Atomic quantum simulator for lattice gauge theories

06/06/2012,
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EU AQUTE

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Joint work with E. Rico and P. Zoller (Innsbruck), M. Mueller (Madrid), P. Stebler, D. Banerjee and U.-W. Wiese (lattice gauge theorists / Bern)

arXiv:1205.6366

Quantum simulation

"Utilize a quantum machine to simulate a quantum problem untreatable on a classical one"

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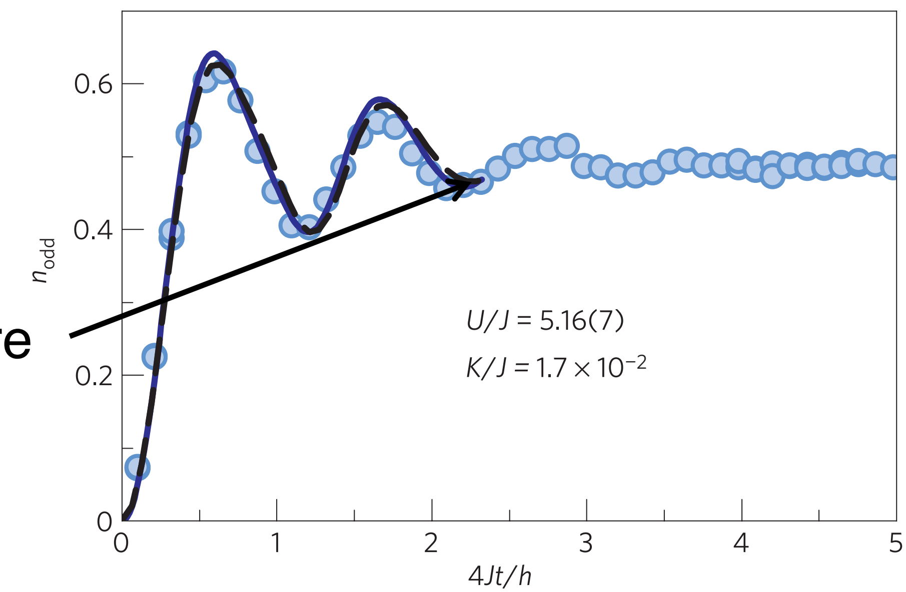
quench

DMRG (theory)
Breaks down here



S. Trotzky et al.,
Nature Physics, 2232 (2012)

...and lattice gauge theories

Gauge theories defined on a discrete lattice structure

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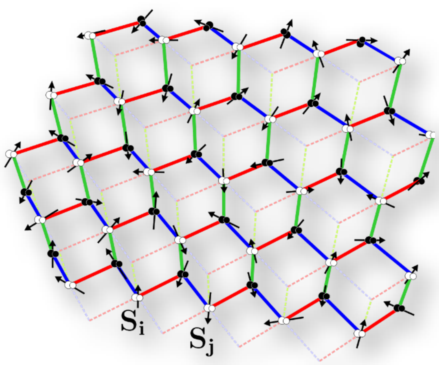
Example: Kitaev Model



$$H = J_1 \sum_{1<i,j>} S_i^{(1)} S_j^{(1)} + J_2 \sum_{2<i,j>} S_i^{(2)} S_j^{(2)} + J_3 \sum_{3<i,j>} S_i^{(3)} S_j^{(3)}$$

$$\to \frac{J_1^2 J_2^2}{16|J_3|^3} \left(\sum_{\text{vertex}} XXXX + \sum_{\text{plaquette}} ZZZZ \right)$$

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Main need / goal: design a *quantum simulator* for *lattice gauge theories* and investigate some relevant phenomenon

Outline of the talk

Poor man view of global versus gauge symmetries and static vs dynamical gauge fields

General strategy for quantum simulation: **quantum link models** vs Wilson LGT

The simplest quantum link model: U(1) symmetries in 1D and QED

Confinement in LGT: **string breaking**

Implementation of quantum link models with both gauge and matter fields in optical lattices: **Bose-Fermi mixtures**

Observability of **confinement phenomena**

Global and gauge symmetries: what makes the difference?

Global symmetries

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Gauss law!

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Static gauge fields:
particles hopping around
a plaquette acquire a
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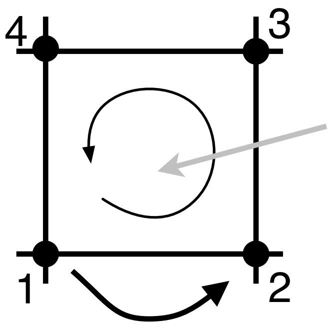

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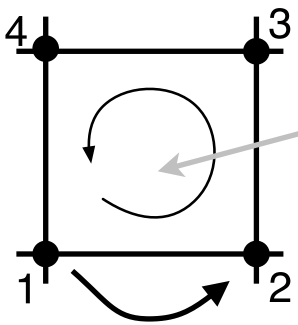

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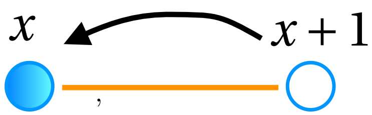

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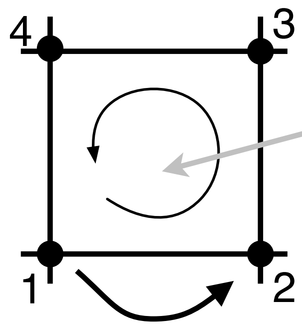

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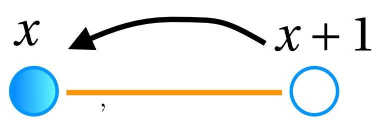

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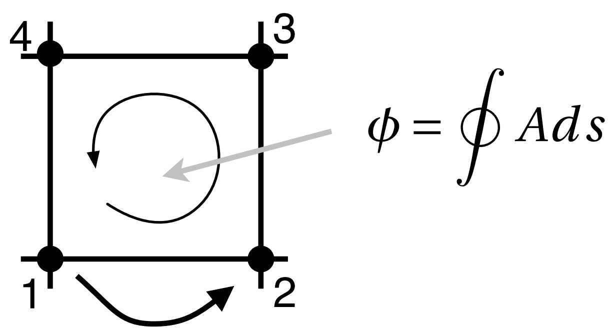

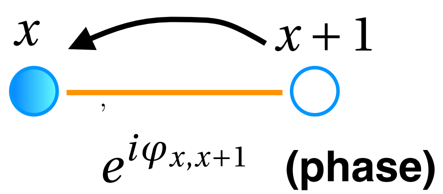

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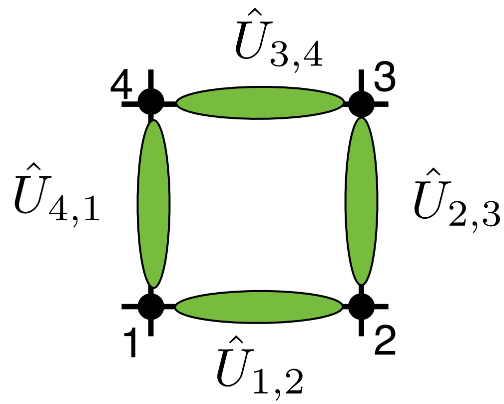

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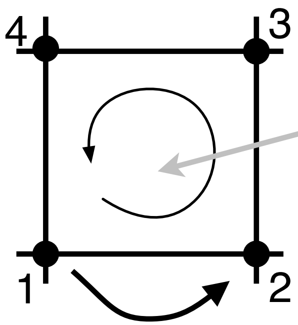

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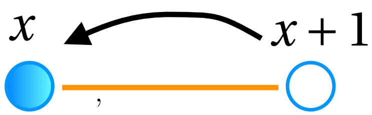

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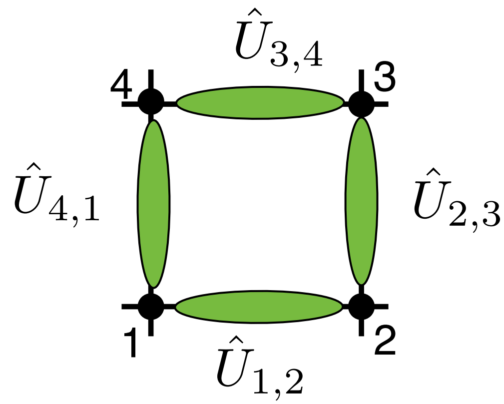

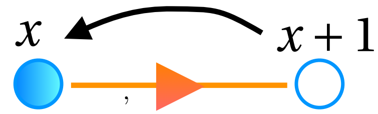

$U_{x,x+1}$ **(operator)**

$$H = -t\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \text{h.c.}$$

See Creutz and Montvay/Muenster books + Kogut (Rev. Mod. Phys. 1979)

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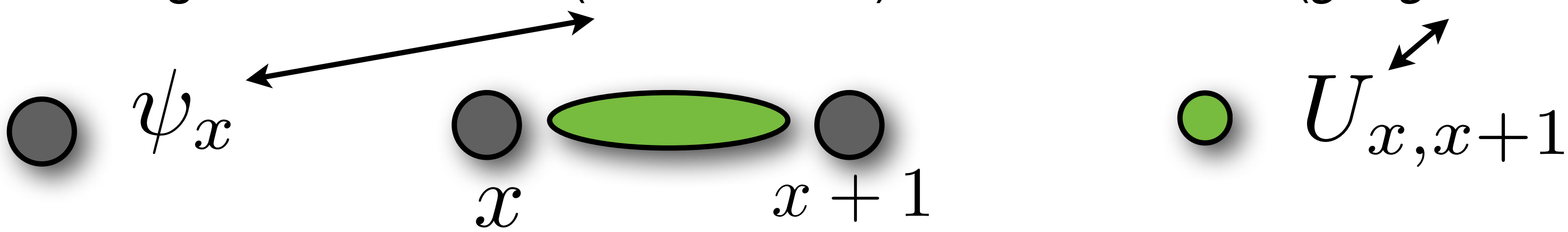
1)**set of fields** acting on the vertices (*matter fields*) and on the links (*gauge fields*)



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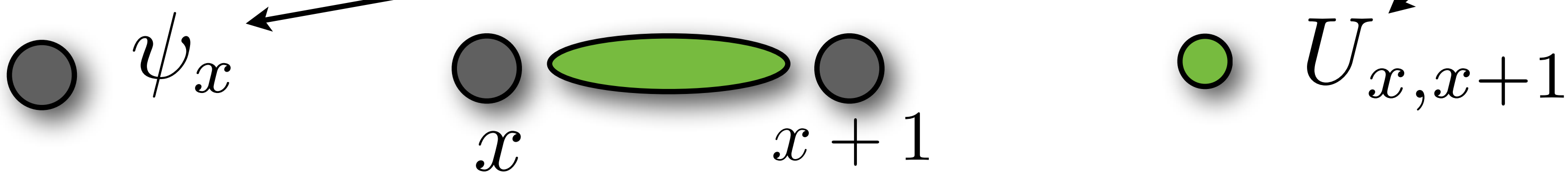

2)**set of *generators*** which define the gauge symmetry, and the physical Hilbert space:

$$[G_x^\alpha, U_{y,y+1}] = \delta_{x,y} U_{x,x+1} \qquad\qquad G_x^\alpha |\Psi_{\text{phys}}\rangle = 0$$

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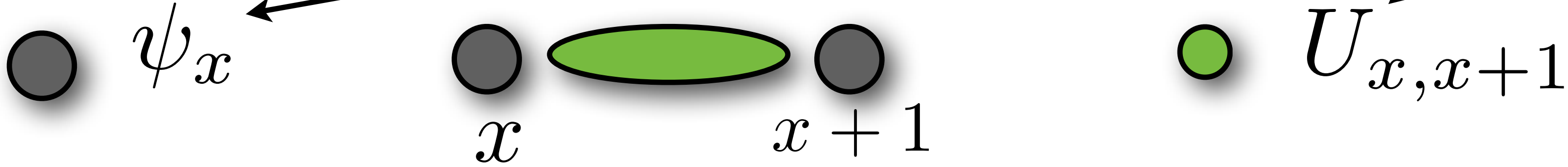

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3)a **Gauge invariant Hamiltonian**:

$$H[\psi_x, U_{x,x+1}], \quad [H, G_x] = 0 \;\forall x$$

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Alternative formulation of lattice gauge theories with *discrete gauge variables,* which are usually *quantum spins*: **Quantum link models**

$$U_{x,x+1} \to S^{+}_{x,x+1} \qquad E_{x,x+1} \to S^{z}_{x,x+1}$$

Quantum link models: D. Horn (1981), Chandrasekharan and Wiese (1996). See also Wen's book.

