Momentum-resolved spectroscopy of one-dimensional Bose gases

Nicole Fabbri

European Laboratory for Nonlinear Spectroscopy
Istituto Nazionale di Ottica-CNR

Theory of Quantum Gases and Quantum Coherence
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Credits

European Laboratory for Non-Linear Spectroscopy

BEC 1 (Rb-87)

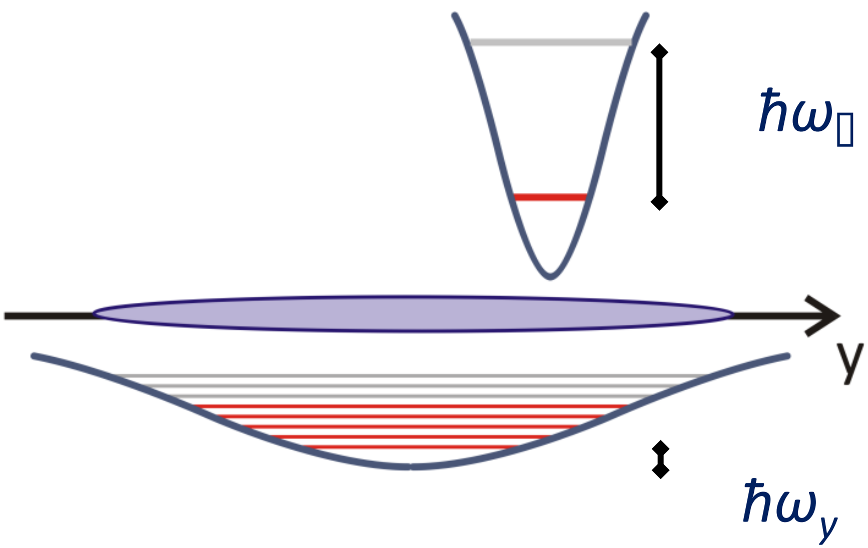
Chiara Fort
Massimo Inguscio
Leonardo Fallani
(now Yb lab, Firenze)
David Clément
(now at Laboratoire Charles Fabry)
N. F.
Sara Rosi
Alain Bernard







One-dimensional systems



A potential forces particles to be localized in two directions

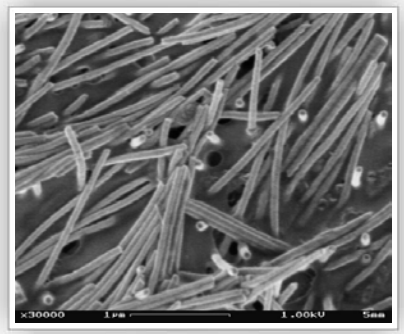
and $\hbar\omega_y \ll E_{int}, k_B T \ll \hbar\omega_\perp$

(the system occupies the transverse ground-state, which can be degenerate by including several longitudinal modes)

One-dimensional systems

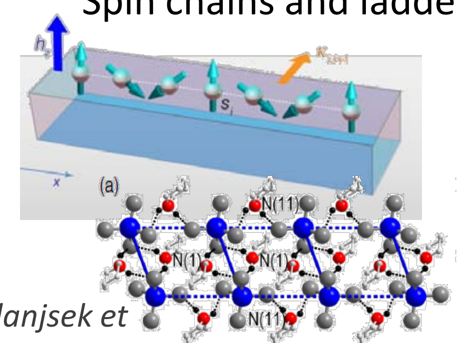
- interactions and large quantum fluctuations + exactly solvable models (Lieb-Liniger...) + powerful numerical methods; time-dependent dynamics, out-of-equilibrium calculations
- many realizations of 1D systems do exist in 3D world

1D Superconductors

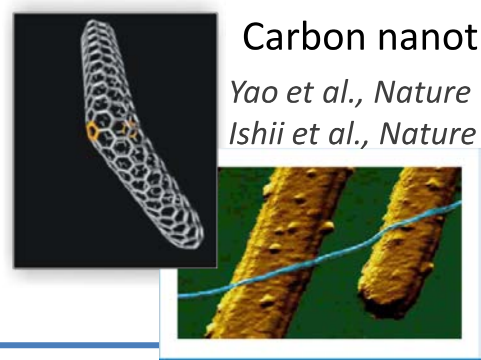


Vodolazov et al., PRL (2003)

Spin chains and ladders

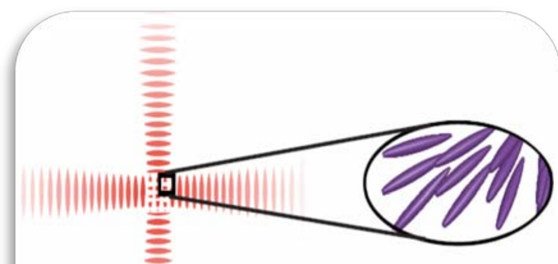


M. Klanjsek et al., PRL (2008)



Carbon nanotubes

Yao et al., Nature (1999)
Ishii et al., Nature (2003)



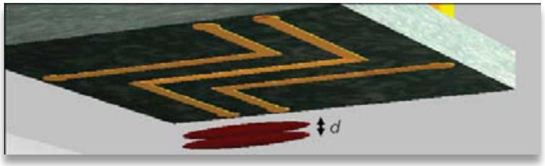
Ultracold atoms

1D physics with cold atoms

Realizations of 1D systems in cold gases:

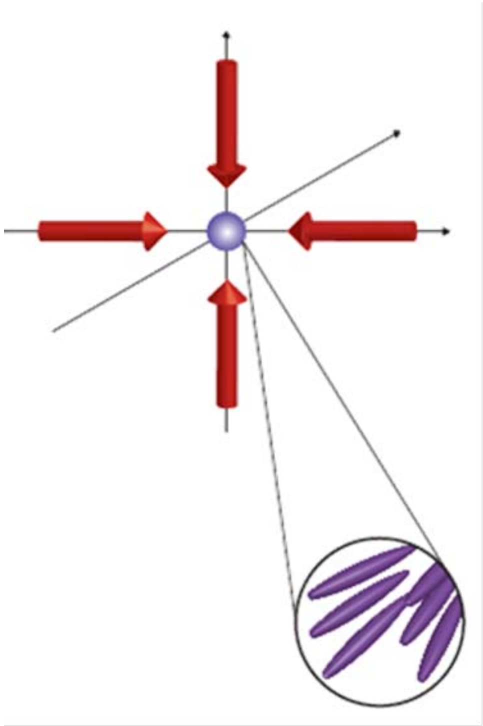
- (Quasi-) 1D gases in elongated magnetic traps (atom chips)

Schmiedmayer @ TU-Wien (Nat. Phys. 2005, PRL 2001, PRA 2009, 2010); Bouchole @ LCF (PRL 2006,2010,2011, PRA 2007,2011); van Druten @ Amsterdam (PRL 2008,2010)…



- Optical lattices to produce array of 1D gases

Esslinger @ ETHZ (Nature 2002, PRL 2003); Porto @ NIST (PRL 2004)…



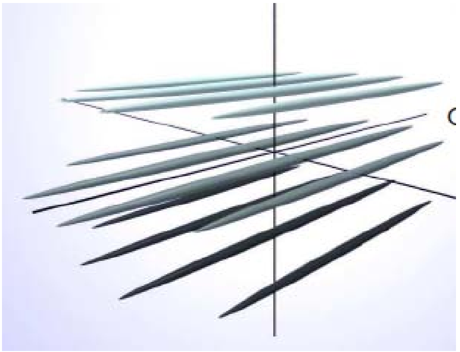
→ Strongly interacting regime

Bloch (Paredes et al., Nature 2004); Weiss (Kinoshita et al., Science 2004); Nägerl @ Innsbruck (Haller et al., Science 2009)…

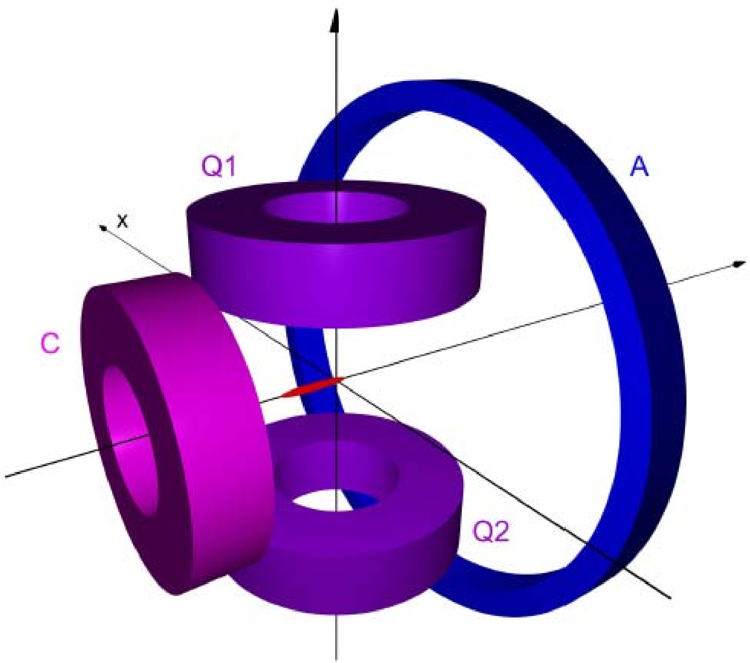
Outline

Study of the momentum distribution of an array of harmonically trapped 1D Bose gas (Rb atoms)

