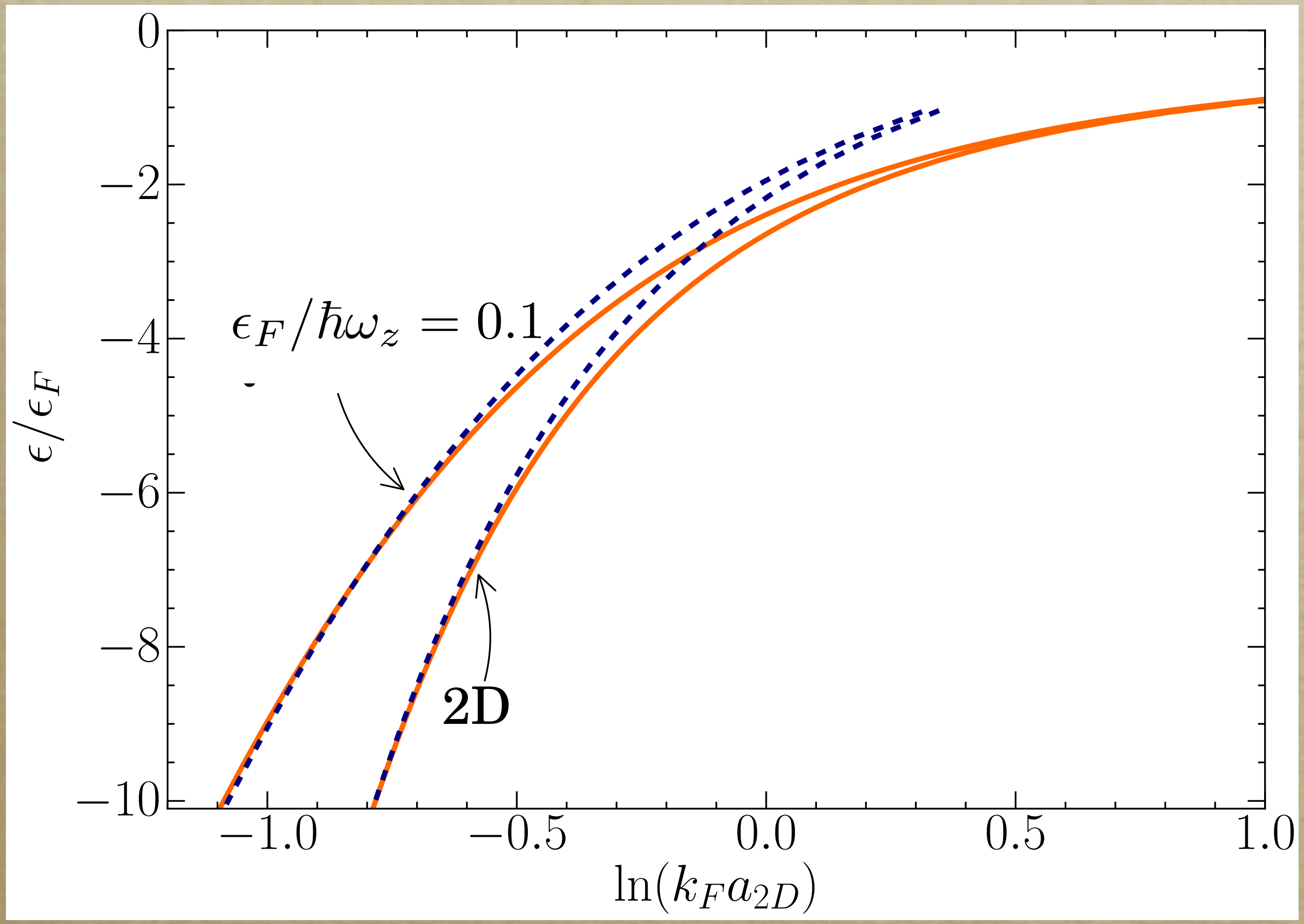
$$|M\rangle = \sum_{nn'\mathbf{k}} \phi_k^{nn'} c^\dagger_{\uparrow\mathbf{k}n} c^\dagger_{\downarrow -\mathbf{k}n'} |\mathrm{FS}, N_\uparrow - 1\rangle + \frac{1}{2} \sum_{\substack{nn'mm' \\ \mathbf{k}\mathbf{k}'\mathbf{q}}} \phi^{nn'mm'}_{\mathbf{k}\mathbf{k}'\mathbf{q}} c^\dagger_{\downarrow \mathbf{q}-\mathbf{k}-\mathbf{k}'n} c^\dagger_{\uparrow\mathbf{k}n'} c^\dagger_{\uparrow\mathbf{k}'m} c_{\uparrow\mathbf{q}m'} |\mathrm{FS}, N_\uparrow - 1\rangle$$