Digital approaches: H. Weimer et al.(2010), Tagliacozzo et al.(2012).

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A snapshot of the (physical) gauge invariant Hilbert space

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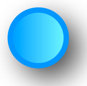
Example: Spin 1 representation

$|0\rangle$  $|1\rangle$ 

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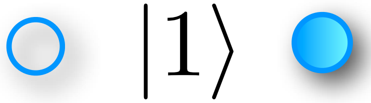

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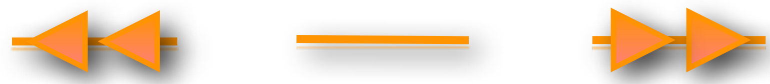
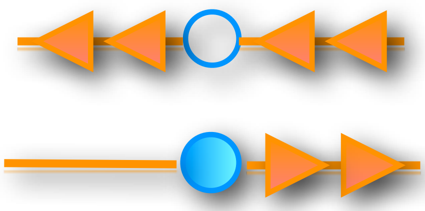


Even sites



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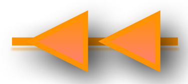
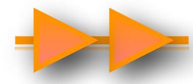
$|-1\rangle$  $|0\rangle$  $|+1\rangle$ 

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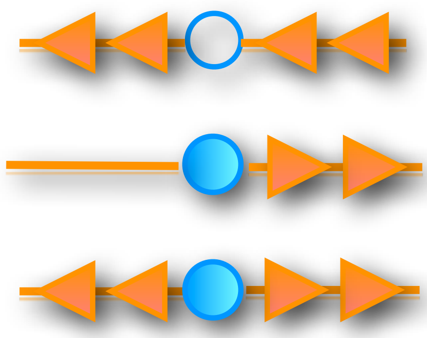

A snapshot of the (physical) gauge invariant Hilbert space

$$G_x = \psi_x^\dagger \psi_x - S^z_{x,x+1} + S^z_{x-1,x} + \frac{(-1)^x - 1}{2} \qquad G_x|\Psi\rangle = 0$$

Example: Spin 1 representation

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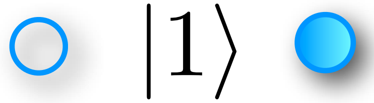

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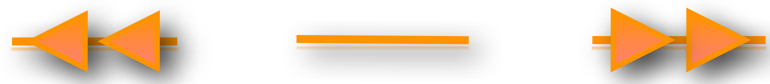

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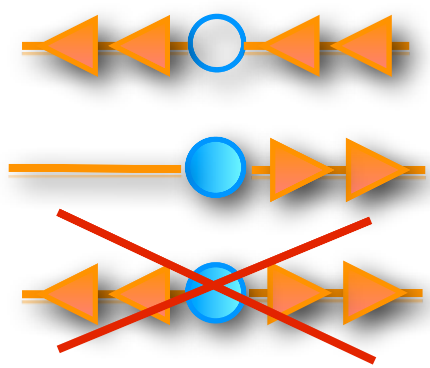

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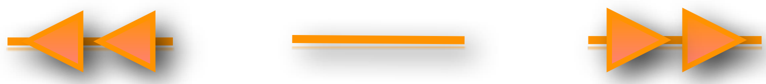

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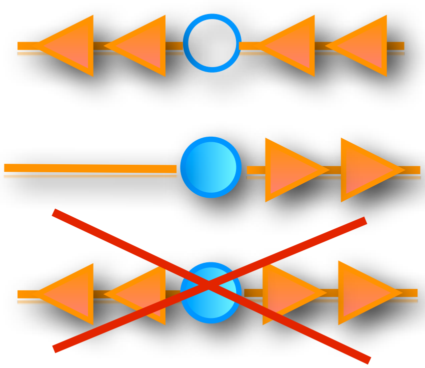
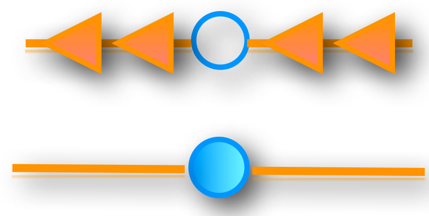


Odd sites



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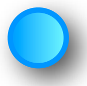
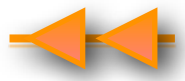
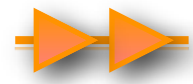
$|0\rangle$  $|1\rangle$ 

$|-1\rangle$  $|0\rangle$  $|+1\rangle$ 

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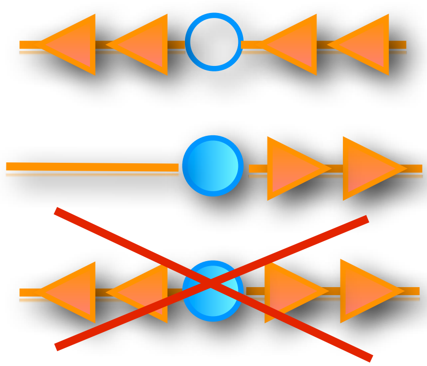

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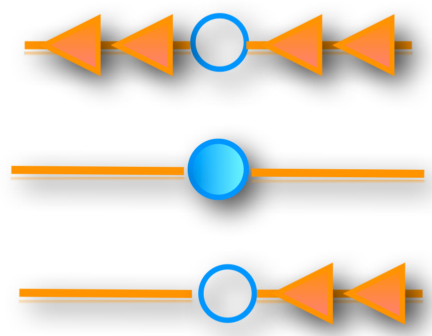

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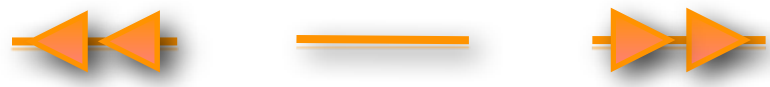

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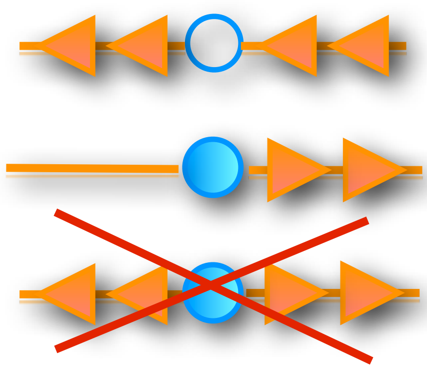

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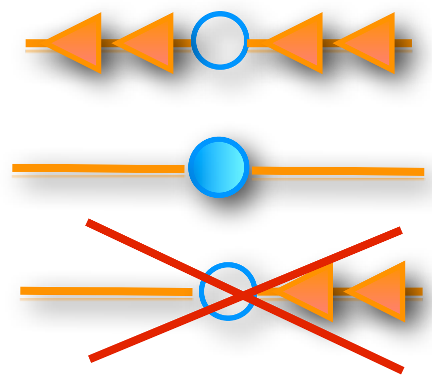

String breaking and confinement

quark *anti-quark* *flux string*

String breaking and confinement

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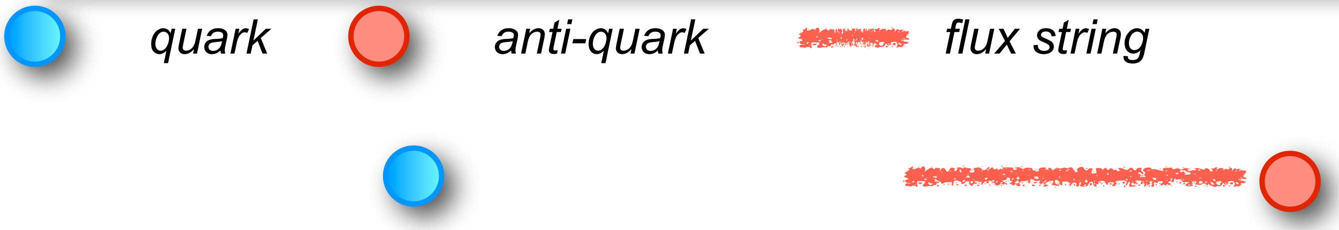

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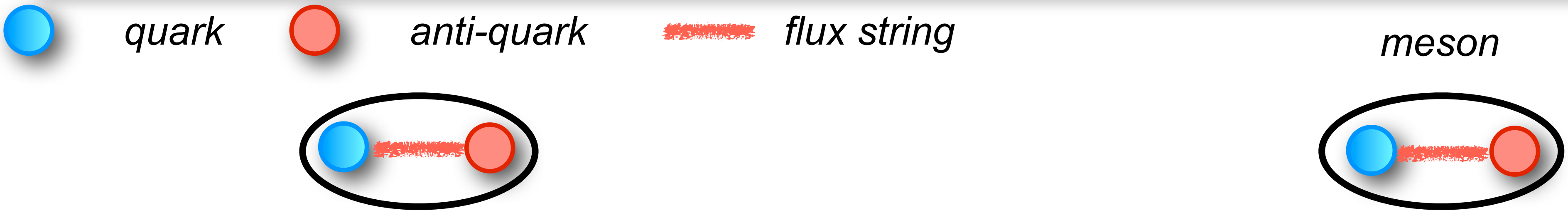

String breaking and confinement

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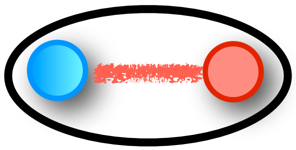
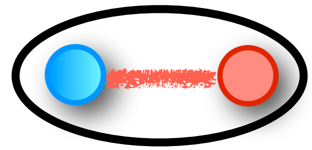
meson





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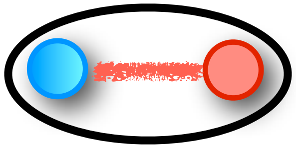

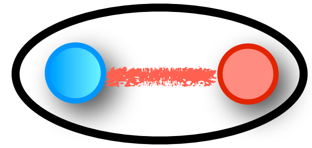

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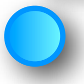
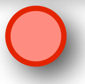
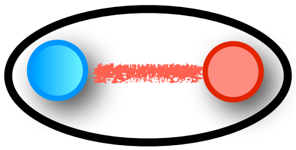
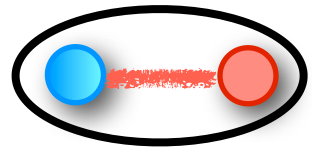
String breaking and confinement

 *quark*  *anti-quark*  *flux string*

meson





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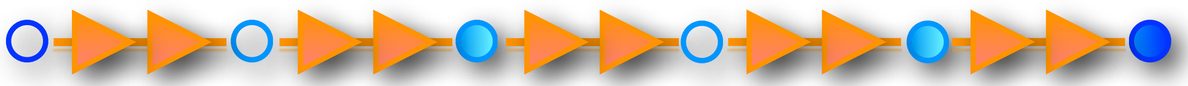
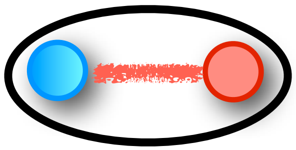
Unbroken String

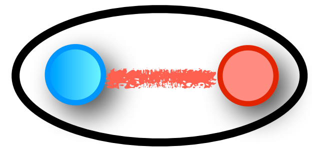


String breaking and confinement

quark *anti-quark* *flux string*

meson





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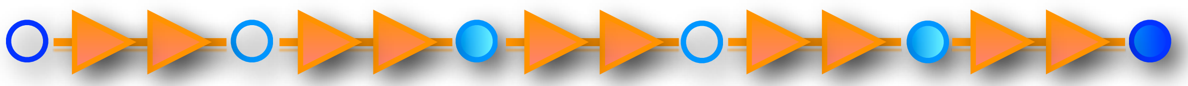
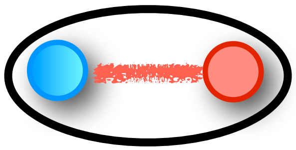


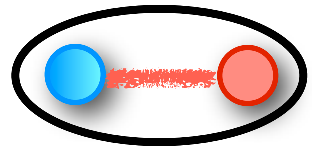
$$E = \frac{g^2}{2}(L-1) - \frac{Lm}{2}$$

String breaking and confinement

quark anti-quark flux string

meson





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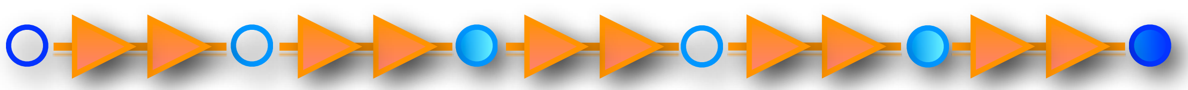