- ▷ inelastic light scattering (Bragg spectroscopy)
- ▷ mapping momentum distribution via density distribution after free evolution



Sample preparation, harmonic trap



Bose Einstein condensate of ^{87}Rb
in the in lowest hf state $|f=1, m_f=-1\rangle$

λ_0=780 nm

$$N_{BEC} \sim 10^5$$

$$T_C \cong 125\, nK$$

$$\mu/h \approx 600\, Hz$$

$$R_\perp \cong 4\, \mu m$$

$$R_y \cong 40\, \mu m$$

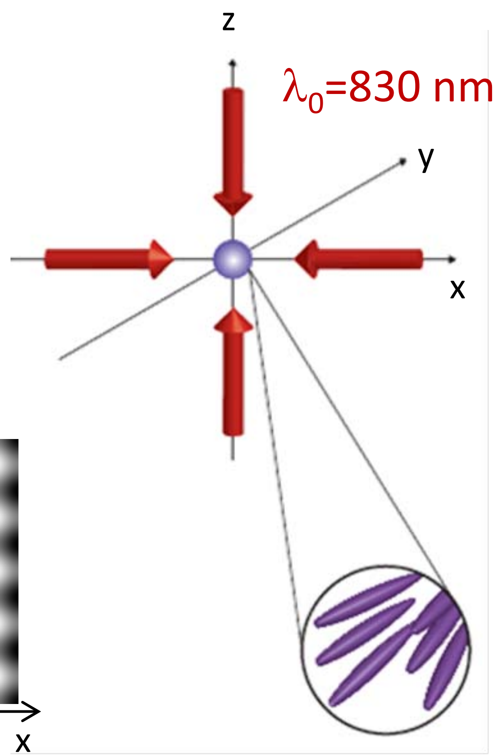
$$\omega_\perp / 2\pi \sim 90 \text{ Hz}$$

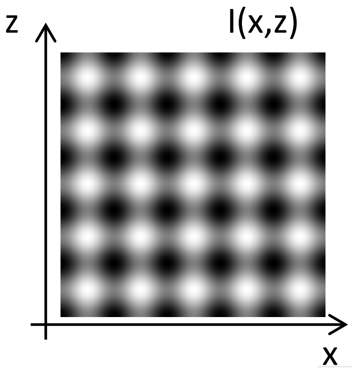
$$\omega y / 2\pi \sim 9 \text{ Hz}$$

Sample preparation, 2D optical lattice

$$V_{dip} \propto \frac{I(x,y,z)}{\omega_L - \omega_0}$$

$$\omega_L = 2\pi/\lambda_L$$





from the interference of two pairs of counterpropagating off-resonant laser beams:

$$V_{dip} \propto s_\perp E_R \left(\sin^2(kx) + \sin^2(kz)\right)$$

Array of 1D gases

Number of 1D gases: $N \approx 1500$
Number of atoms per tube:150
Size 30 μm × 50 nm
Density ≈ 7 μm^{-1}

$$@\, s_\perp = 40$$

tunneling time ≈ 3s

$\omega_\perp / 2\pi \approx 50\ kHz$ $\quad$ $\omega_{//} / 2\pi \approx 50\ Hz$

$\mu_{1D} / h \approx 1\ kHz$ $\quad$ $k_B T / h \approx 2\ kHz$

$(T \approx 100\ nK)$

$$\boxed{\hbar\omega_y < E_{int}\, , k_B T \ll \hbar\omega_\perp .}$$

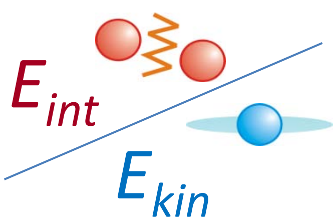
One-dimensional systems

Lieb-Liniger model:

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial y_i^2} + g_{1D}\sum_{i<j=1}^{N}\delta(y_i - y_j)$$

many-body problem of bosons in 1D with pairwise interactions
- T=0
- uniform
- contact interactions

E.H. Lieb e W. Liniger, PR 130 (4), 1963



$$\gamma = \frac{m\, g_{1D}}{\hbar^2\, \rho}$$

$\gamma \rightarrow 0$ non-interacting bosons

$\gamma \gg 1$ gas of impenetrable bosons
interactions mimic Pauli exclusion principle: $|\psi|$ akin spinless fermions
Tonks Girardeau gas
M. Girardeau, J. Math. Phys. (1960)

$$g_{1D} = \frac{2\hbar^2}{ma_\perp}\frac{a_s/a_\perp}{1 - C(a_s/a_\perp)}$$

$$a_\perp = \sqrt{\hbar/(m\omega_\perp)}$$

In 1D interactions dominate in the low density limit!

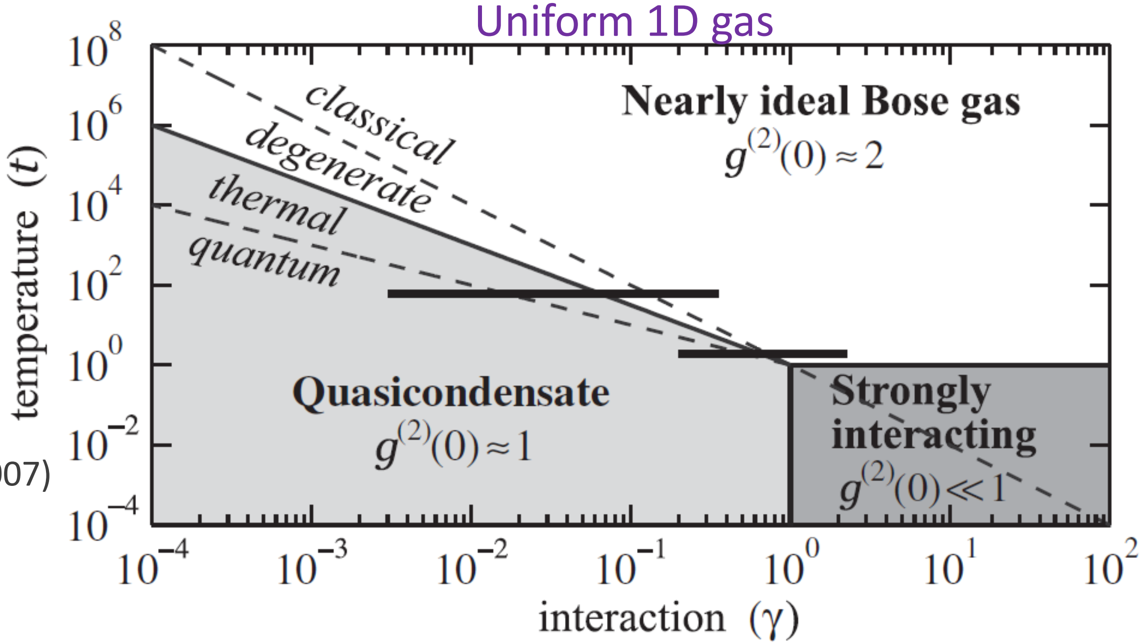
Phase diagram of 1D Bose gases

Ideal gas
Quasicondensate
Strongly interacting regime

$$t = \frac{2\hbar^2 k_B T}{m g_{1D}^2} \qquad \gamma = \frac{m g_{1D}}{\hbar^2 \rho}$$

Petrov, Shlyapnikov, Walraven, PRL (2000)
Bouchoule, Kheruntsyan, Shlyapnikov, PRA (2007)
Jacqmin et al. PRL (2011)