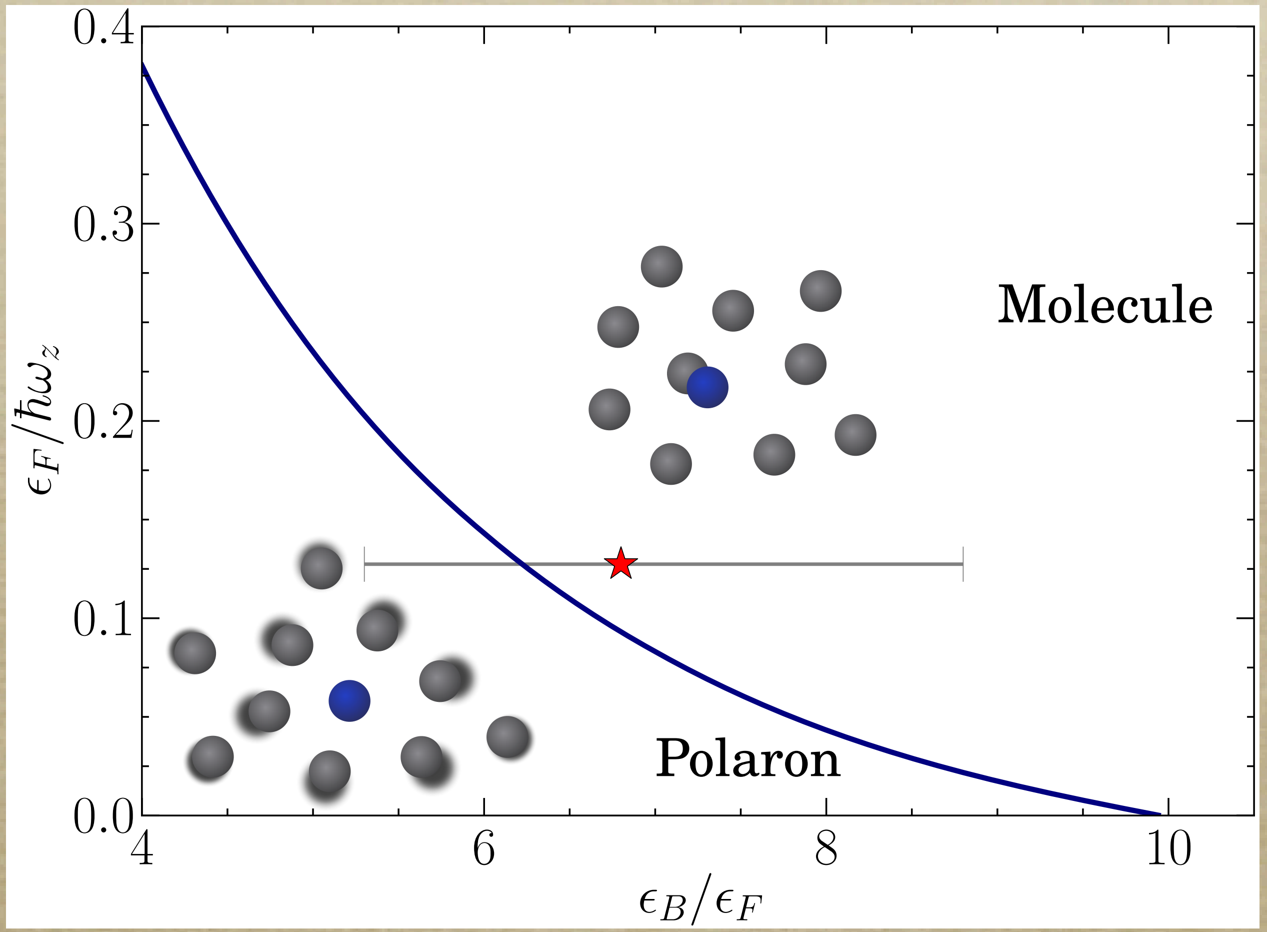




$$\begin{aligned} H^{n_1 n_2 n_3}_{\mathbf{q}\mathbf{k}} = & \sum_{n_1' n_2'} T^{n_1 n_2}_{n_1' n_2'}(\mathbf{q}-\mathbf{k}, \epsilon + \epsilon_{\mathbf{q}0} - \epsilon_{\mathbf{k}n_3})[1 - n_{F\uparrow}(\mathbf{k}, n_3)] \\ & \times \left\{ \sum_{\mathbf{k}'} \frac{H^{n_3 n_2' n_1'}_{\mathbf{q}\mathbf{k}'}}{E^{n_1'+n_2'+n_3}_{\mathbf{k}\mathbf{k}'\mathbf{q}}} [1 - n_{F\uparrow}(\mathbf{k}', n_1')] - \delta_{0n_1'} \sum_{\mathbf{q}'} \frac{H^{0 n_2' n_3}_{\mathbf{q}'\mathbf{k}}}{E^{n_2'+n_3}_{\mathbf{k}}} \right. \\ & \left. + \delta_{0n_1'} \sum_{n_2'' n_3'} \frac{T^{n_2' n_3}_{n_2'' n_3'}(\mathbf{0}, \epsilon)}{E^{n_2'+n_3}_{\mathbf{k}}} \sum_{\mathbf{q}'\mathbf{k}'} \frac{H^{0 