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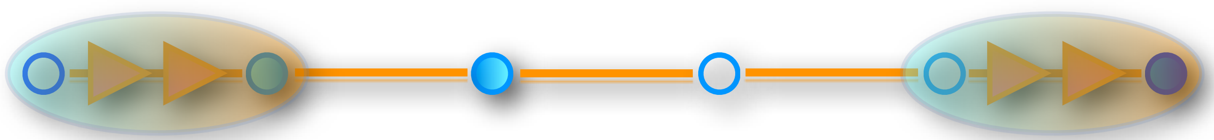
Mesons + vacuum



String breaking and confinement

quark *anti-quark* *flux string* *meson*

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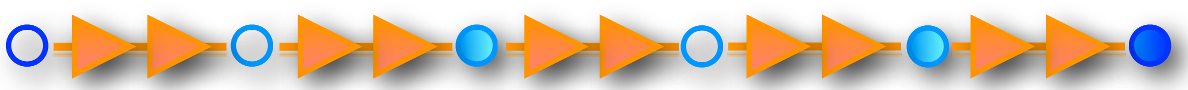

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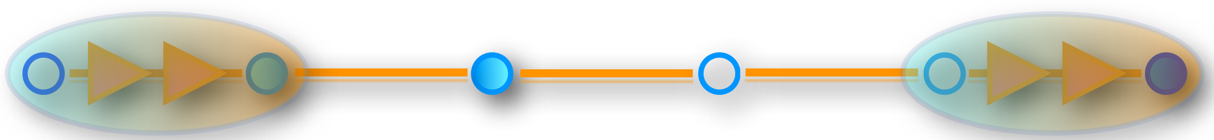
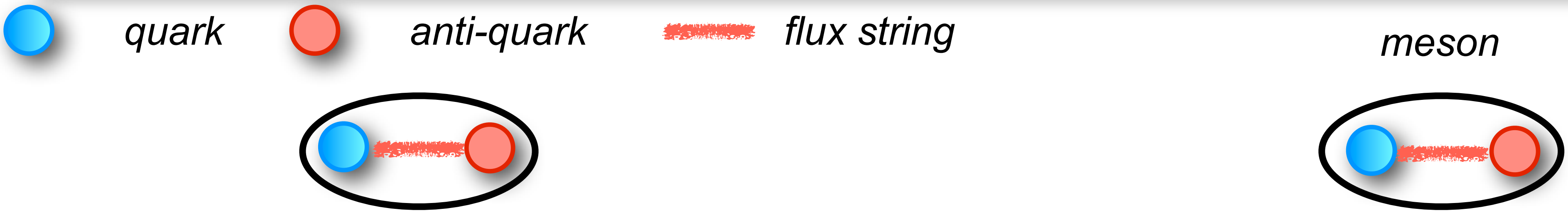


$$E^{(m)} = g^2 - \frac{(L-2)m}{2}$$

String breaking and confinement



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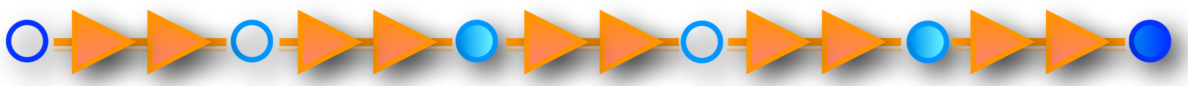

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Mesons + vacuum

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Critical string length $L^{(c)} = 2 + 2m/g^2$

QLM and cold atomic gases

Main ingredients

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QLM and cold atomic gases

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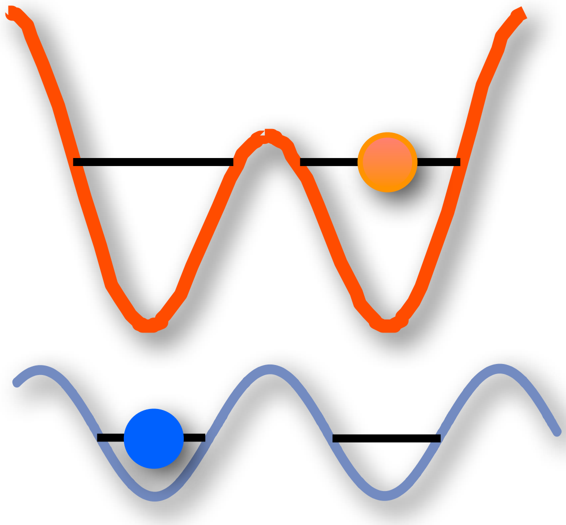
Strategy: use **Mott insulator (like) condition** to enforce **effective gauge invariant dynamics**

Building block: Single-link physics



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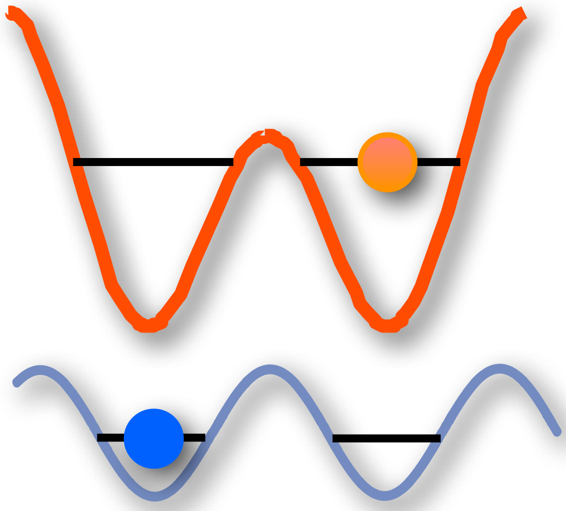

$$H = -t_f(c_1^\dagger c_2 + h.c.) - t_B(b_1^\dagger b_2 + h.c.) + U\sum_j n_{F,j} n_{B,j}$$

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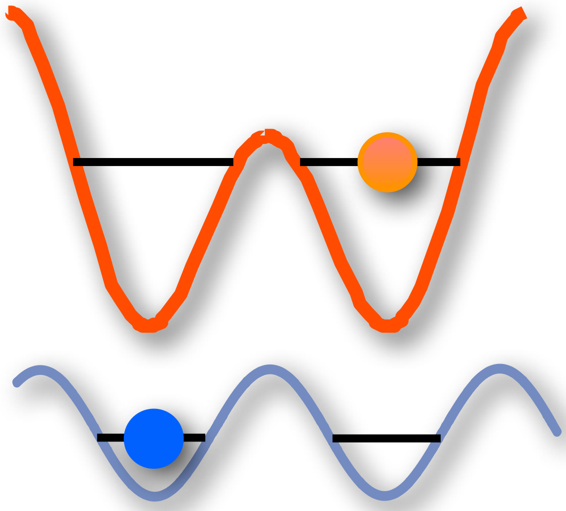

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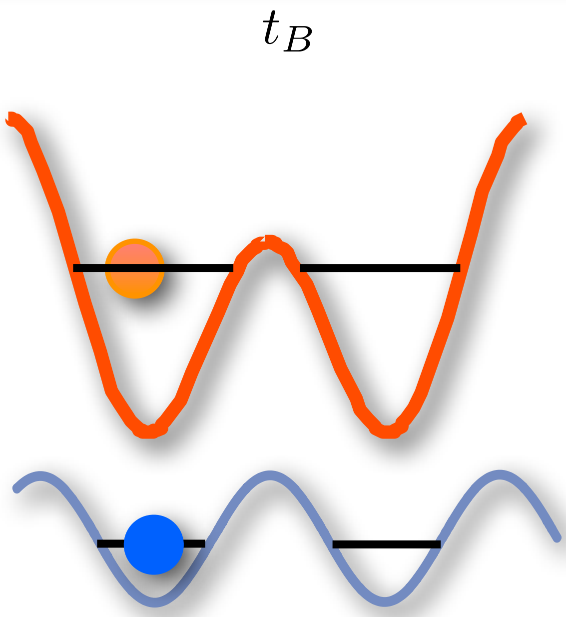

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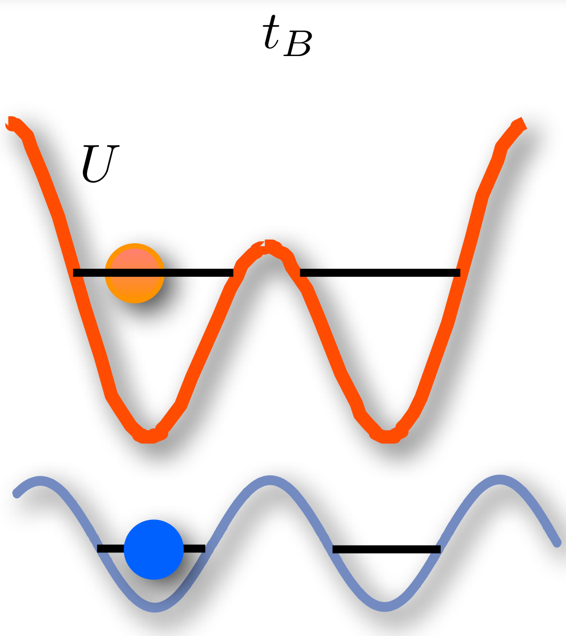

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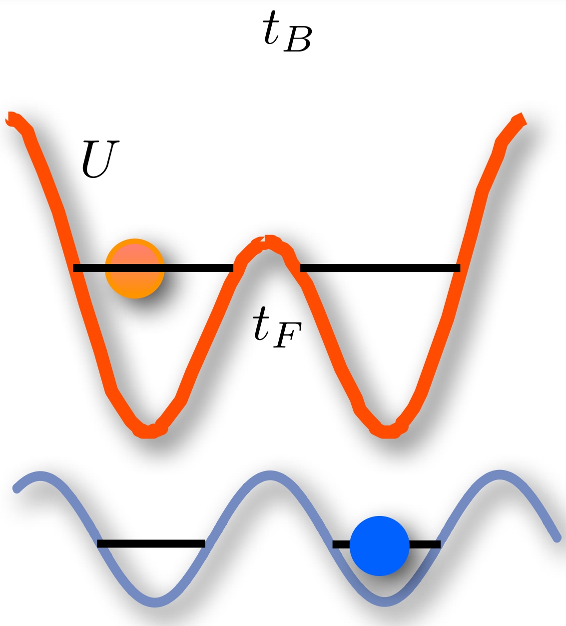

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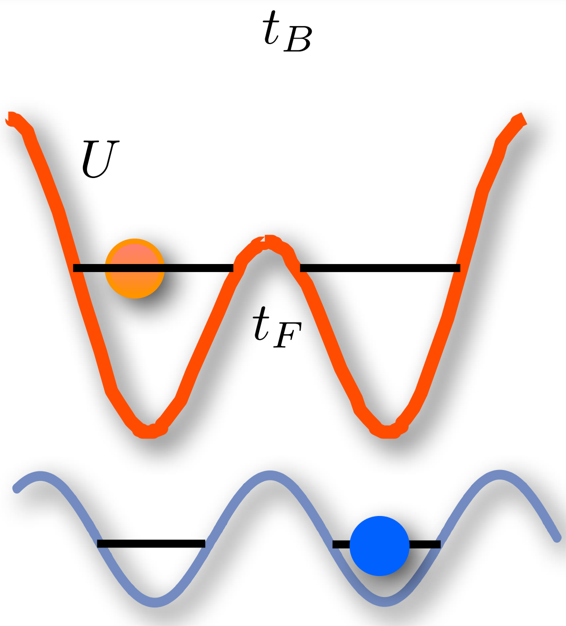

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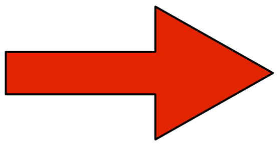
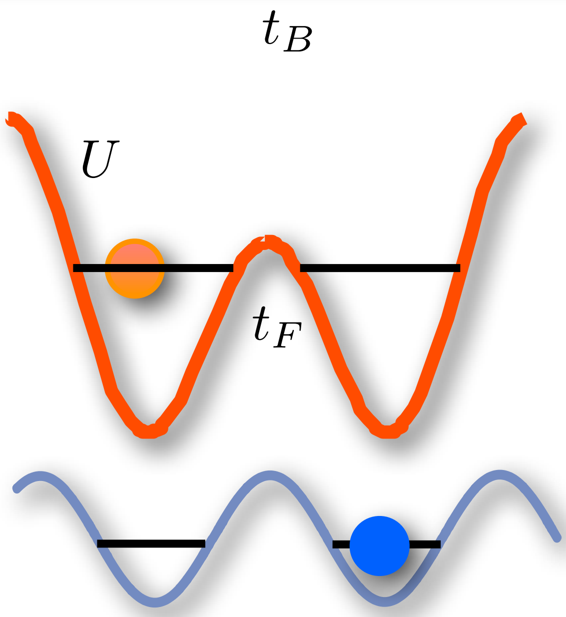


