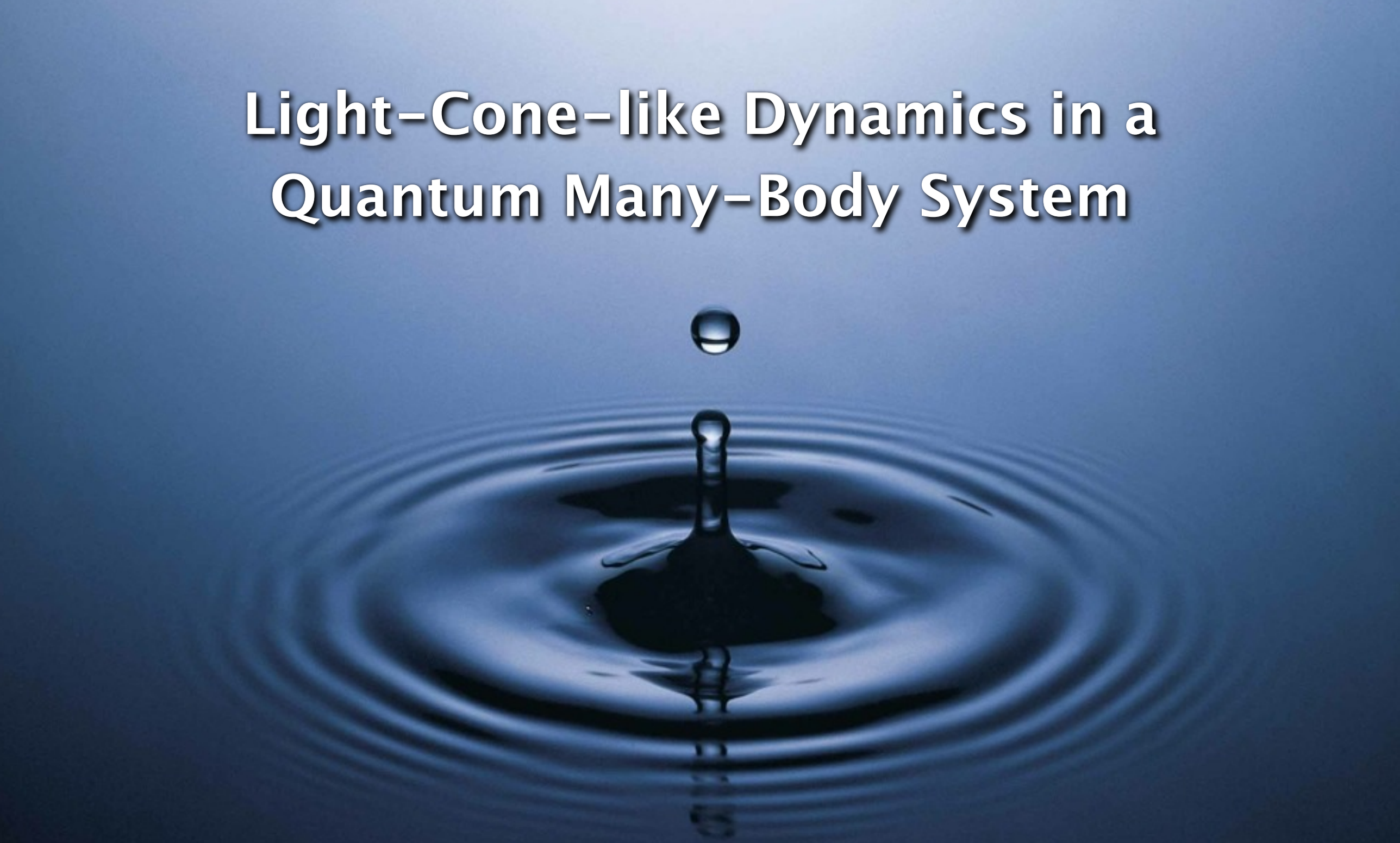
Light-Cone-like Dynamics in a Quantum Many-Body System