Uniform 1D gas



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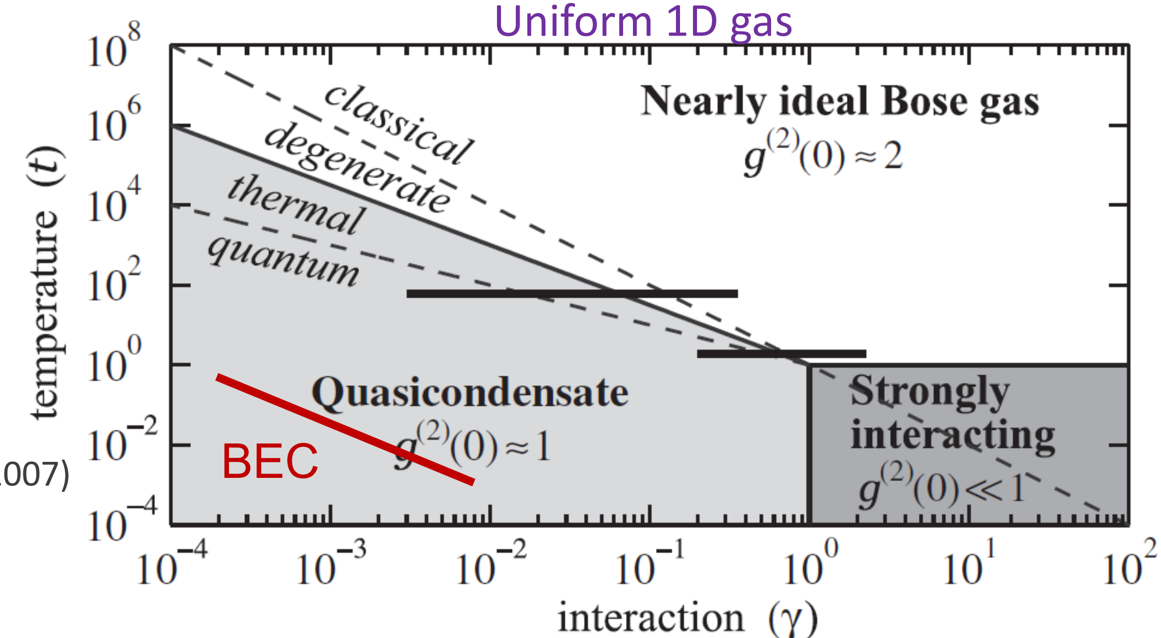
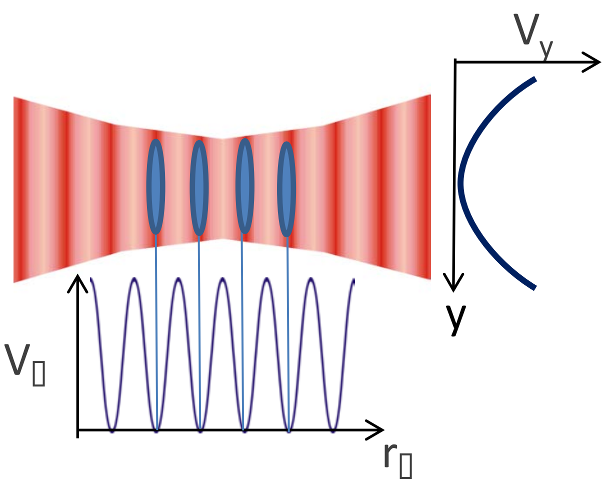


In practice: 1D gas in a trap (tranverse+longitudinal)



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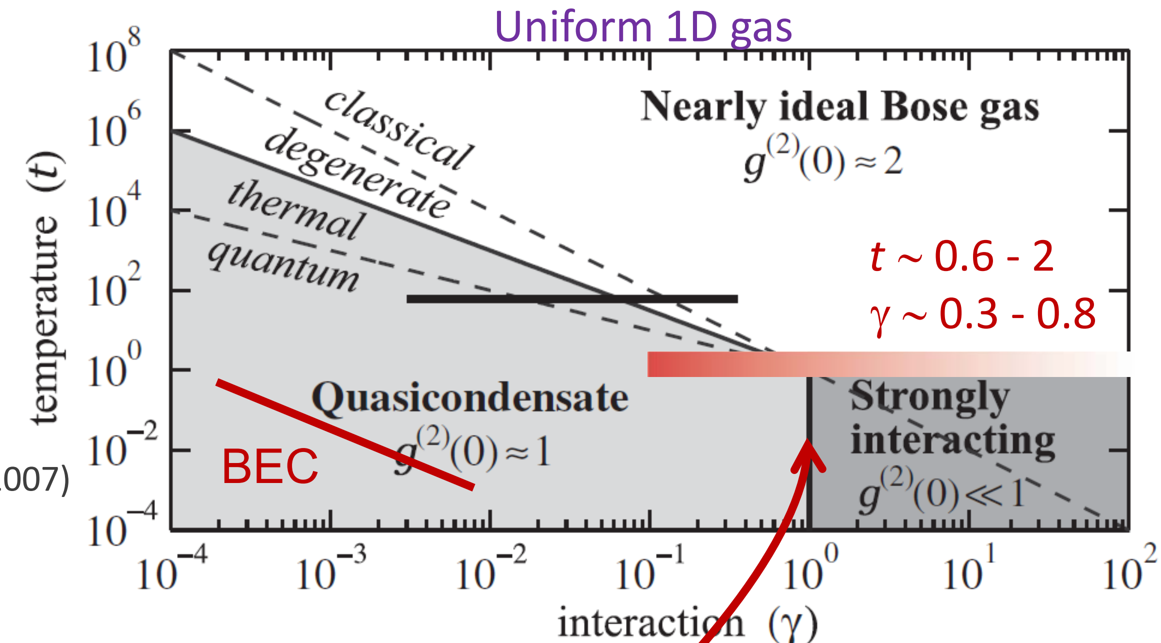

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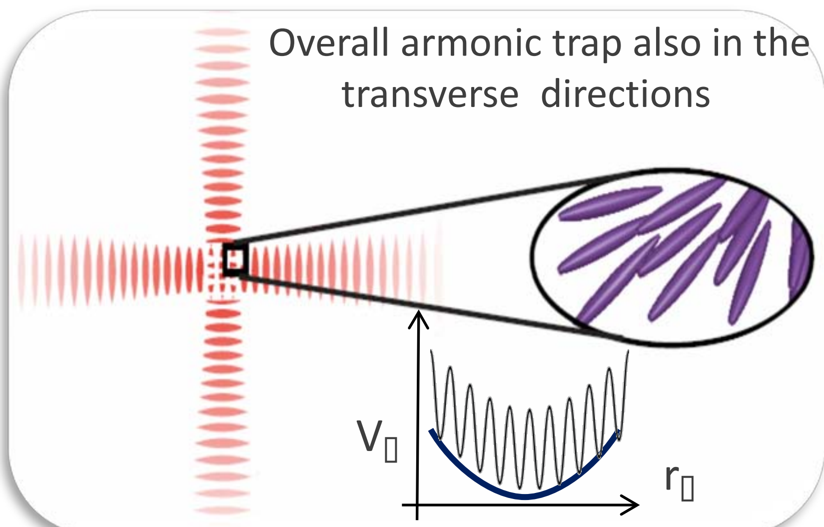
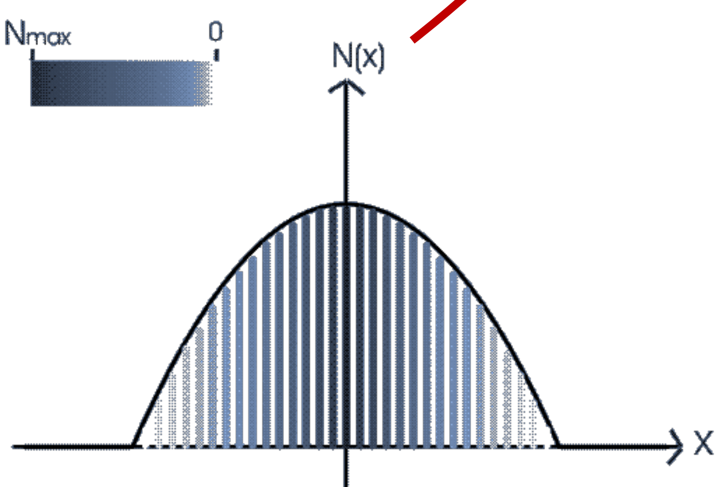
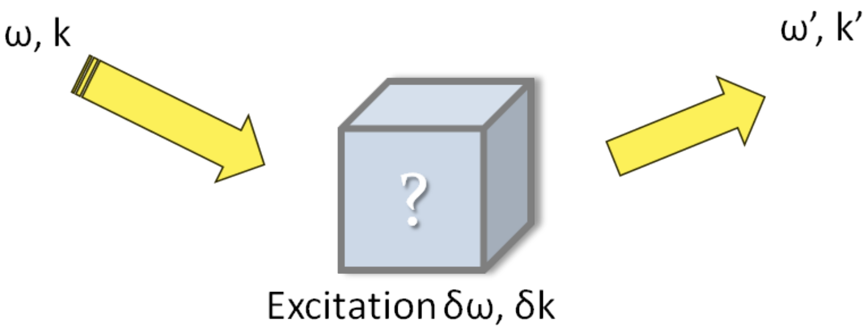