Schwinger representation:

$$S^z_{1,2} = \frac{n_{B,1} - n_{B,2}}{2}$$

$$S^+_{1,2} = b_2^\dagger b_1$$

Building block: Single-link physics



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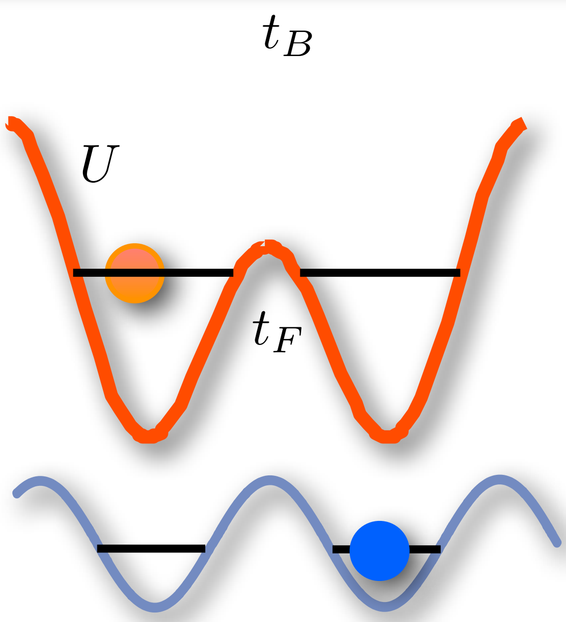
effective exchange Hamiltonian

$$H_{\text{hop}} = J\sum_x \left(\psi^\dagger_{x+1} S^\dagger_{x,x+1} \psi_x + \text{h.c.}\right)$$

Building block: Single-link physics



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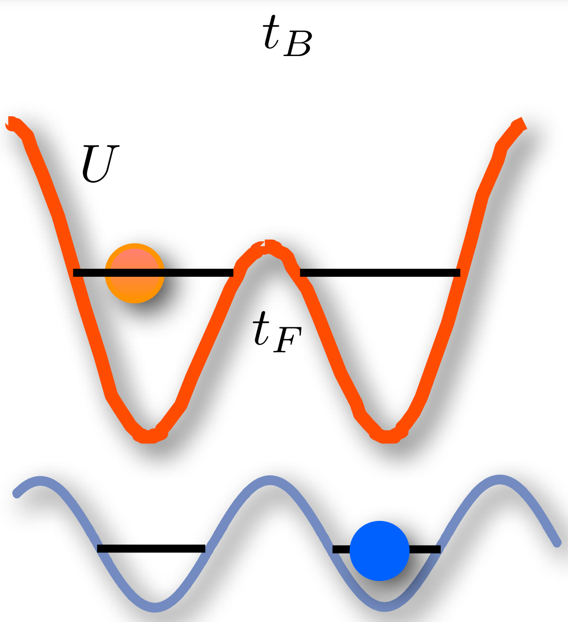
effective exchange Hamiltonian

$$H_{\text{hop}} = J\sum_x \left(\psi^\dagger_{x+1} S^\dagger_{x,x+1}\psi_x + \text{h.c.}\right)$$

Local conserved quantity!

$$G_x = n_{F,x} + n_{B,x}$$

Building block: Single-link physics



$$H = -t_f(c_1^\dagger c_2 + h.c.) - t_B(b_1^\dagger b_2 + h.c.) + U \sum_j n_{F,j} n_{B,j}$$

$$U \gg t_F, t_B$$

Schwinger representation:

$$S_{1,2}^z = \frac{n_{B,1} - n_{B,2}}{2}$$

$$S_{1,2}^+ = b_2^\dagger b_1$$

effective exchange Hamiltonian

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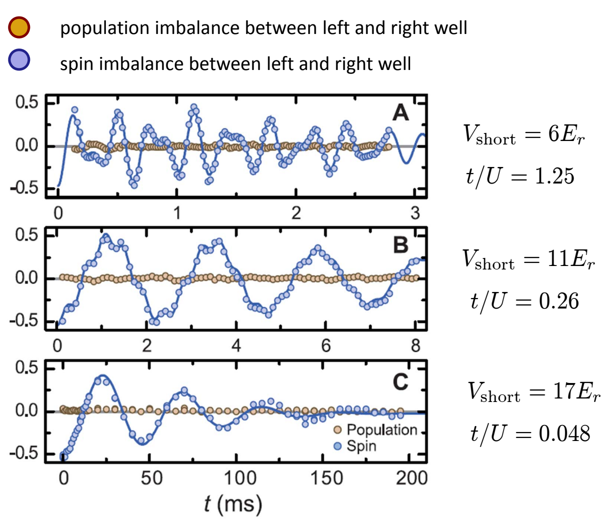
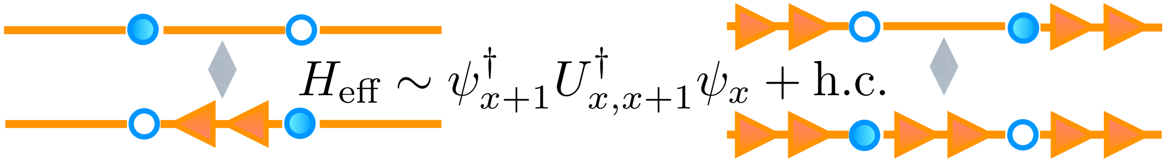


Experimentally demonstrated (with bosons) (Munich, JQI)

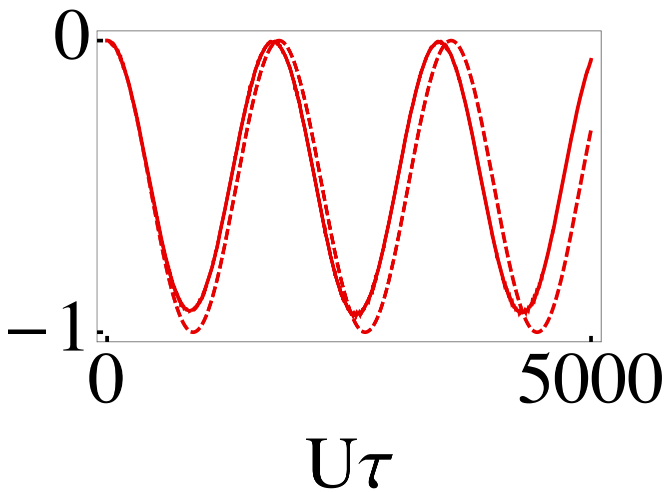
Validation of the building block



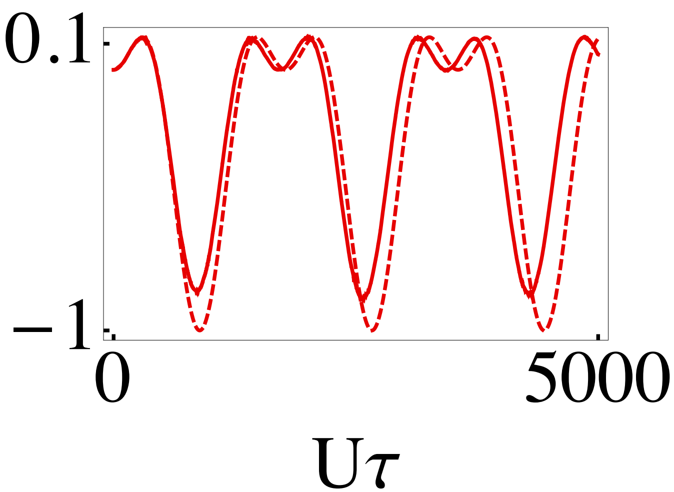
$$H_{\rm eff} \sim \psi^\dagger_{x+1} U^\dagger_{x,x+1} \psi_x + \text{h.c.}$$

$$t_b = t_F = U/20$$

$$\left\langle \left(\psi^\dagger_x \psi_x - \psi^\dagger_{x+1}\psi_{x+1} \right) E_{x,x+1} \right\rangle$$



$$\langle \psi^\dagger_x \psi_x - \psi^\dagger_{x+1}\psi_{x+1} \rangle \langle E_{x,x+1} \rangle$$



Coherent oscillations --> gauge invariant processes!

Probability of remaining in the gauge invariant subspace after a quench: **98%** (S=1), **99.88%** (S=1/2)

Many-body full implementation

Now that we know the precursor....

Many-body full implementation

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Bosons (2 internal states) in state-dependent superlattice

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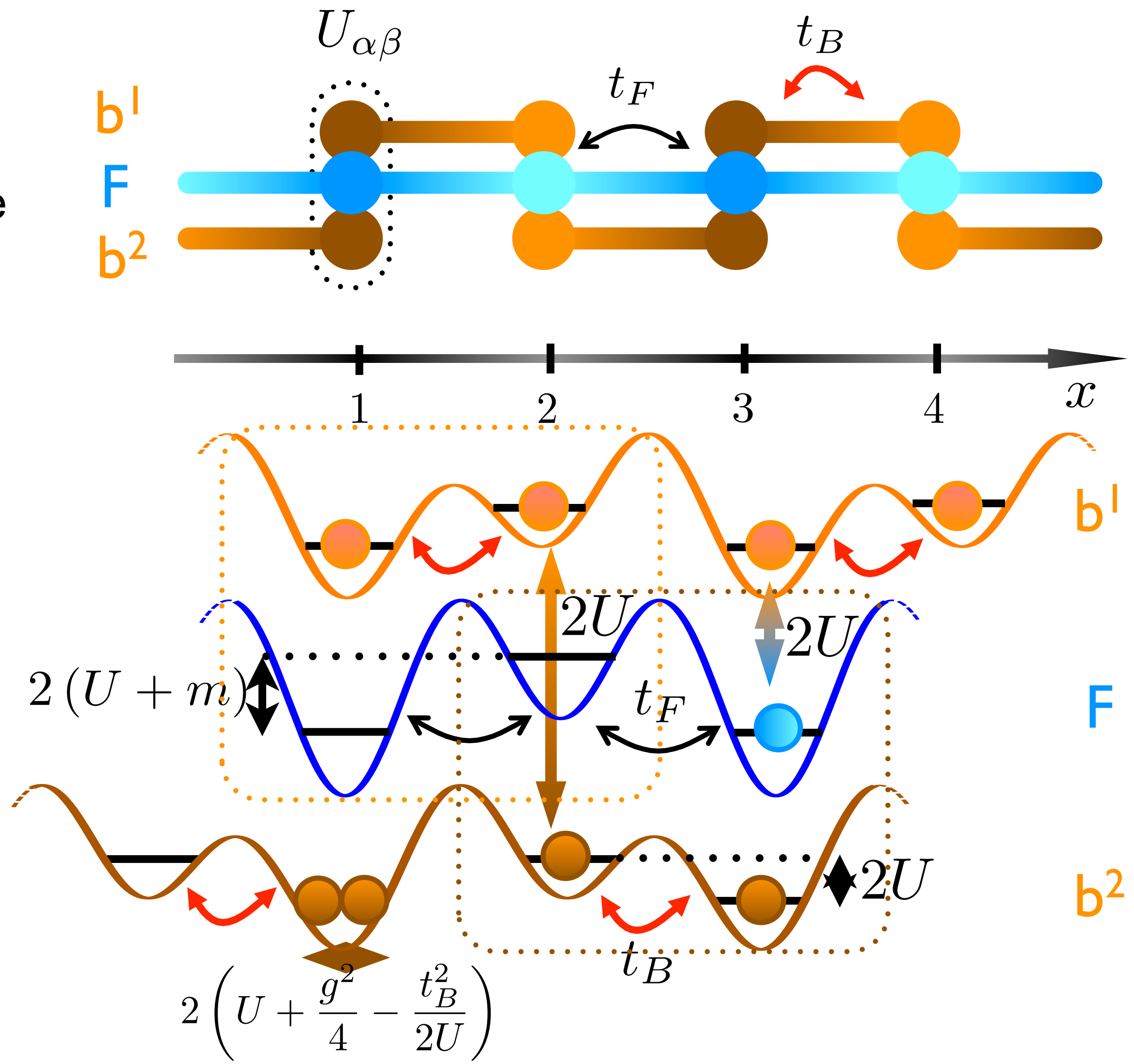