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MPI for Quantum Optics

8 June 2012 – Lyon

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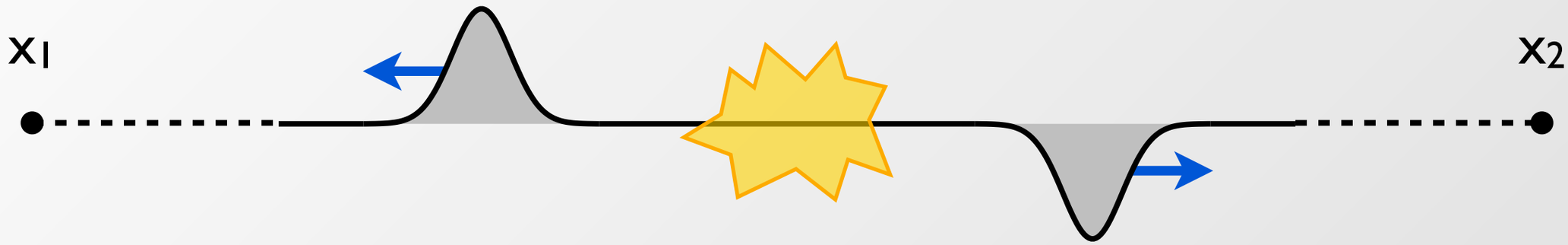
Publications: Nature **481**, 484 (2012)
PRA **85**, 053625 (2012)

Spreading of correlations: Why bother?

Out-of-equilibrium dynamics



- **Relaxation dynamics** of a quantum many-body system:

$$\langle O(x_1)\, O(x_2)\rangle_{\text{initial}} \qquad \langle O(x_1)\, O(x_2)\rangle_{\text{final}}$$

redistribution of correlations

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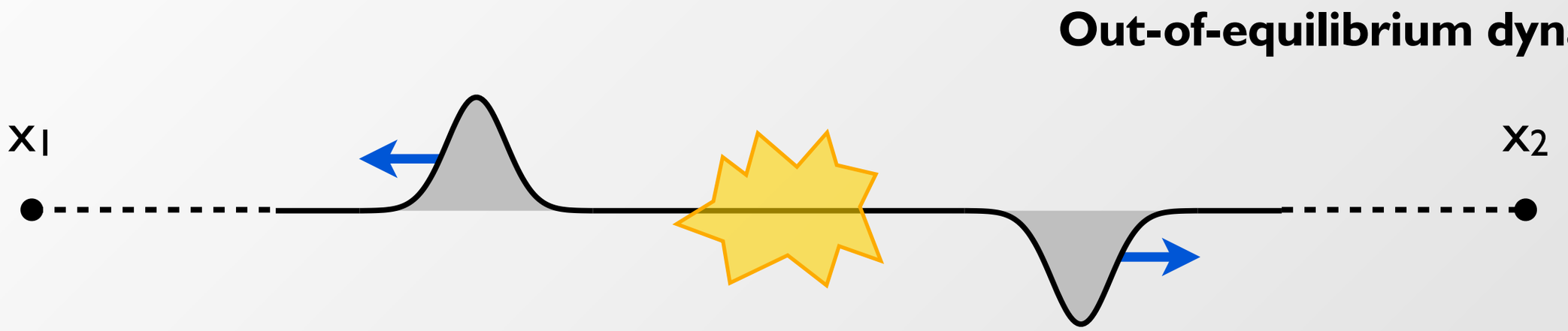

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redistribution of correlations

- **Transport of information** in quantum channels

How fast can correlations propagate?

Equation of motion for the many-body field operator:

$$i\hbar\partial_t\hat{\Psi}(x) = \frac{-\hbar^2}{2m}\partial_x^2\hat{\Psi}(x) + \int dx'\, V(x'-x)\hat{\Psi}^\dagger(x')\hat{\Psi}(x')\,\hat{\Psi}(x)$$

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In some cases

Lorentz invariant effective Lagrangian

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c = sound velocity

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In the generic case

Is the dynamics even local?

Lieb–Robinson Bounds

E. H. Lieb and D. W. Robinson, Comm. Math. Phys. 28, 251 (1972)

Quantum spins with finite-range interactions:

$$\left\langle [O(x_1, 0), O(x_2, t)] \right\rangle \leq \alpha \exp(\beta(v t - |x_1 - x_2|))$$

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"this propagation has many features in common with the propagation of waves in continuous matter"

Consequences of Lieb–Robinson bounds

"**Non-relativistic theories** defined by **local Hamiltonians**, such as the ones used in condensed matter physics, have a dynamics that **does not violate locality**"

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Two examples:

- **Goldstone's theorem**
  for any continuous symmetry spontaneously broken, there exists a massless particle

  W. Wreszinski, Fortschr. Phys. 35, 379 (1987)

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  W. Wreszinski, Fortschr. Phys. 35, 379 (1987)

- **Exponential clustering theorem**
  the vacuum of massive particles has exponentially decaying spatial correlations

  M. Hastings, T. Koma, Commun. Math. Phys. 265, 780 (2006)
  B. Nachtergaele, R. Sims, Commun. Math. Phys. 265, 119 (2006)