Probe the dynamical properties of the system

Inelastic light scattering

Inelastic scattering of waves or particles has been widely used to gain information on the structure of matter



non-zero momentum transfer

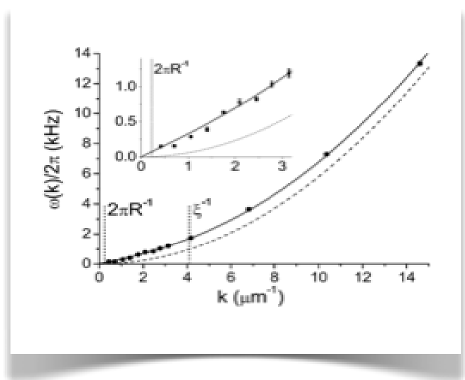
weak perturbation $\rightarrow$ LINEAR RESPONSE

In cold atoms (Bragg scattering): Two-photon transition between two motional states of the same internal ground state

First experiments:

M. Kozuma et al. PRL 82 871 (1999)

J. Stenger et al. PRL 82, 4569 (1999)



Weakly interacting 3D BEC

Excitation spectrum:

J. Stenger et al., PRL (1999)

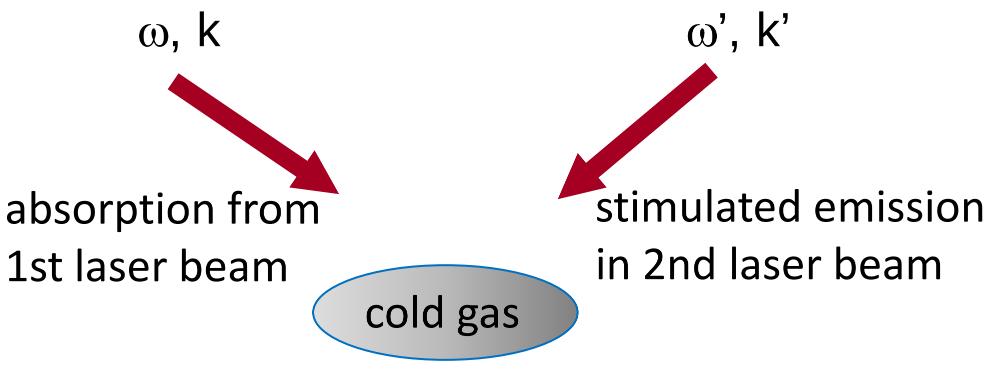
J. Steinhauer et al., PRL (2002)

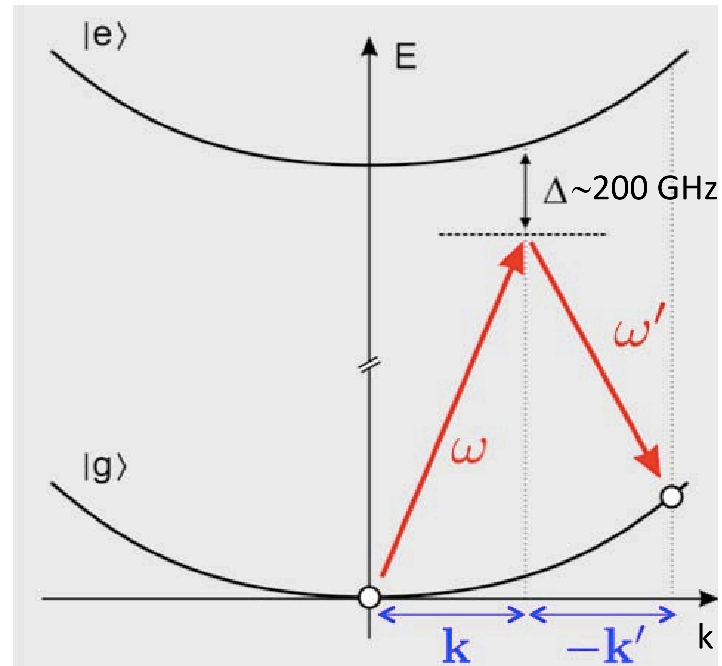
Phase fluctuations:

S. Richard et al., PRL (2003)

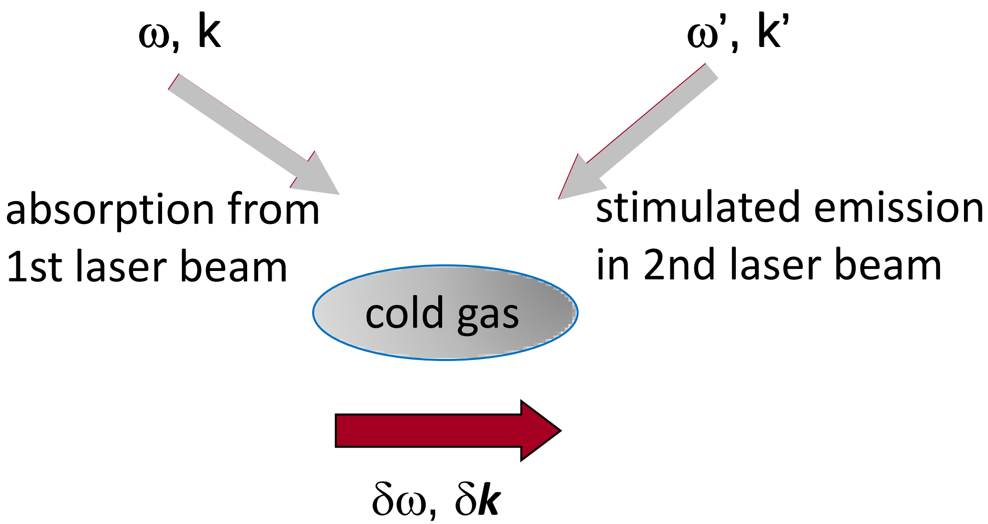
D. Helweg et al., PRL (2003)

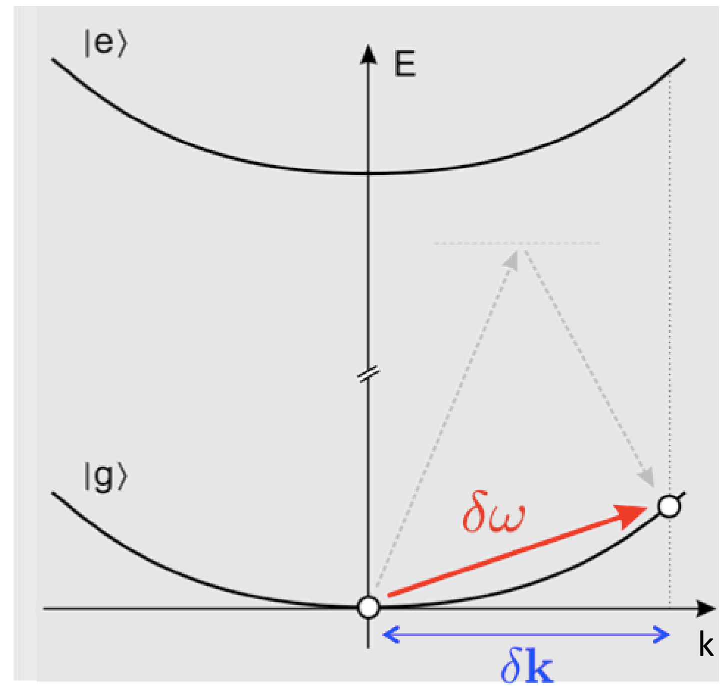
two simultaneous off-resonance laser pulses





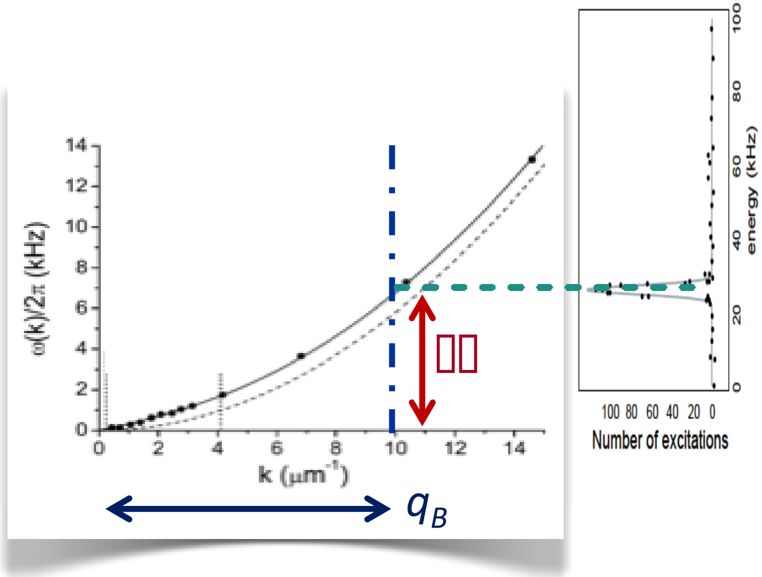
Bragg spectroscopy





Energy of the excitation $\delta\omega = \omega - \omega'$
Momentum of the excitation $\delta \boldsymbol{k} = \boldsymbol{k} - \boldsymbol{k}'$