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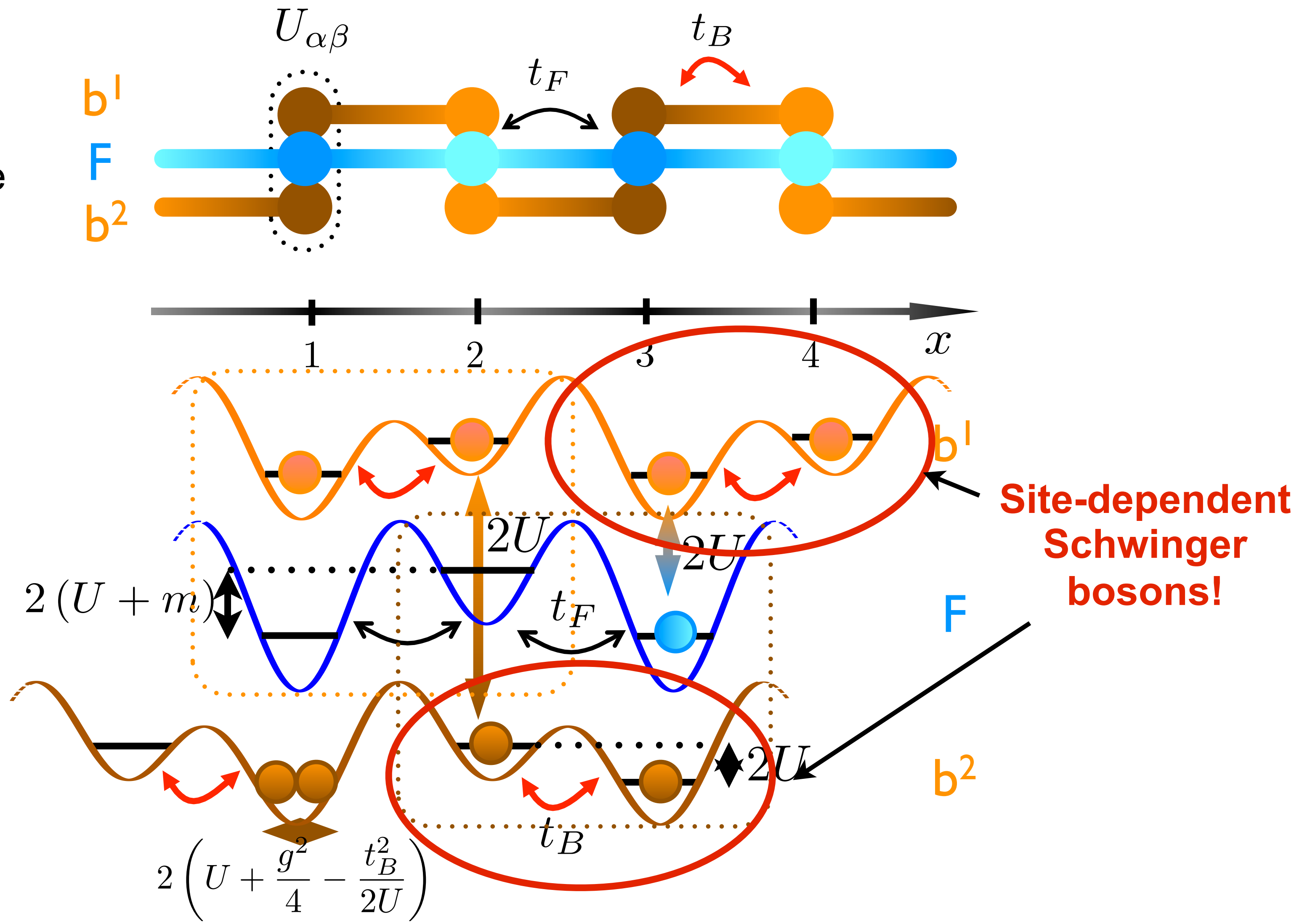

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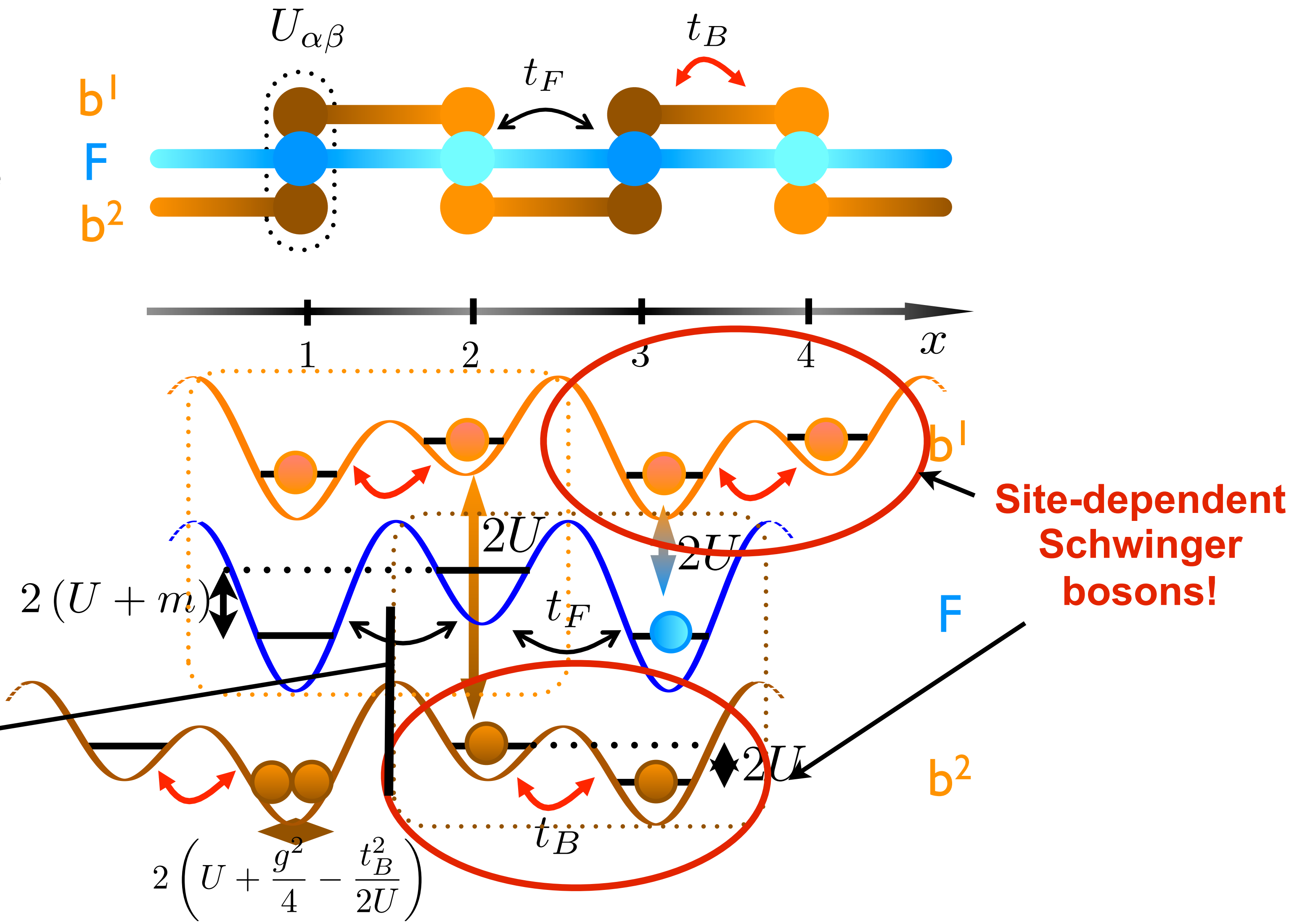
Fermions in superlattice

Tunable interactions

Suppressed tunneling between even-odd sites



Some details

Gauge generators

$$\widetilde{G}_x = n_x^F + n_x^1 + n_x^2 - 2S + \frac{1}{2}\left[(-1)^x - 1\right].$$

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Pictorial gauge invariant subspace: “*Super-Mott*” states

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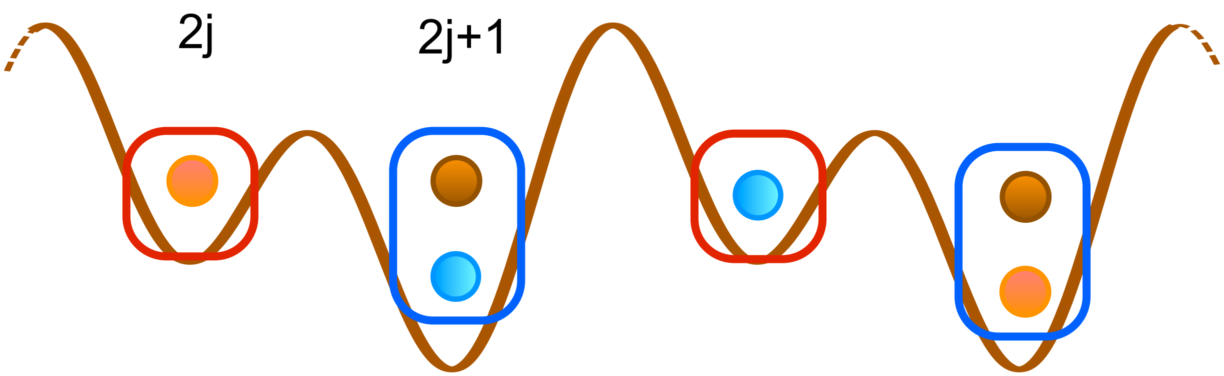