n_2'' n_3'}_{\mathbf{q}'\mathbf{k}'}}{E^{n_2''+n_3'}_{\mathbf{k}'}} [1 - n_{F\uparrow}(\mathbf{k}', n_3')] \right\} \quad (9) \end{aligned}$$

Polaron-molecule transition in quasi-2D





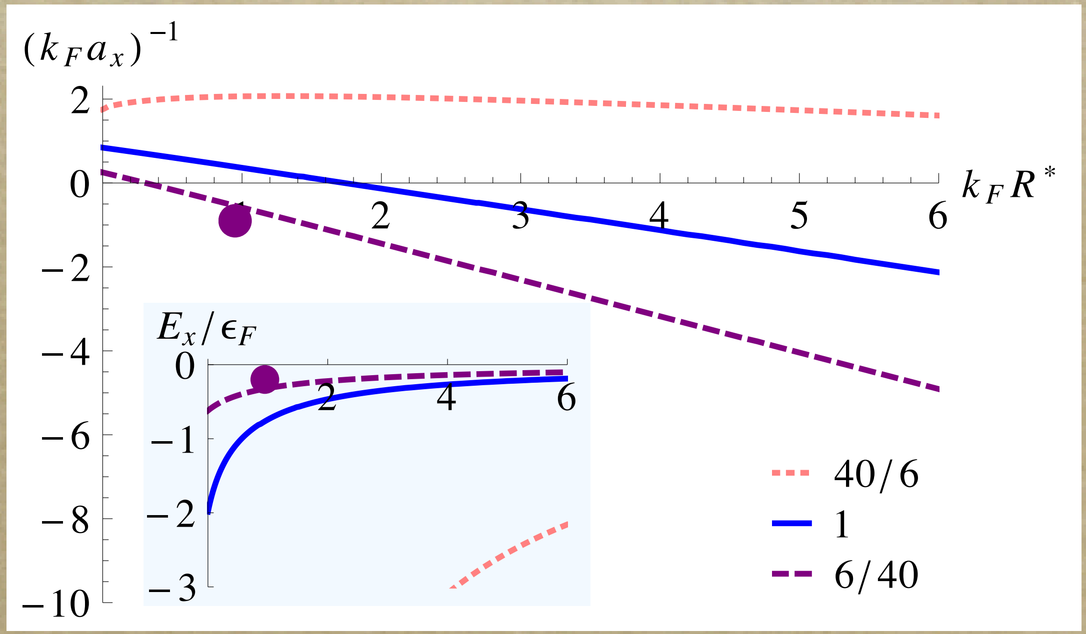
Data: Koschorreck et al, arXiv:1203.1009

- Transition shifted to lower ϵ_b/ϵ_F for increasing density
- Good agreement with experimental result

Analogy with narrow Feshbach resonance

- Narrowness associated with large effective range
  - Polaron-molecule transition recently studied close to narrow FR
  - Large effective range moves transition towards BCS limit, as in quasi-2D