Probe the dynamical structure factor



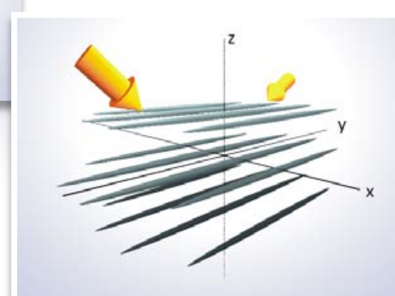
We choose a fixed $\delta k=q_B$ and we scan $\delta\omega$

We measure the energy transferred to the system

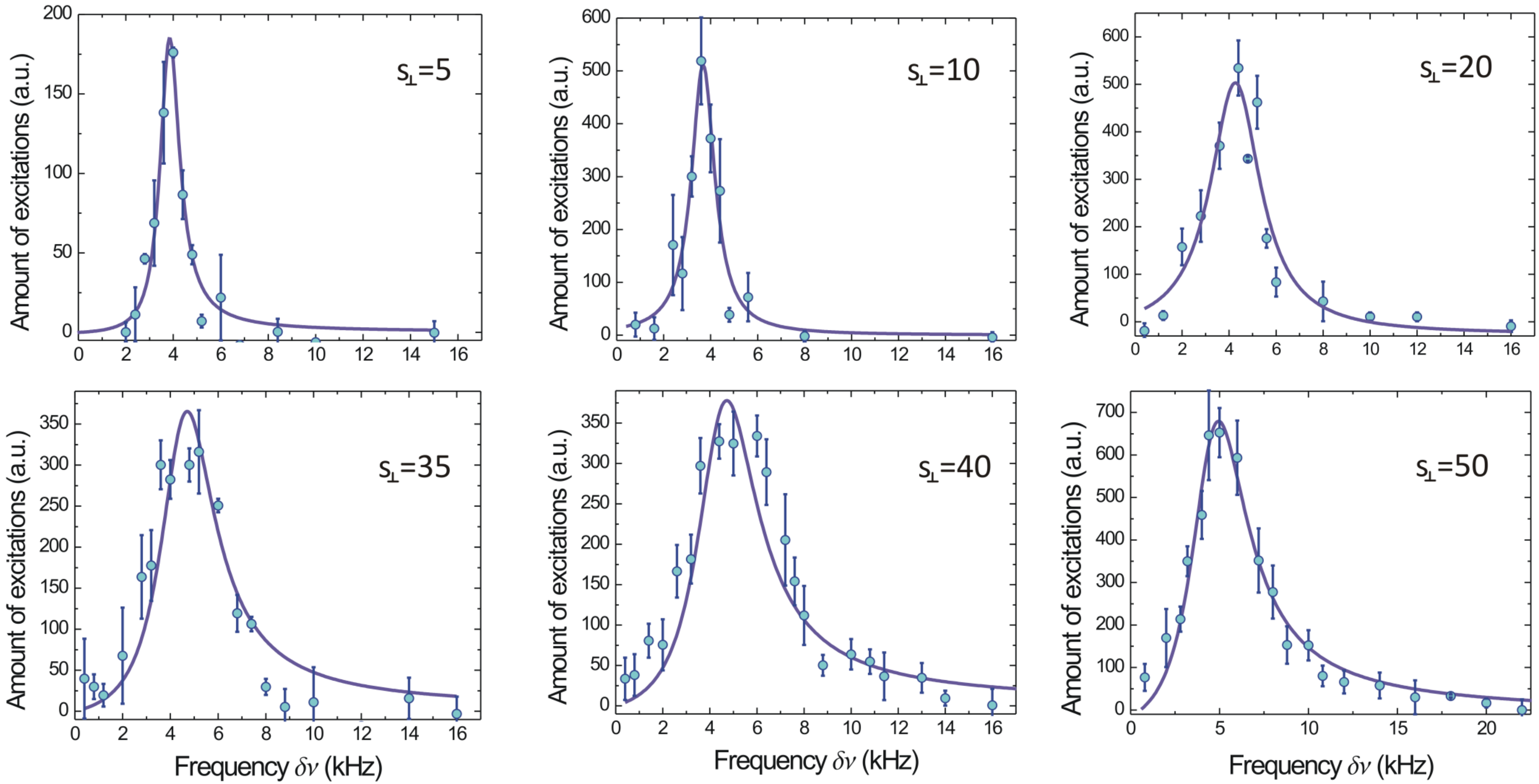
In the linear response theory:

$$\Delta E \propto \omega\, S_{qB}(\omega)$$

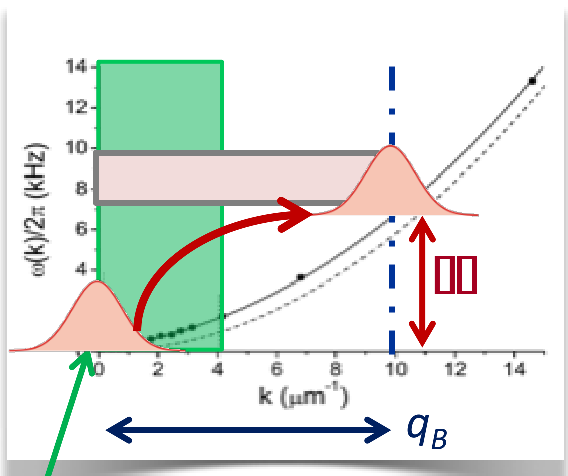
Brunello et al. PRA 2001



Bragg spectra of 1D gases



Probe the momentum distribution



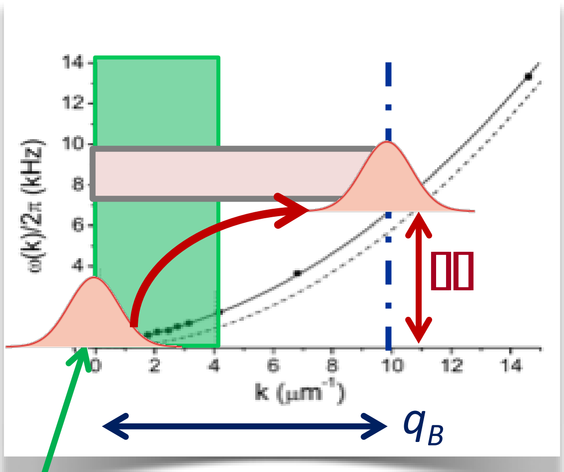
phononic collective regime

Doppler regime:
momentum transfer $q_B \gg 1/\xi = \sqrt{\frac{2mgn}{\hbar^2}}$

mean-field picture (Bogoliubov approx)
Bragg condition for energy and momentum conservation

$$\hbar\delta\omega = 4E^B_{Rec} + \alpha\mu + \hbar q_B v_{at}$$

Probe the momentum distribution



phononic collective regime

Doppler regime:
momentum transfer $q_B \gg 1/\xi = \sqrt{\frac{2mgn}{\hbar^2}}$

Bragg is sensitive to the initial momentum distribution $S_{qB}(\omega) \propto n(p)$

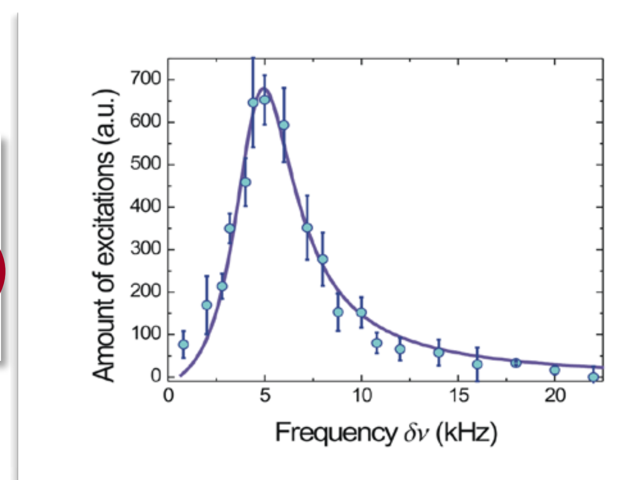
$$\Delta p = \frac{2\pi m}{q_B}\Delta\nu$$