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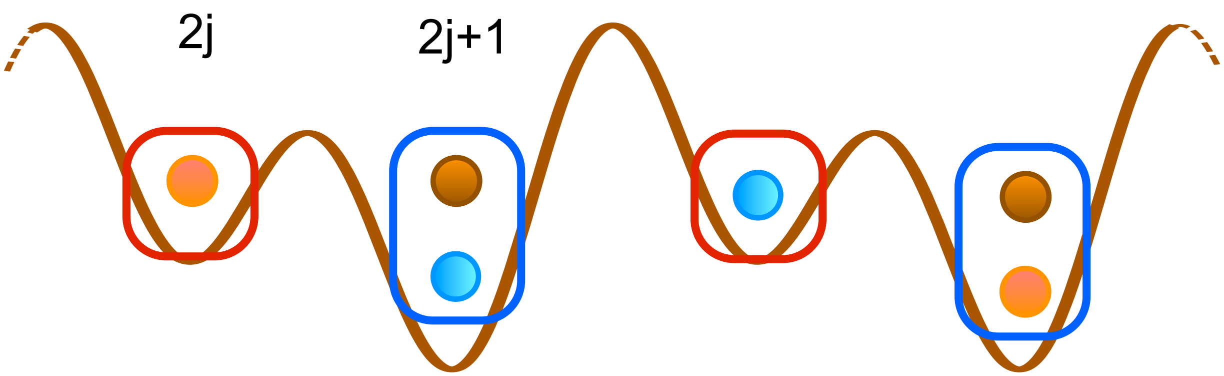
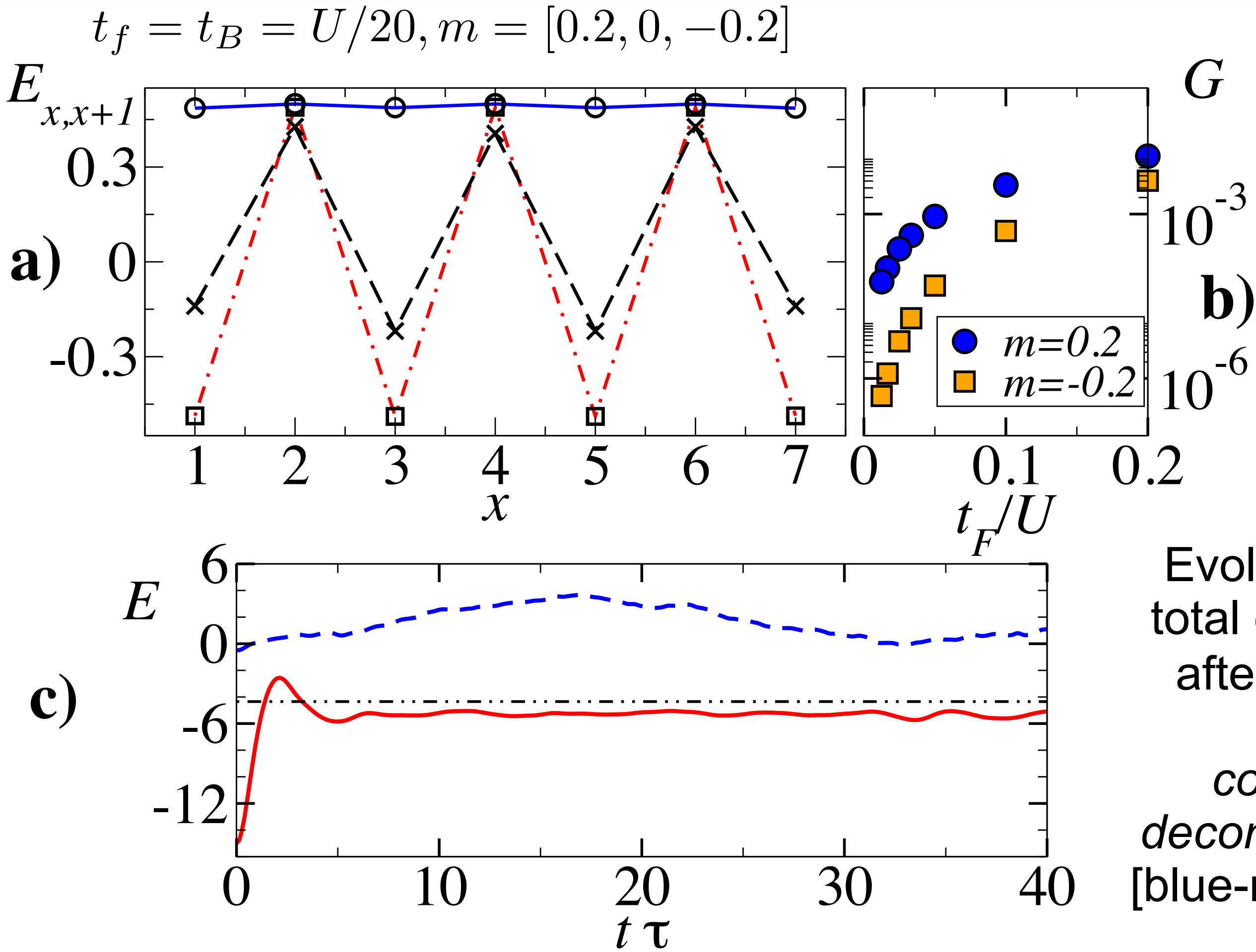


Odd sites:
2 atoms

Even sites:
1 atom

Many-body validation and dynamics

Electric field value: micro (lines) vs gauge invariant model (symbols)



Ground state “gauge invariance”

Evolution of the total electric field after a quench from *confined* to *deconfined* phase [blue-red: S=1/2-1]

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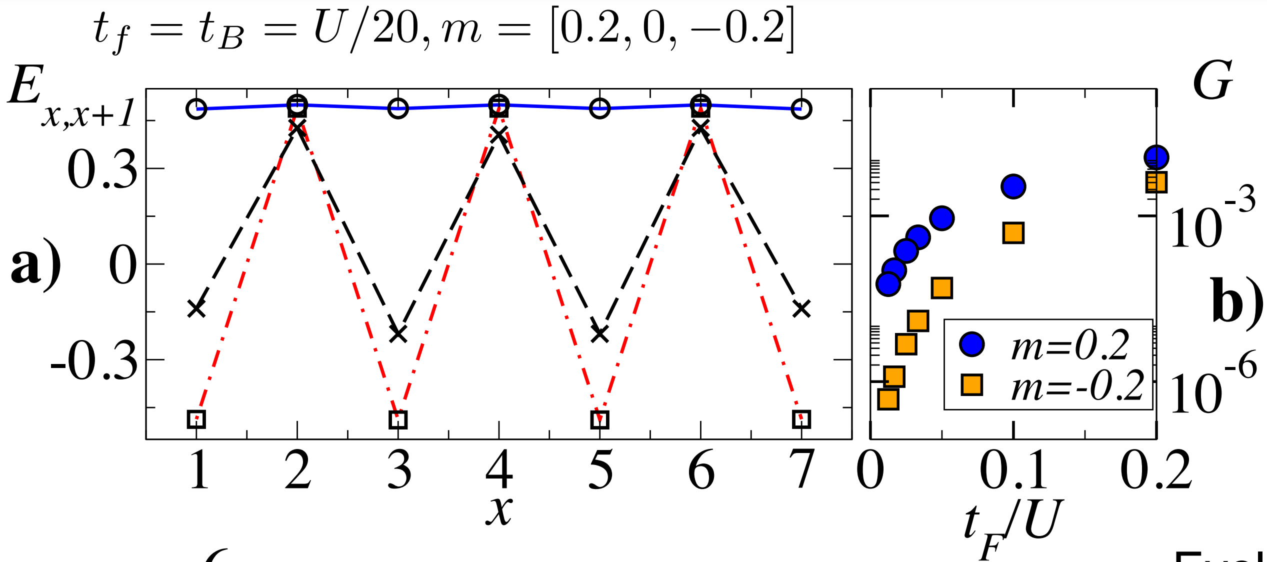

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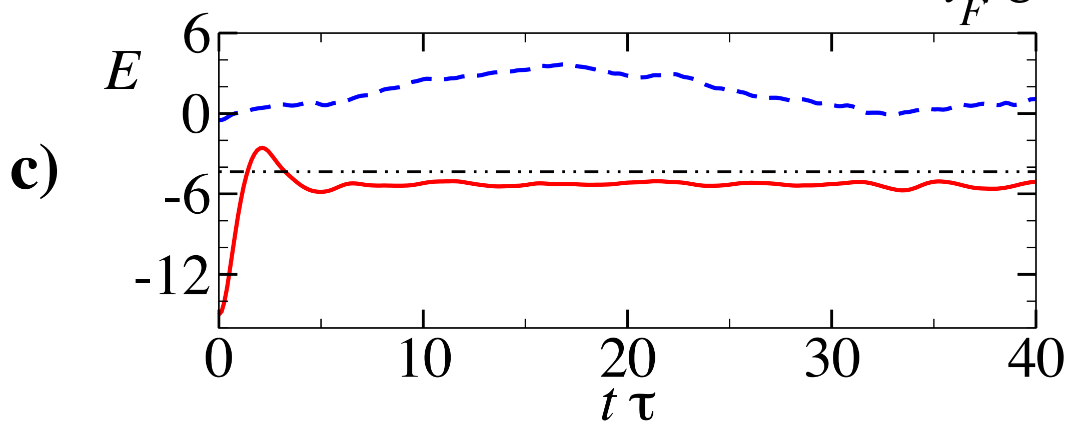

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1)Notable effects of string breaking phenomena

(disclaimer: for S=1/2, false vacuum decay...)

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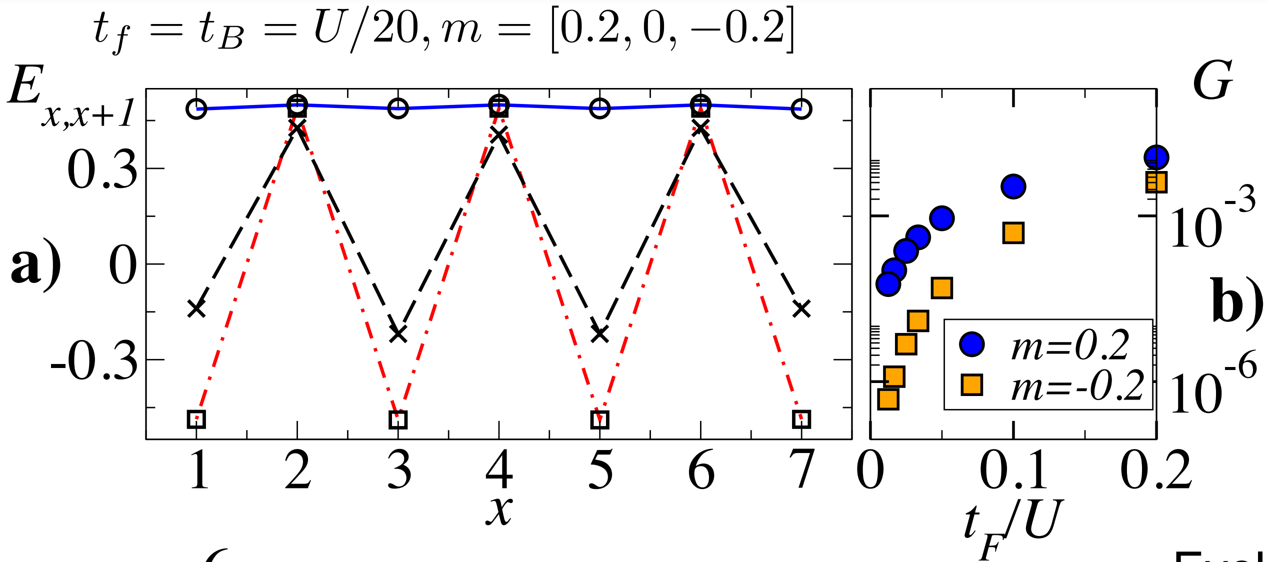

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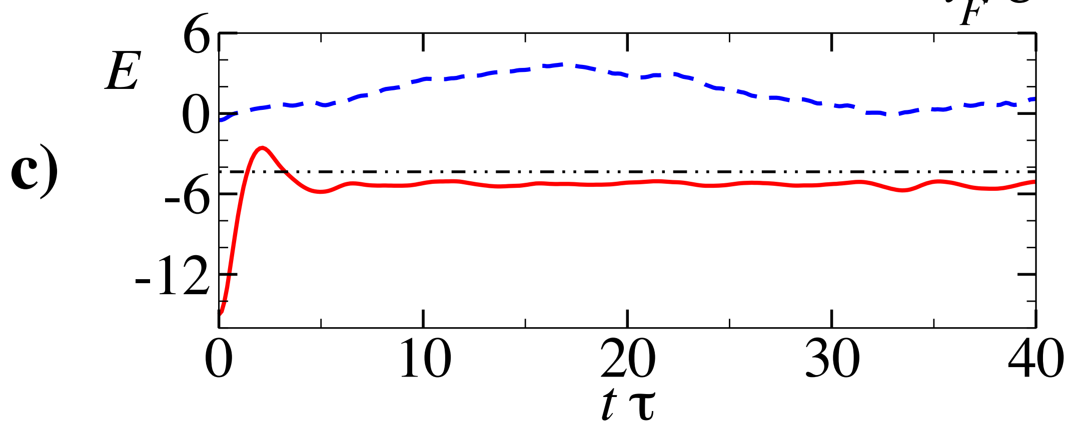

Evolution of the total electric field after a quench from *confined* to *deconfined* phase [blue-red: S=1/2-1]

2)Relaxation dynamics in gauge theories (crucial for understanding heavy-ion collisions) *can be captured by atomic simulators*

Experimental probes

Critical quantity for confinement phenomena: electric flux configuration

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Quantum link: Spin configuration!