Beyond quantum spins

- The proof for the existence of a Lieb–Robinson bound usually relies on 3 conditions: **finite-range interactions**, **lattice** and **finite local Hilbert space**

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$$\hat{H} = -J \sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

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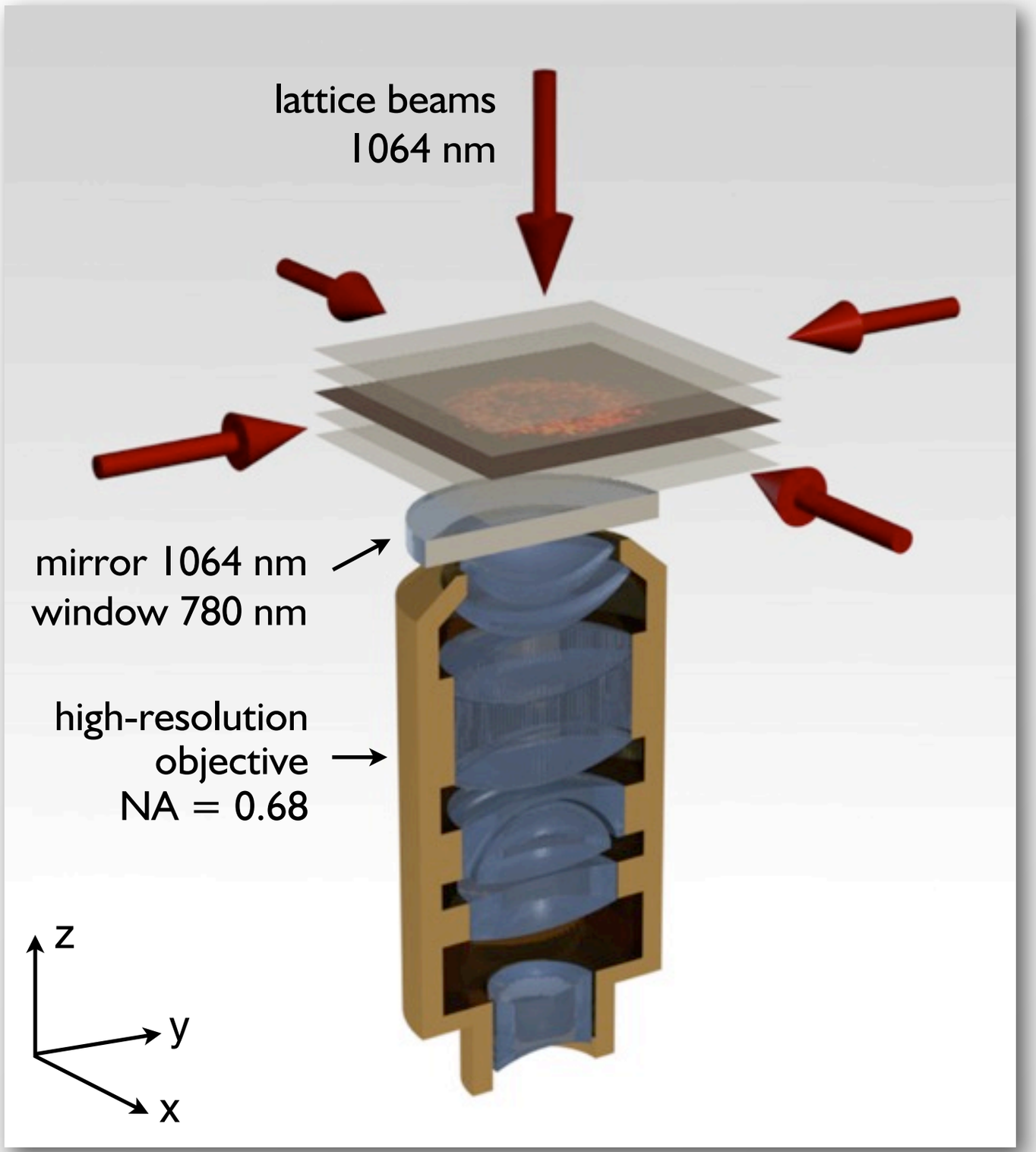
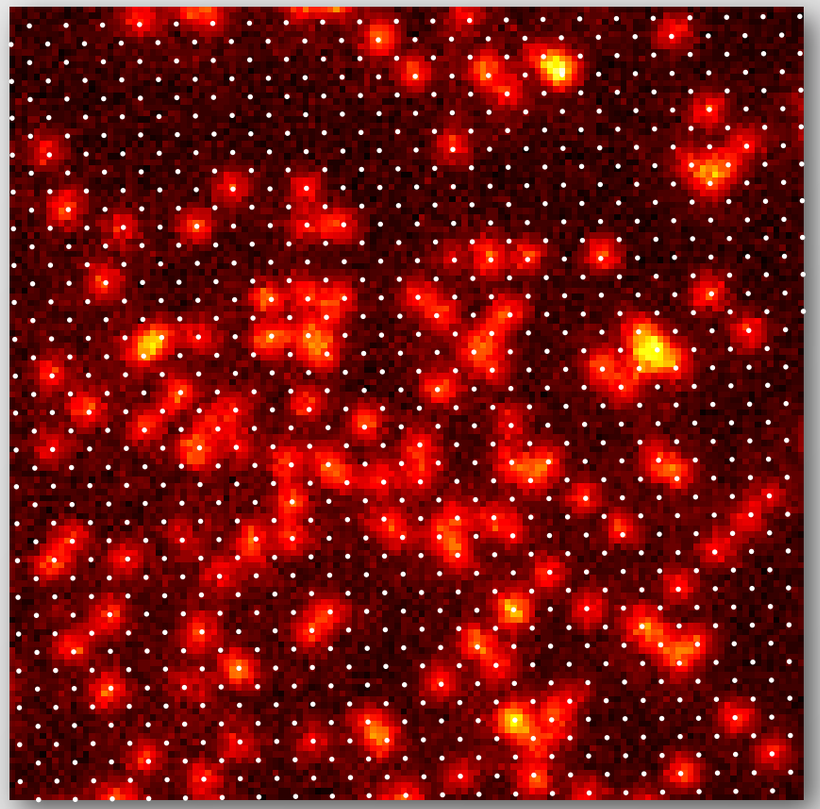
No strict LR bound has been found so far in the BH model