Momentum distribution of (not too strongly) interacting 1D gases at finite T:

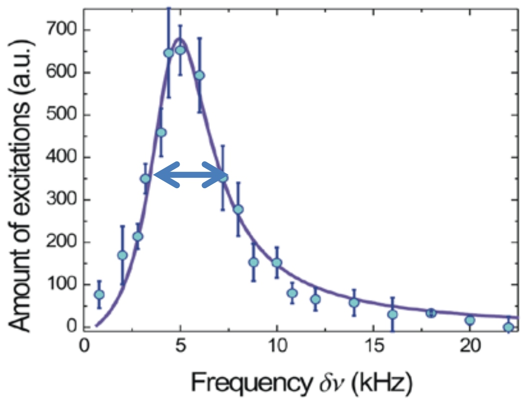
$$n(p;T) \simeq 4\left(\frac{K^2}{\pi}\right)^{\frac{1}{2K}} (\rho\,\Delta p)^{1-1/2K} \frac{1}{1+(2p/\Delta p)^2}$$

Cazalilla, New Journal of Physics (2006)

Lorentzian shape

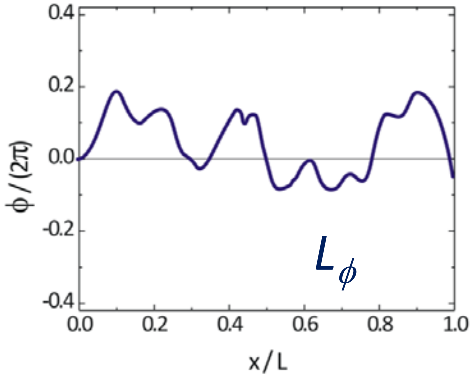


Coherence length



Lorentzian shape: width related to the inverse of the coherence length L_ϕ

$$L_\phi = \frac{\hbar^2 \rho}{m k_B T}$$



For trapped 1D gases

Gerbier, PhD thesis (2003)

$$\Delta p = 0.635 \frac{\hbar}{L_\phi}$$

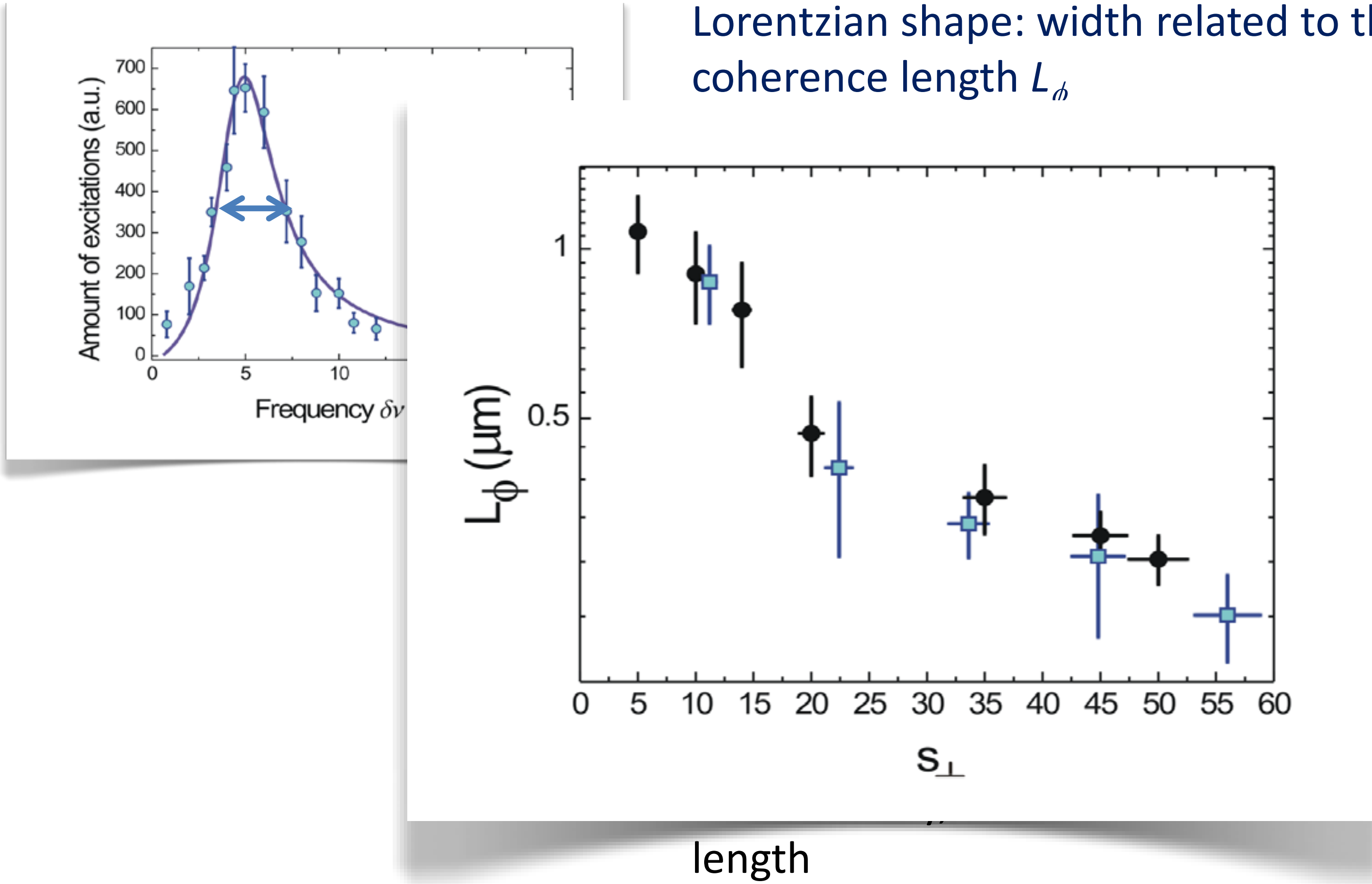
> In the array, we extract an effective global coherence length

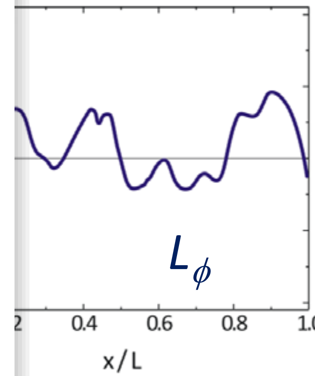
> We study how this varies by increasing the strenght of the transverse confining potential

N. Fabbri et al., PRA 83, 031604(R) (2011)

Coherence length

Lorentzian shape: width related to the inverse of the coherence length L_ϕ





$$.635\frac{\hbar}{L_\phi}$$

global coherence length

> We study how this varies by increasing the strenght of the transverse confining potential

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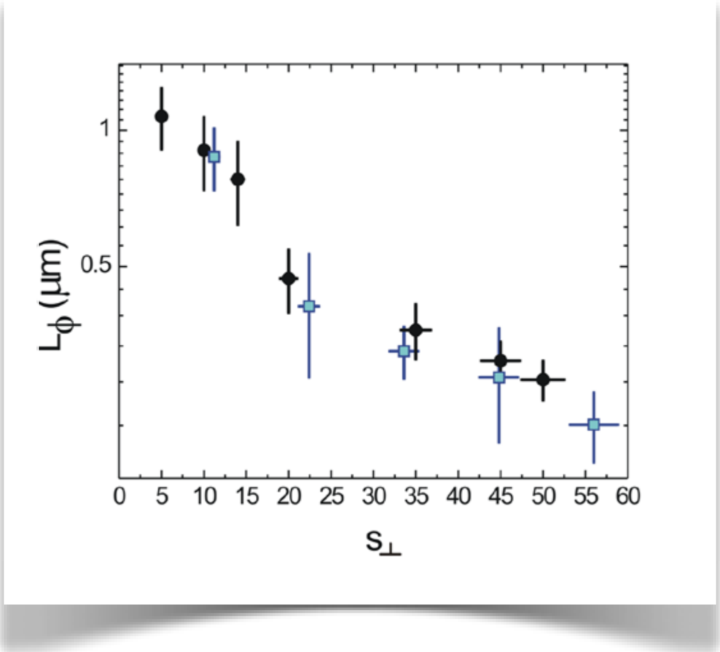
Coherence length