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↓

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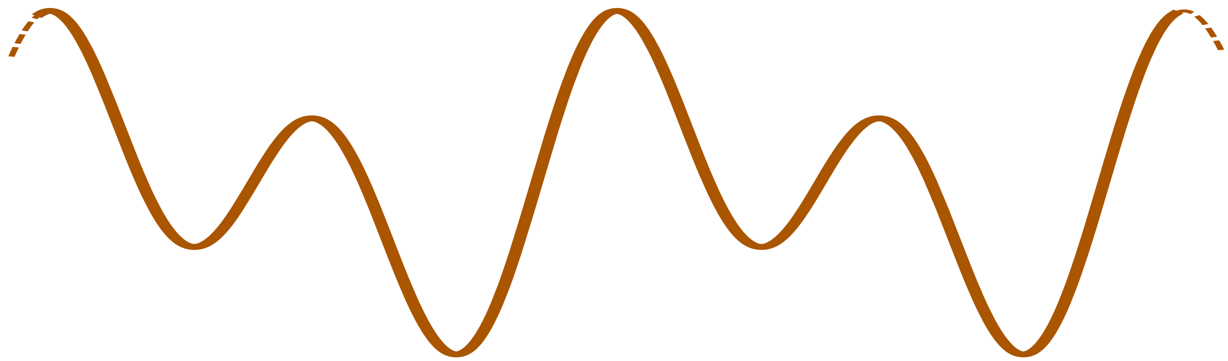

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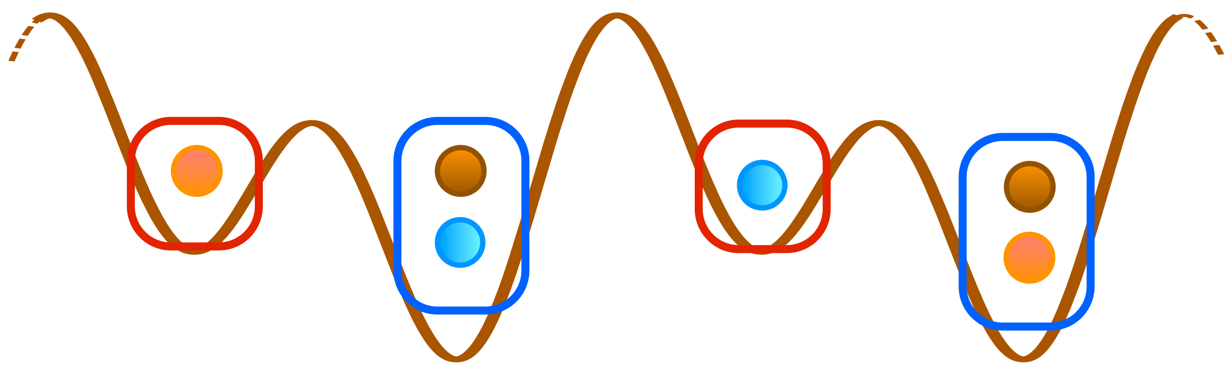

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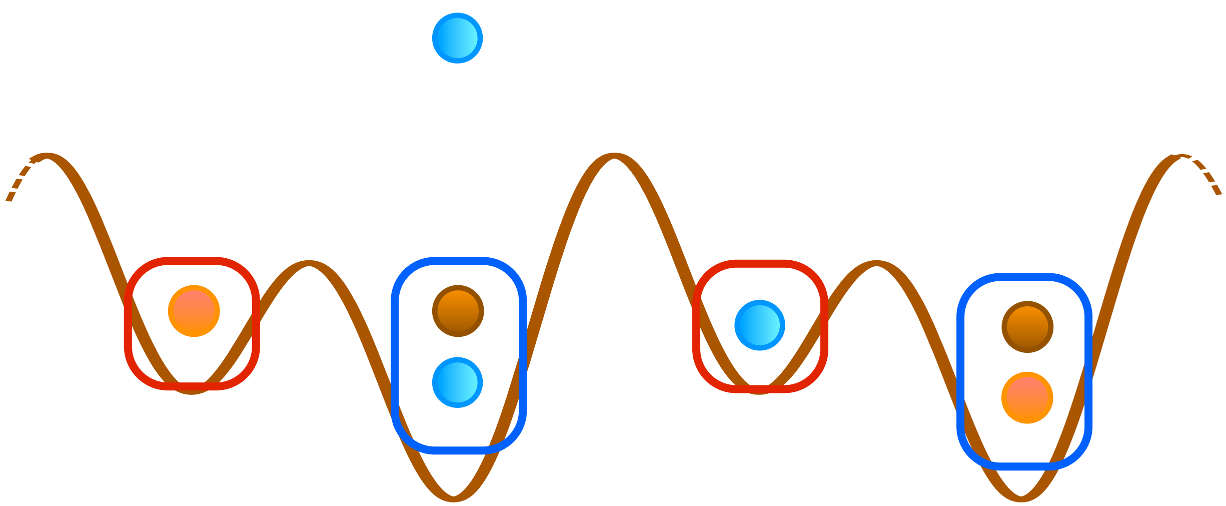

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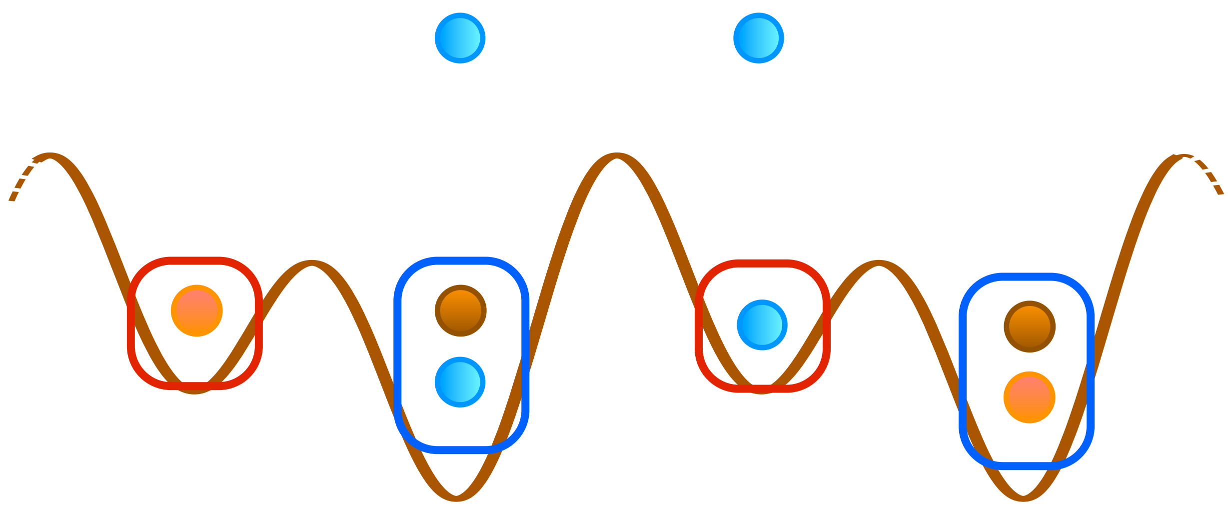

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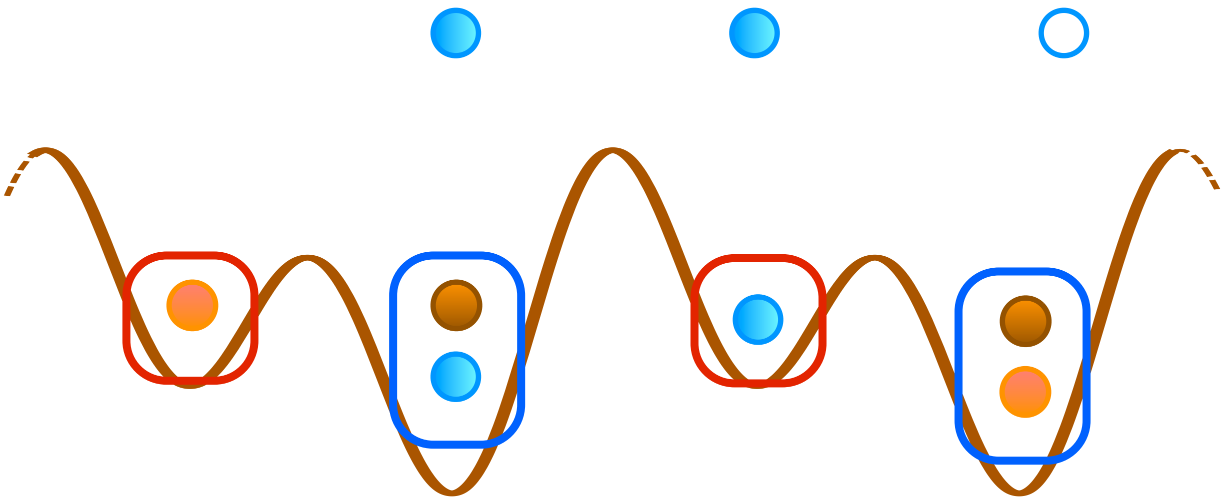

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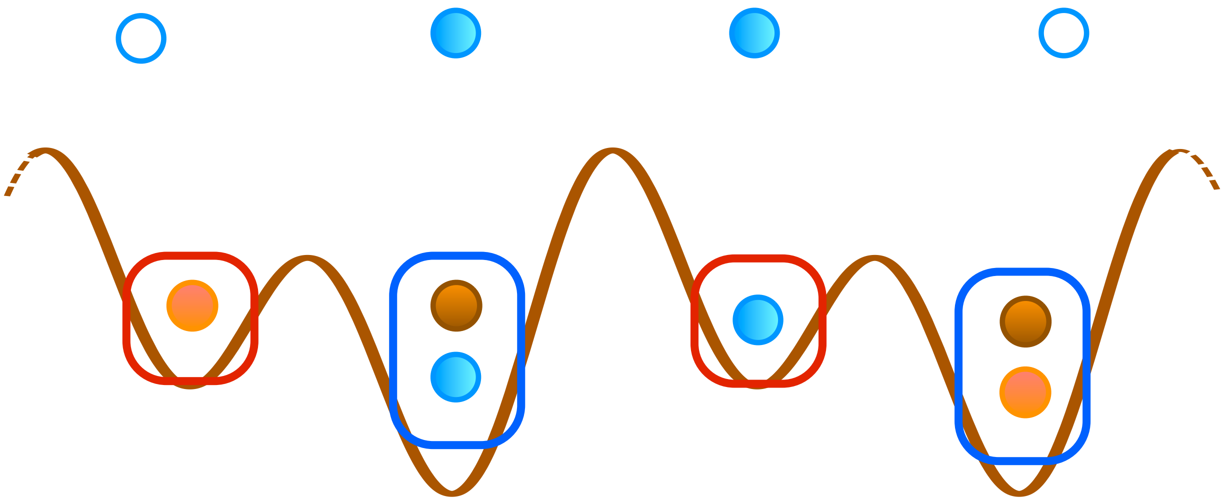

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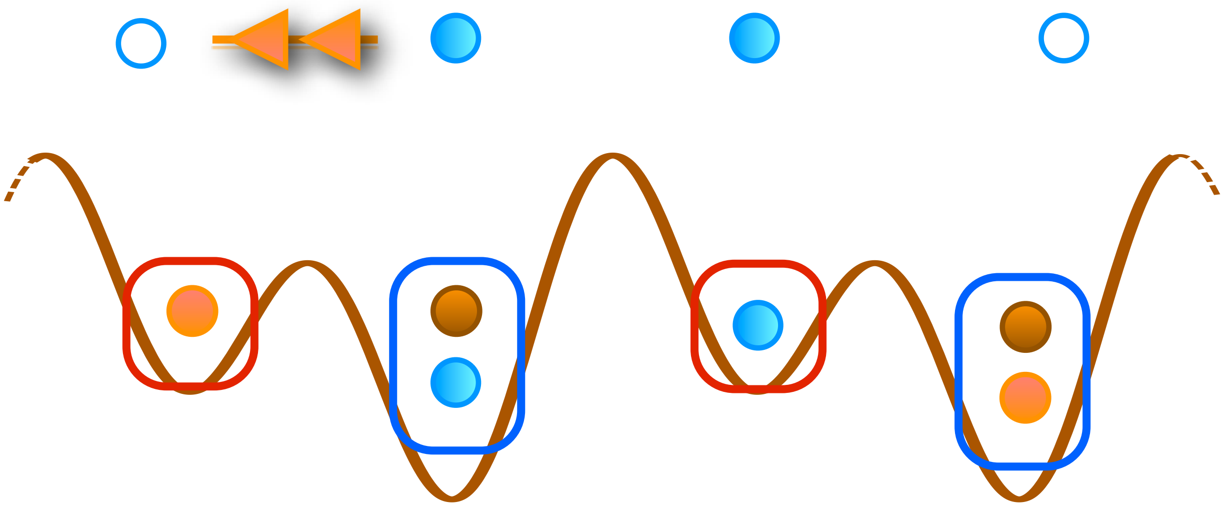

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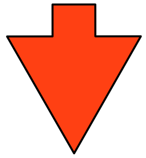

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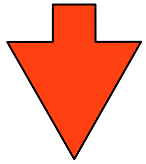