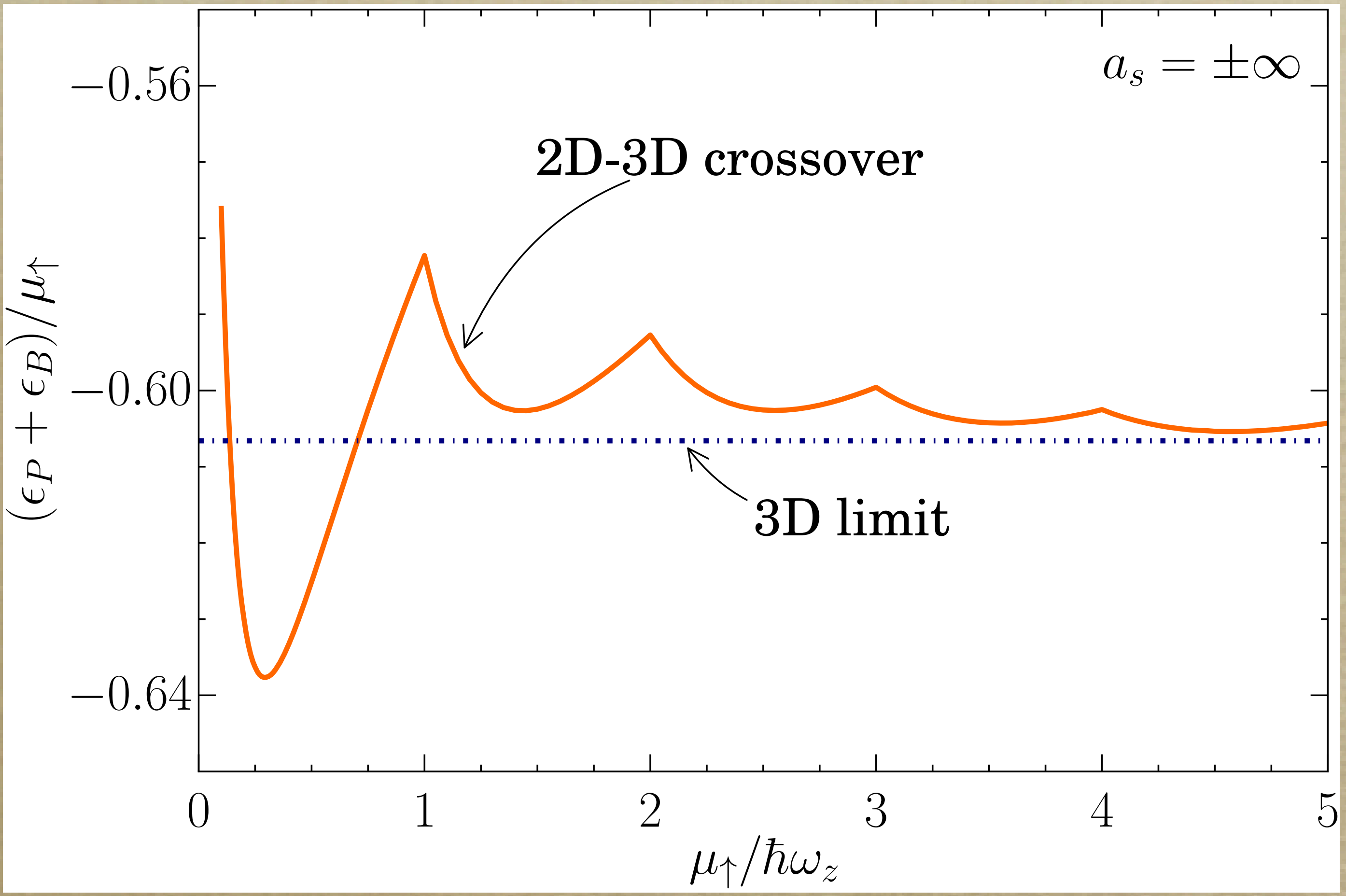
Massignan arXiv 2011, Trefzger & Castin arXiv 2011



(Massignan)