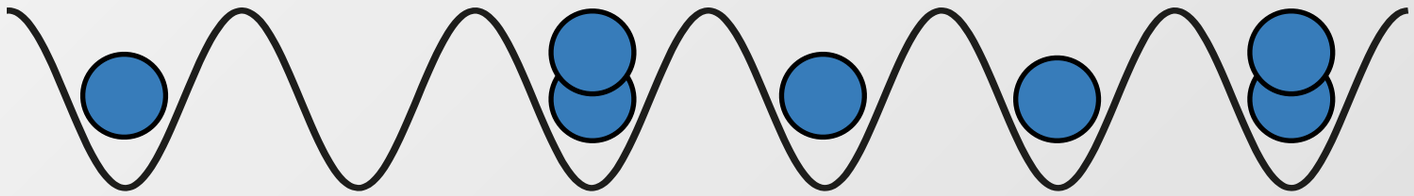
Our apparatus



- optical lattice
- 2D or 1D geometry
- few 100 atoms (bosons)
- **in-situ** fluorescence imaging **single-site** and **single-atom** resolution



Fluorescence imaging in an optical lattice



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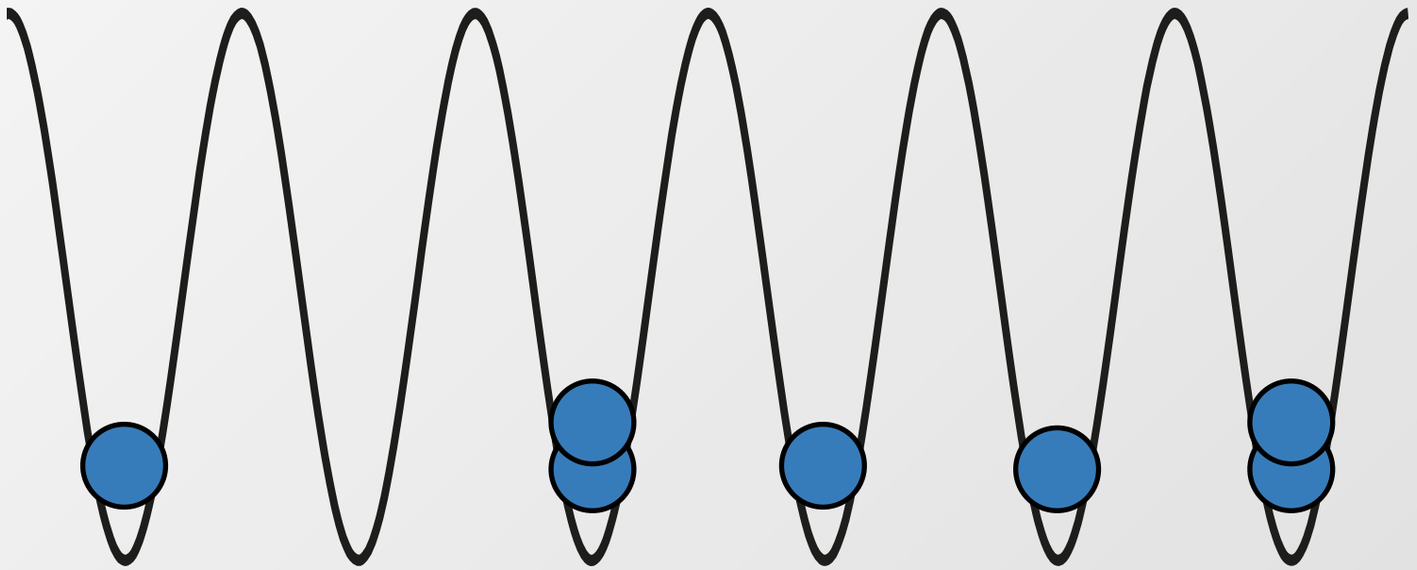