$$L_\phi = \frac{\hbar^2 \rho}{m k_B T}$$



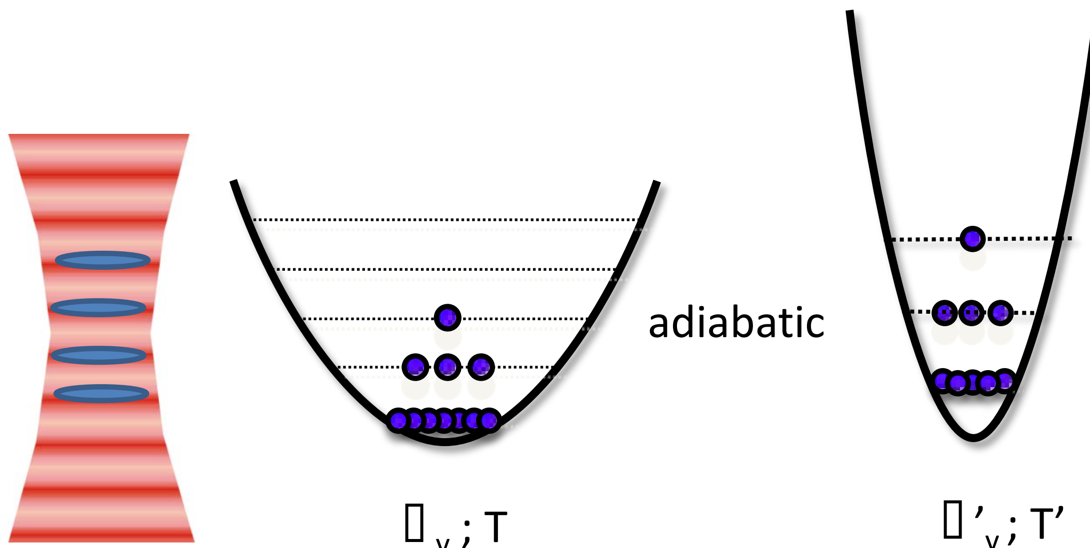
increasing the transverse confinement:

- anisotropy increases by □10%
- linear density decreases by □15%
- **T increases due to axial compression** (harmonic confinement along the tubes due to the radial effects of the optical lattice beams)

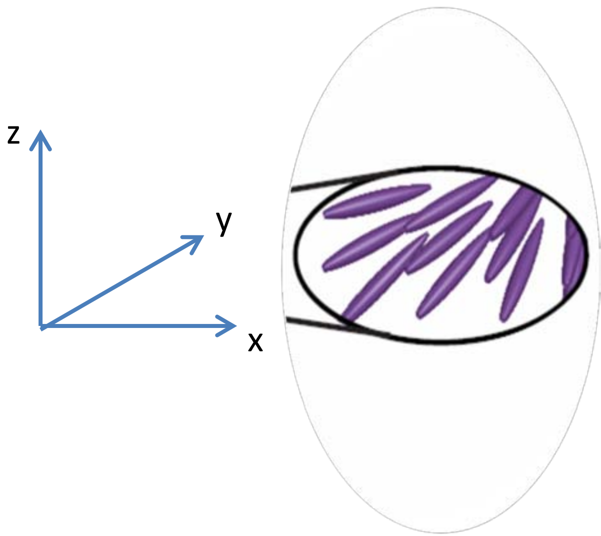
$$k_B T \gg \hbar\omega_y$$

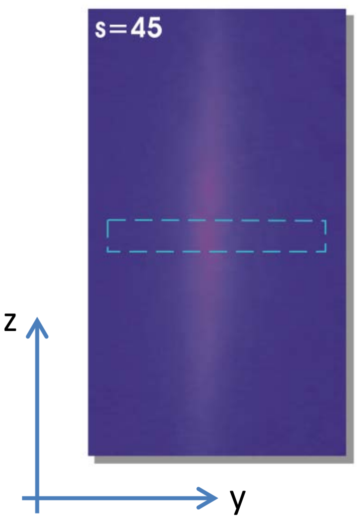
$$T \sim \sum_j n_j \epsilon_j$$

$$\epsilon_j = \hbar\omega_y \sqrt{\frac{j(j+3)}{4}}$$



Measuring momentum distribution via TOF imaging





When the trap is switched off, the spatial density distribution of the atomic cloud after time-of-flight (TOF) reflects the *in-trap momentum distribution*

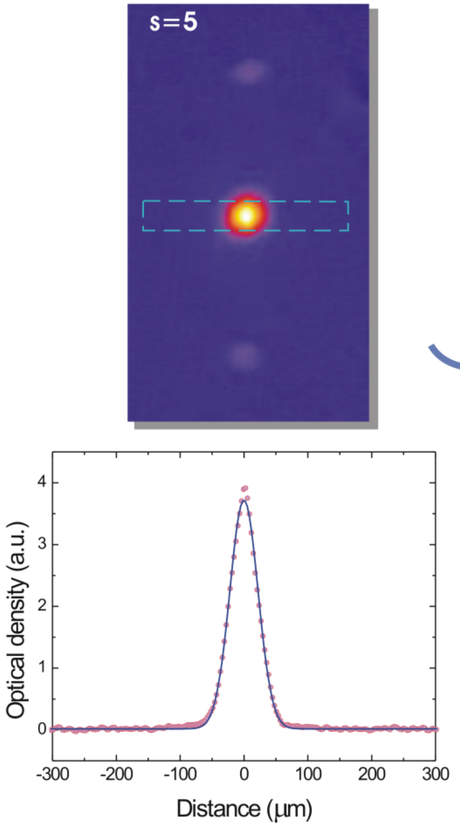
The expansion of the gas is governed by two kinds of kinetic energy:
- interactions converts into kinetic energy
- local phase gradients produce a velocity field

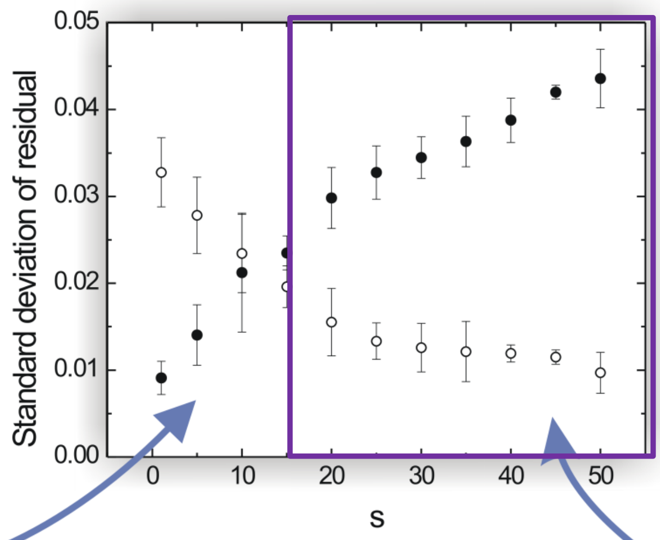
$$\mathbf{v}_\phi = (\hbar/m)\nabla\phi$$

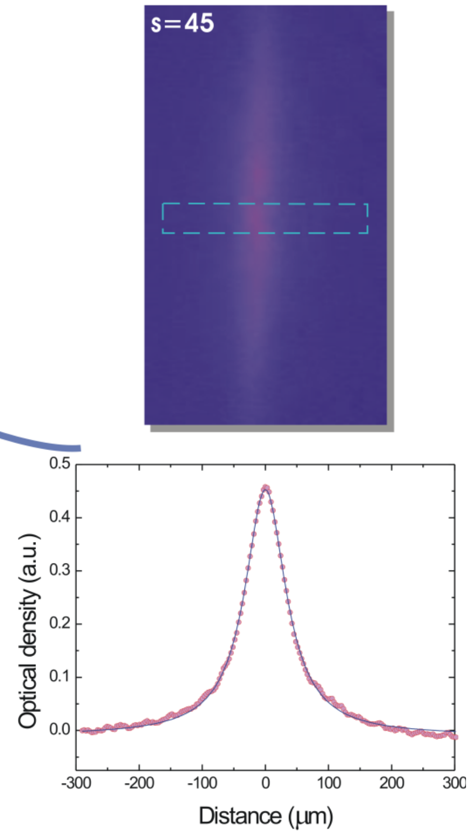
When strong enough, initial phase fluctuations dominate longitudinal dynamics during TOF

$$\frac{R^{\phi}_{tof}}{R^{\phi}_{tof}} \sim \frac{\hbar t_{tof}}{m\,L_\phi\,R^{int}_{tof}} > 1$$