2D-3D crossover

- Polaron energy at 3D FR: $(\epsilon_b = 0.244\hbar\omega_z)$

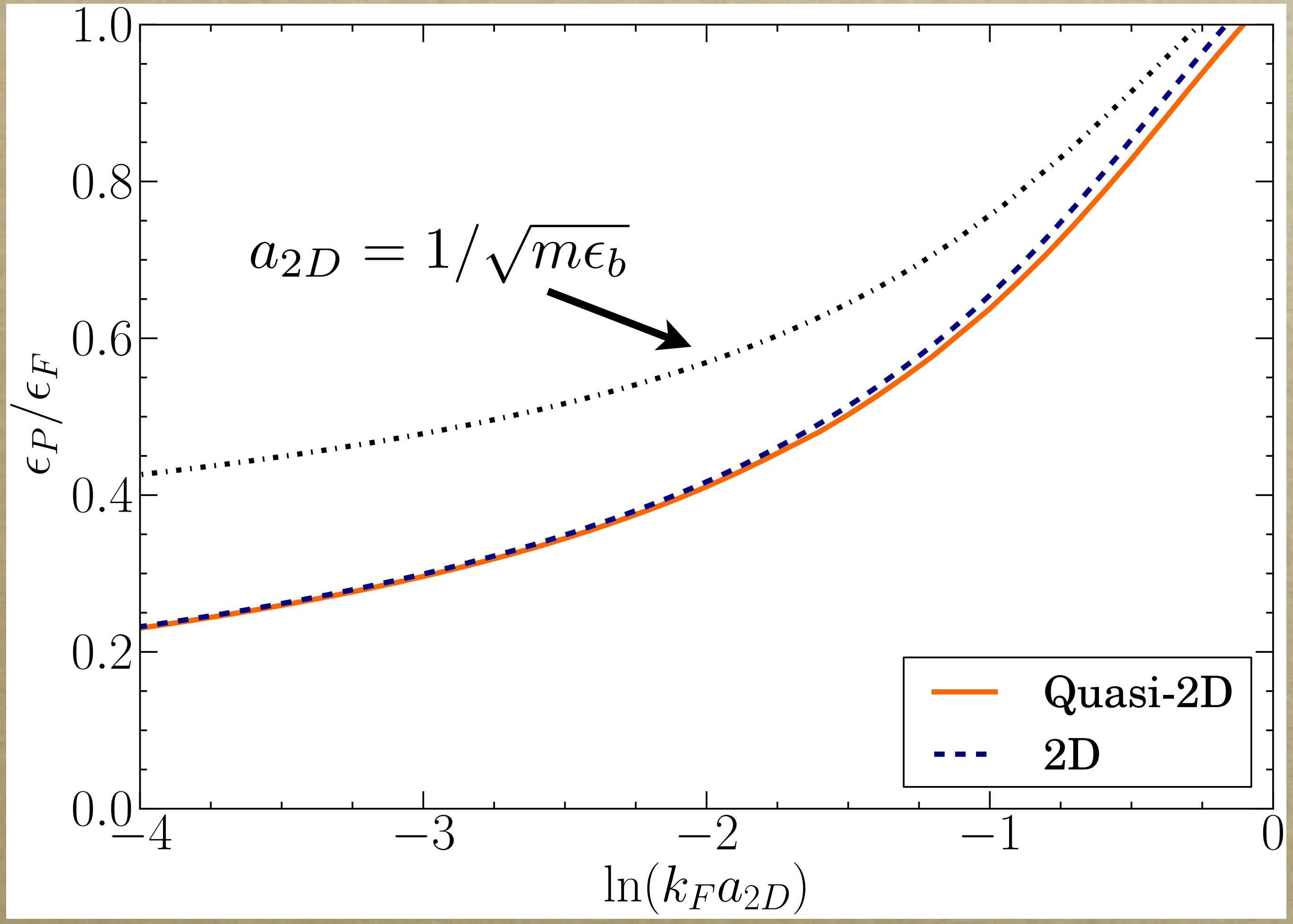


- Series of cusps due to change in density of states
- Quickly saturates close to 3D value $\epsilon = -0.607\epsilon_F$

Chevy PRA 2006, Combescot et al PRL 2007

Repulsive polaron

- Energy of repulsive polaron accurately described by 2D theory provided $\ln(k_f a_{2D})$ is calculated within the quasi-2D theory



$$\epsilon_F/\hbar\omega_z = 0.1$$

Note: Energy correctly described, however width requires precise description of decay channels

2D theory: Schmidt, Enss, Pietilä, Demler PRA 2012, Ngampruetikorn, JL, Parish, EPL 2012

Conclusions

- Quantitative description of current quasi-2D experiments requires taking confinement into account
  - Changes position of polaron-molecule transition towards smaller binding energy
  - Agrees with experiment
- Can study 2D-3D crossover quantitatively
  - Cusps in energy, quick convergence towards 3D limit
- Upper branch may be described by 2D theory provided correct scattering length is used

Stefan Baur

JL, S.K. Baur
arXiv:1202.6564

Thank you!