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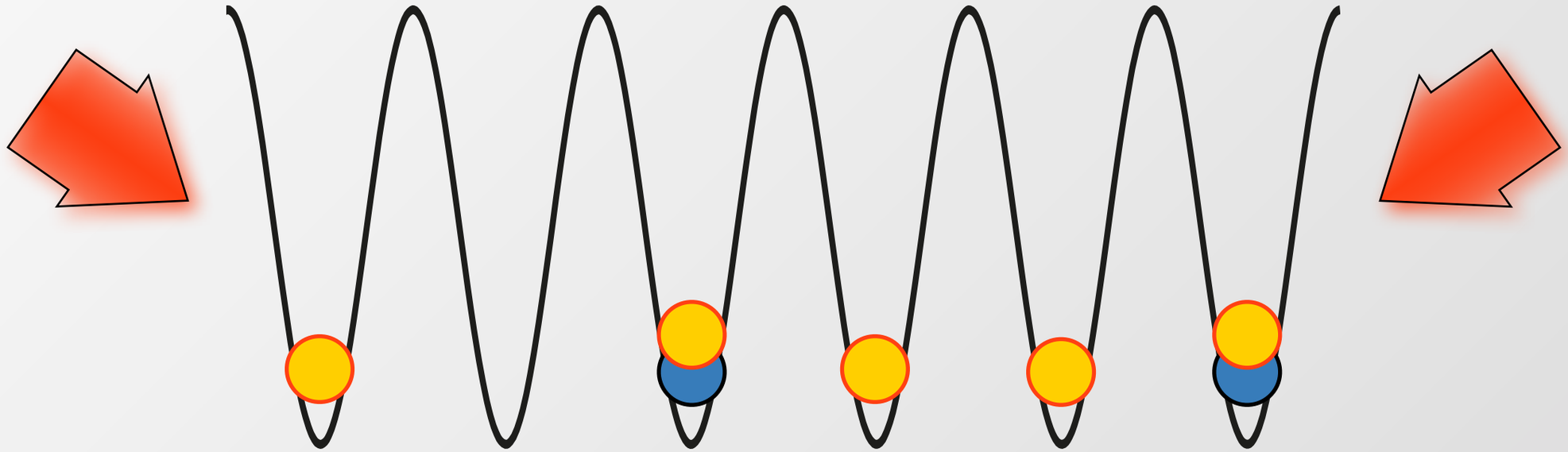