Measuring momentum distribution via TOF imaging







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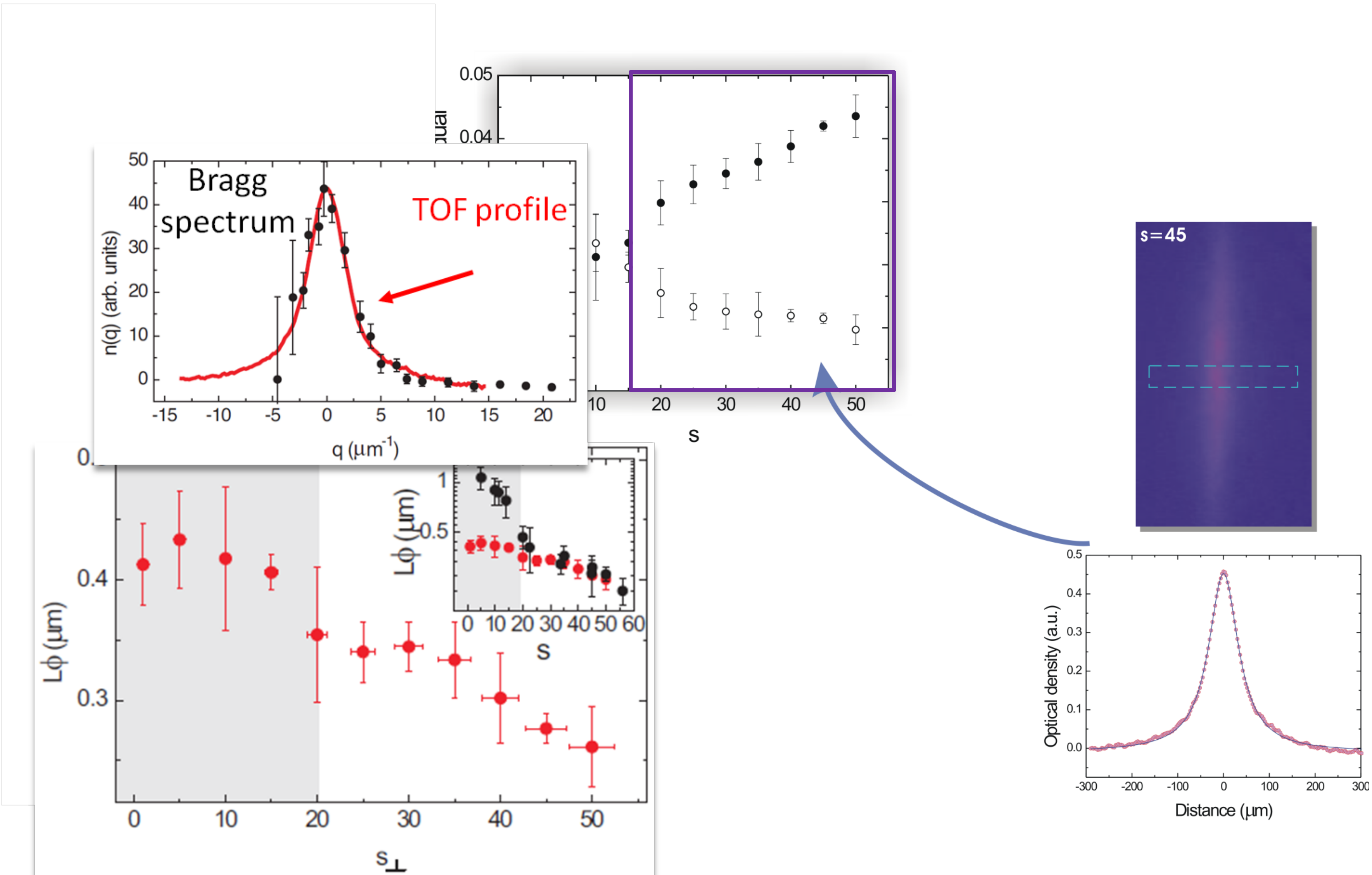
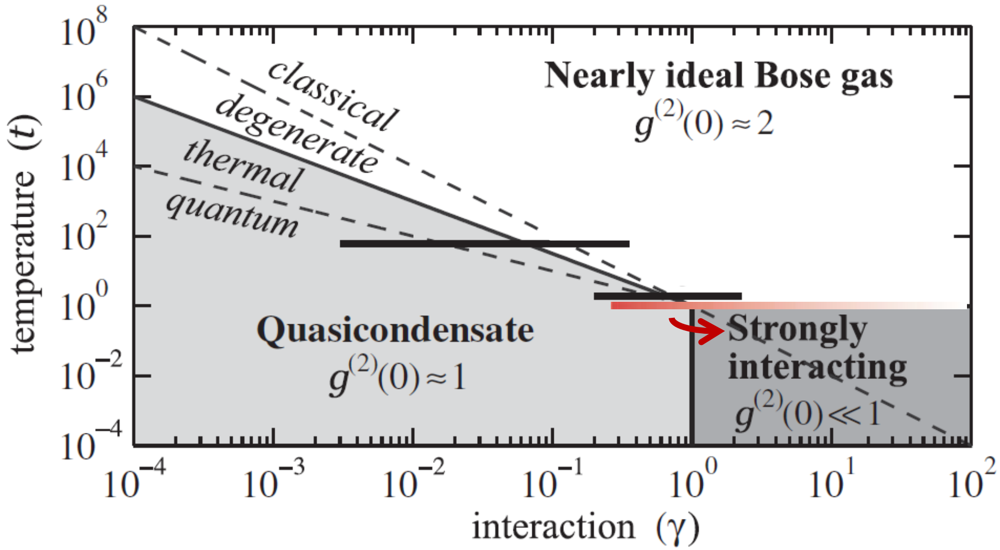
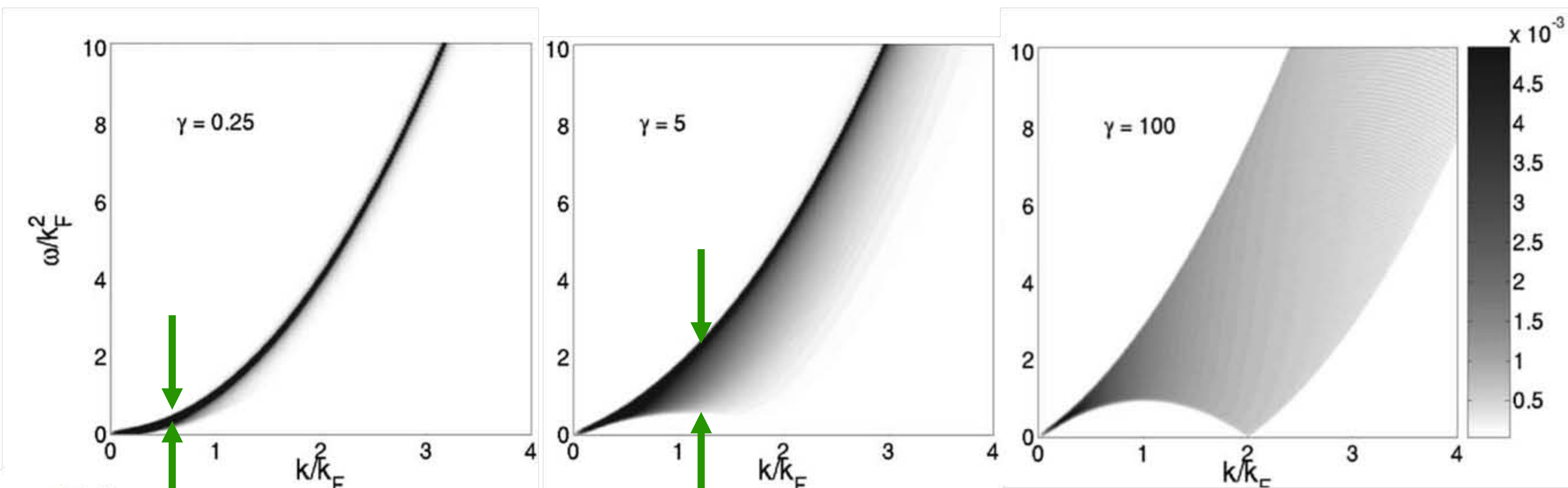


Current plans



$$t = \frac{2\hbar^2 k_B T}{m\, g_{1D}^2}$$

$$\gamma = \frac{m\, g_{1D}}{\hbar^2 \rho}$$



$k_F = \pi \rho$

J.S. Caux and P. Calabrese PRA 74, 031605R (2006)

Summary

Bragg spectroscopy is a powerful tool to investigate the linear response of the system at nonzero momentum transfer

We measured the momentum distribution of an array of 1D gases: Lorentzian shape, as predicted for a single uniform 1D gas; Measured the coherence length as a function of the transverse confinement.

In systems with short-range phase fluctuations: mapping momentum distribution via density distribution after free evolution reveals the coherence properties of the system

- Strongly interacting systems
- Strongly interacting disordered systems (debate on the role of thermal phase fluctuations in the nature of the superconductor-insulator transition…)

Thank you!