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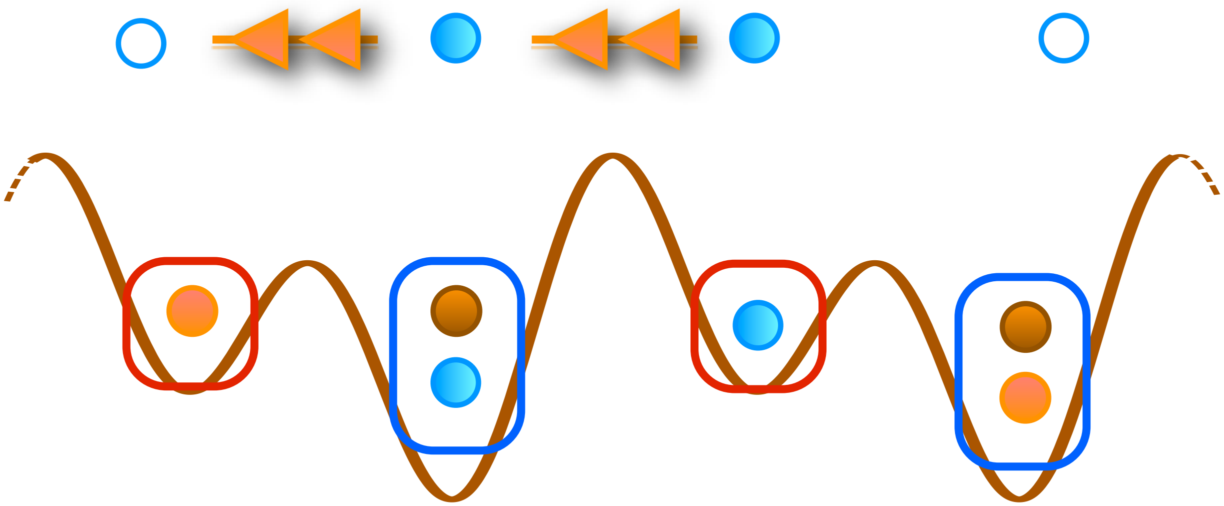

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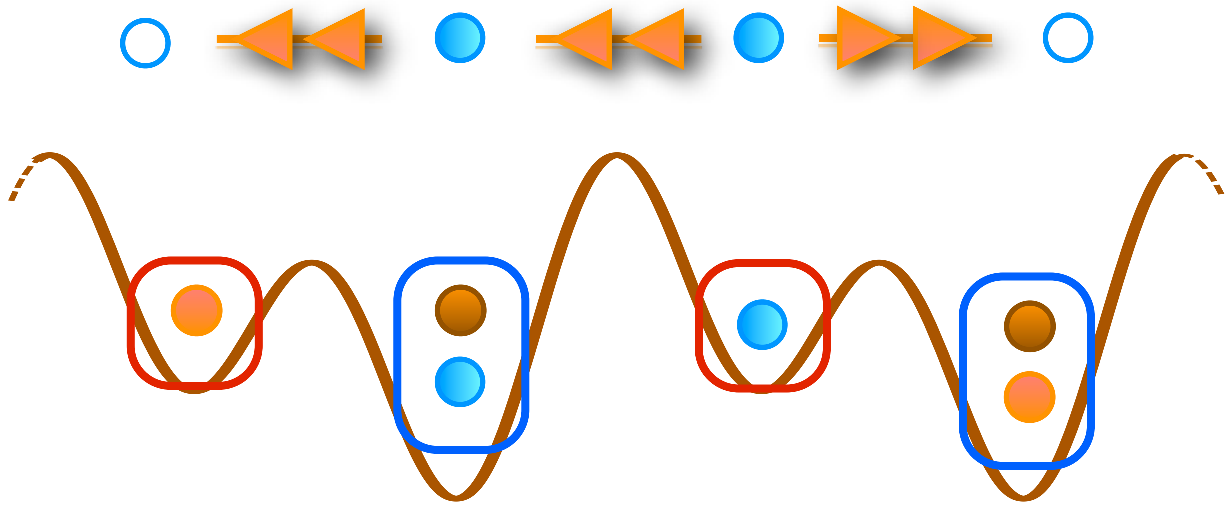

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Observability of confinement phenomena with state-of-the-art techniques

Conclusions and perspectives

Conclusions and perspectives

Thanks to E. Rico, P. Zoller (Innsbruck),
M. Mueller (Madrid), P. Stebler, D. Banerjee
and U.-J. Wiese (Bern)

Conclusions and perspectives

Non-Abelian extensions? Far from being trivial in quantum link formulation (8-body interactions...)

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Finite-temperature confinement/deconfinement phase transition (Abelian quark-gluon plasma?)

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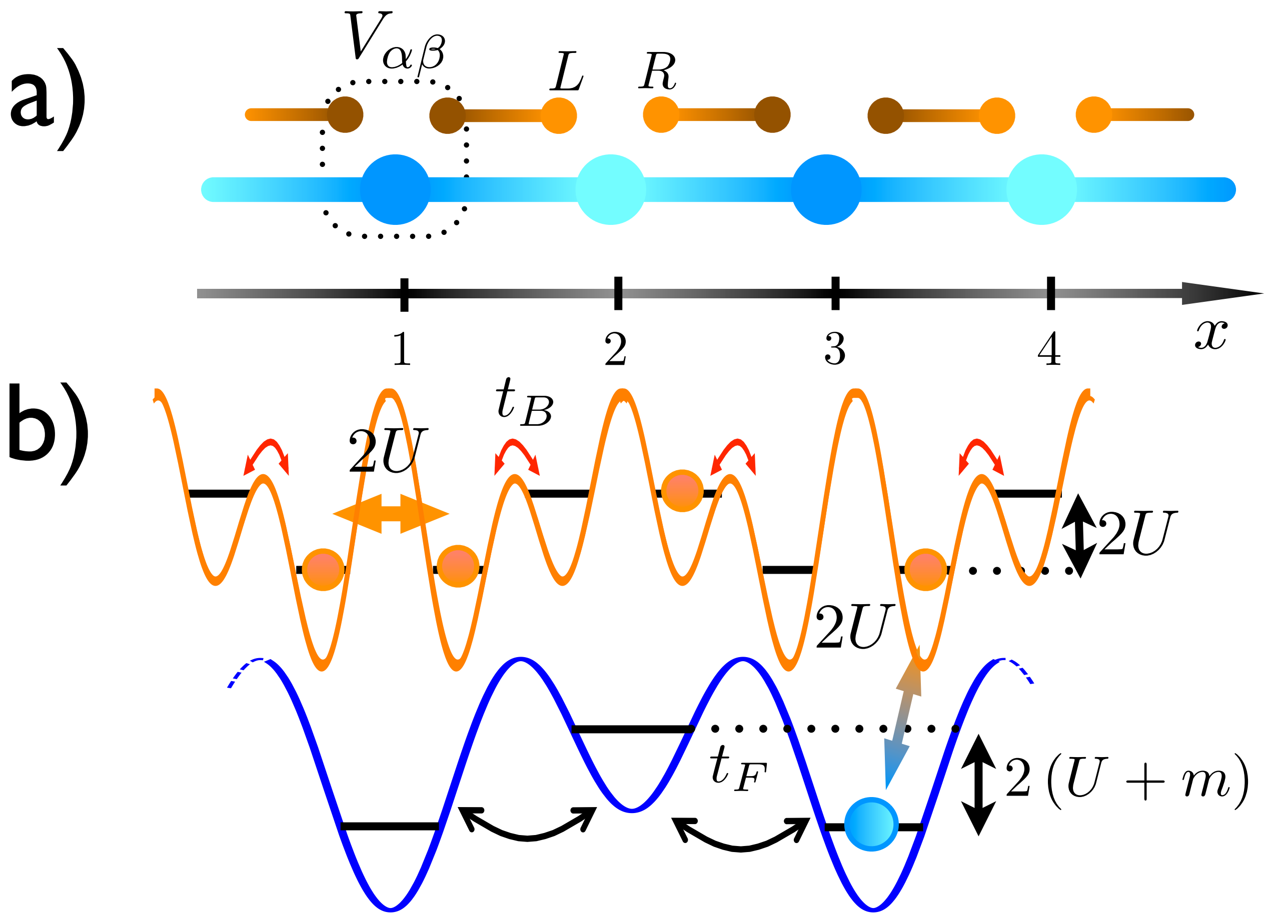
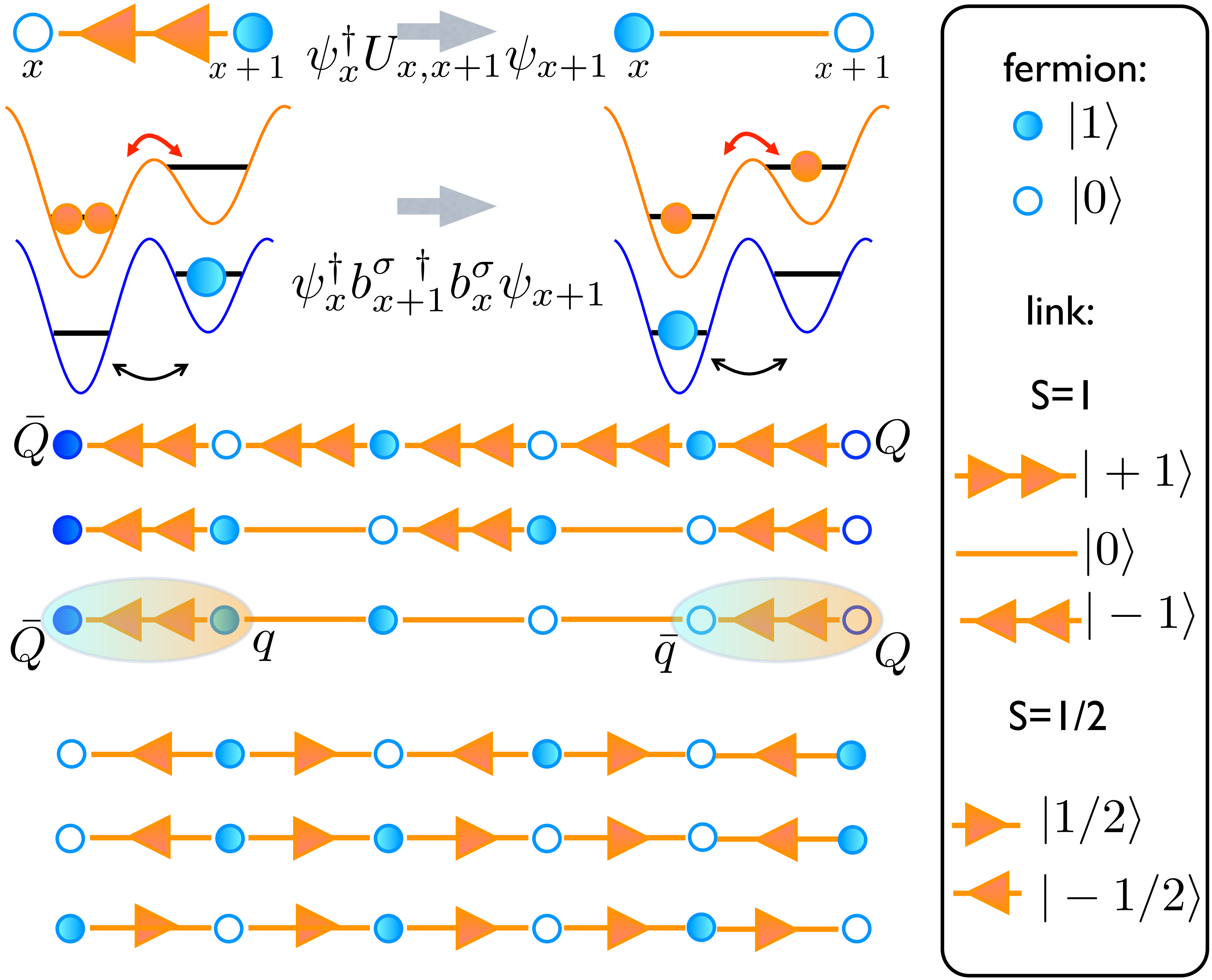
Alternative setup: dipole interactions



Building block: additional info



x
$x+1$
$\psi_x^\dagger U_{x,x+1} \psi_{x+1}$
x
$x+1$
$\psi_x^\dagger b_{x+1}^{\sigma\ \dagger} b_x^\sigma \psi_{x+1}$
$\bar{Q}$
Q
$\bar{Q}$
q
$\bar{q}$
Q
fermion:
$|1\rangle$
$|0\rangle$
link:
S=1
$|+1\rangle$
$|0\rangle$
$|-1\rangle$
S=1/2
$|1/2\rangle$
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