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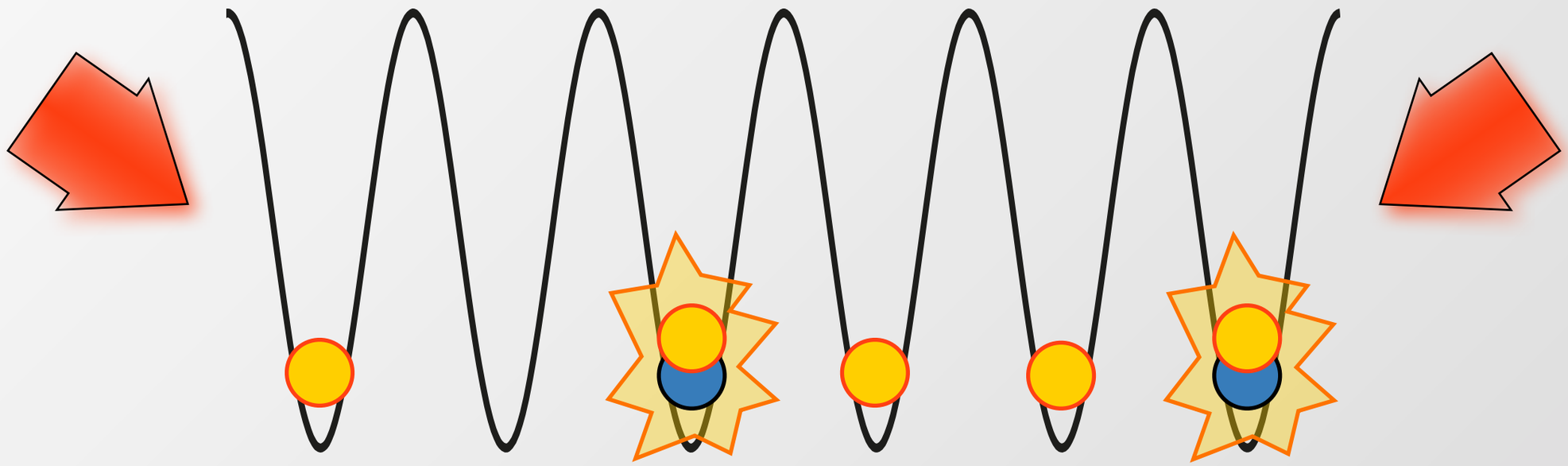

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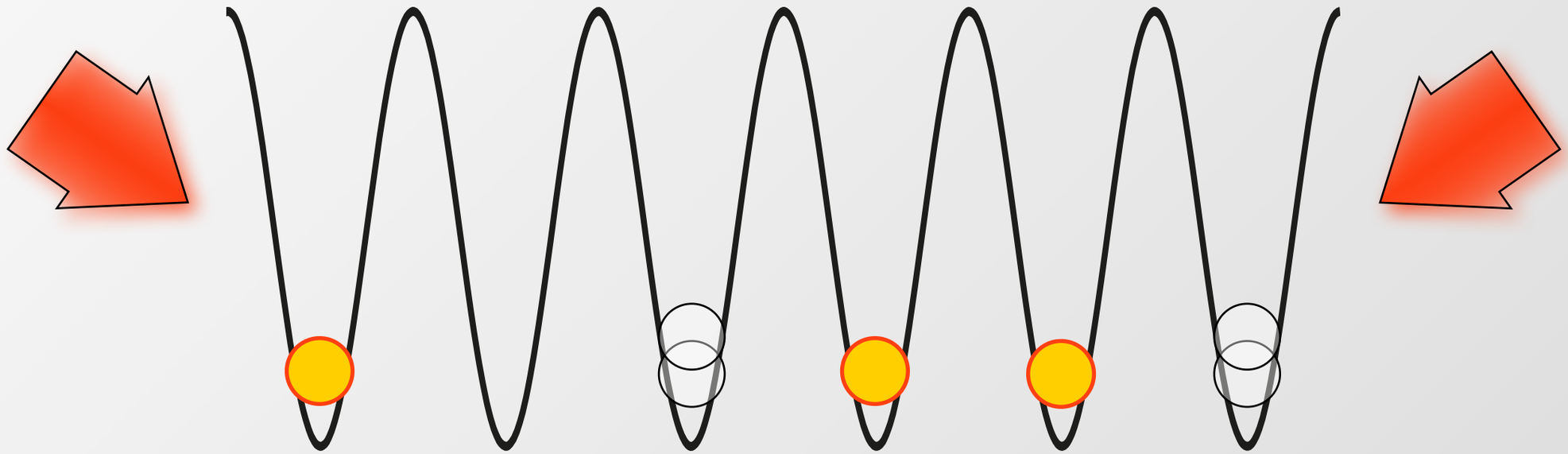

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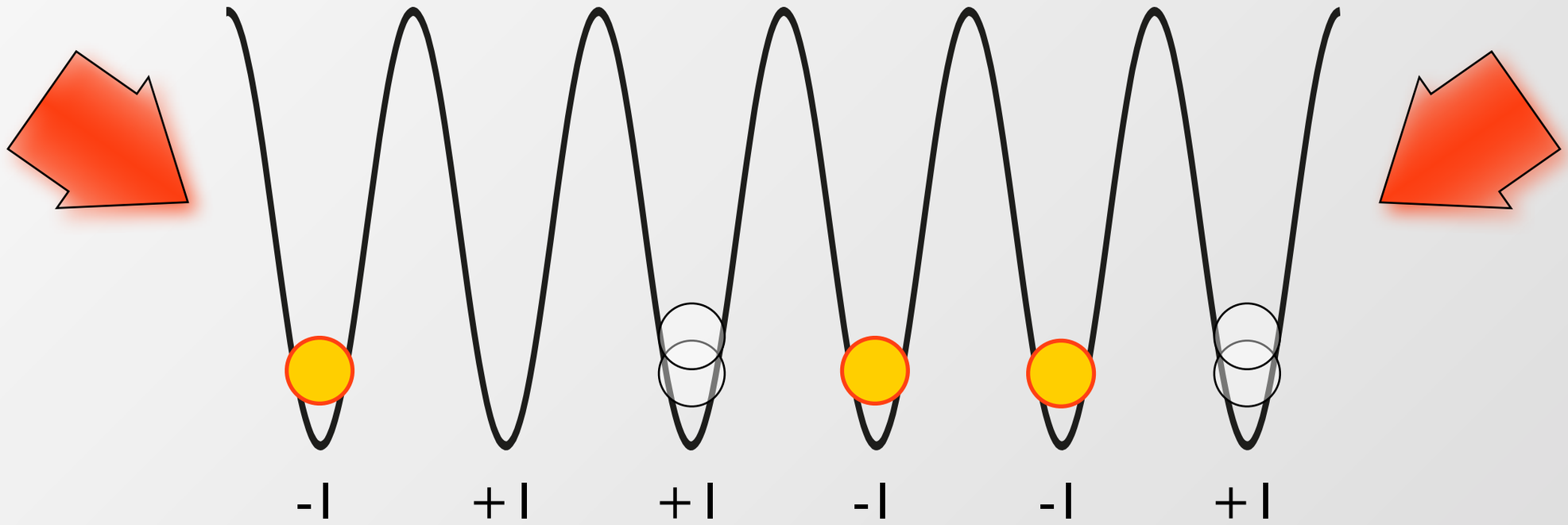
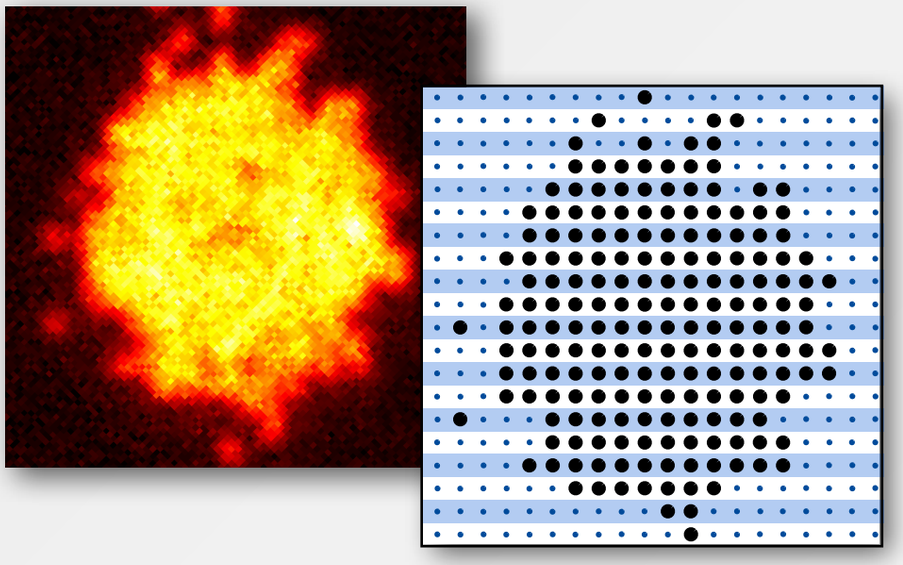


Access only the parity of the initial distribution

Direct access to spatial correlations

deep in the Mott insulator phase



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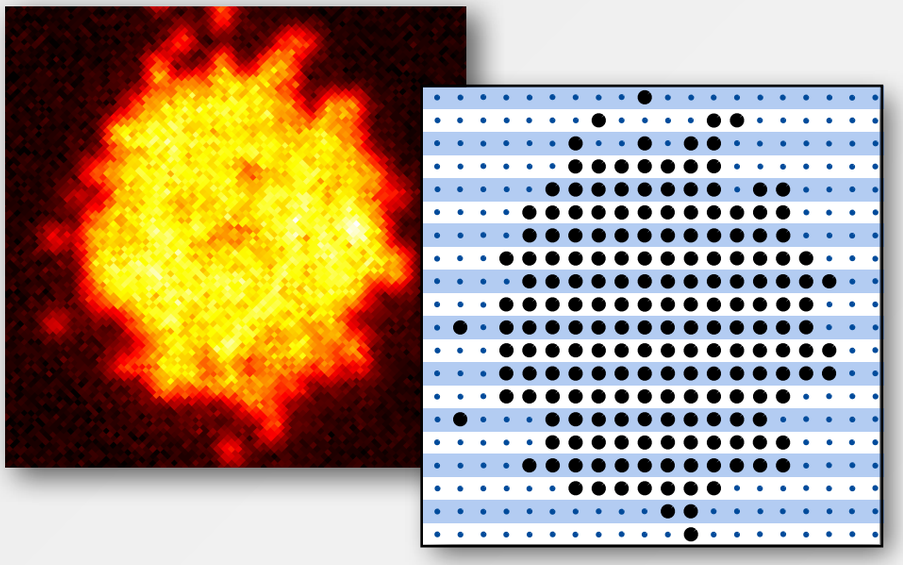
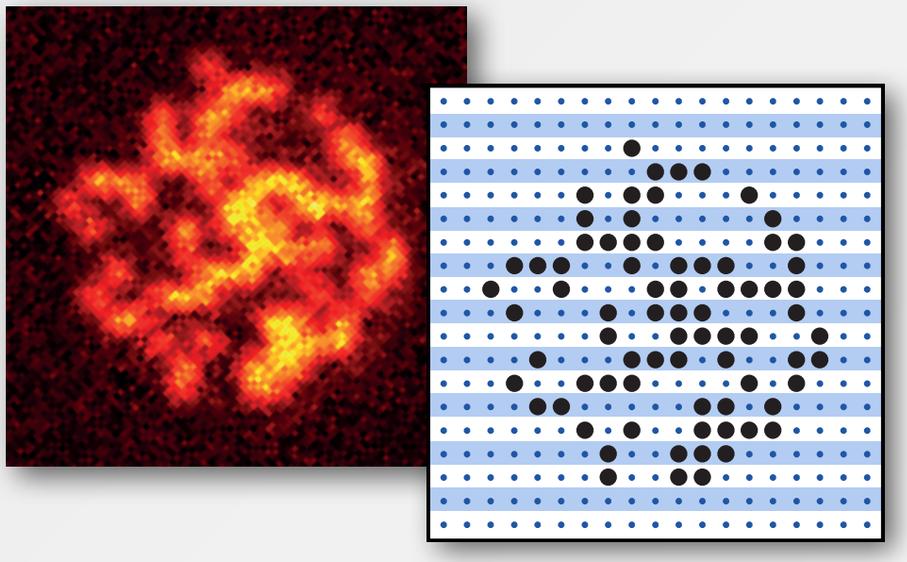


close to the superfluid transition



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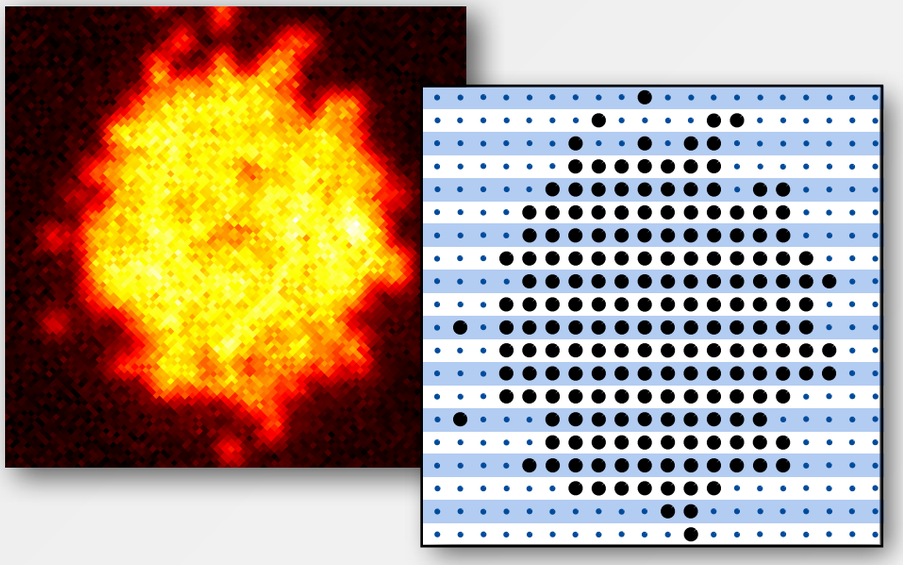

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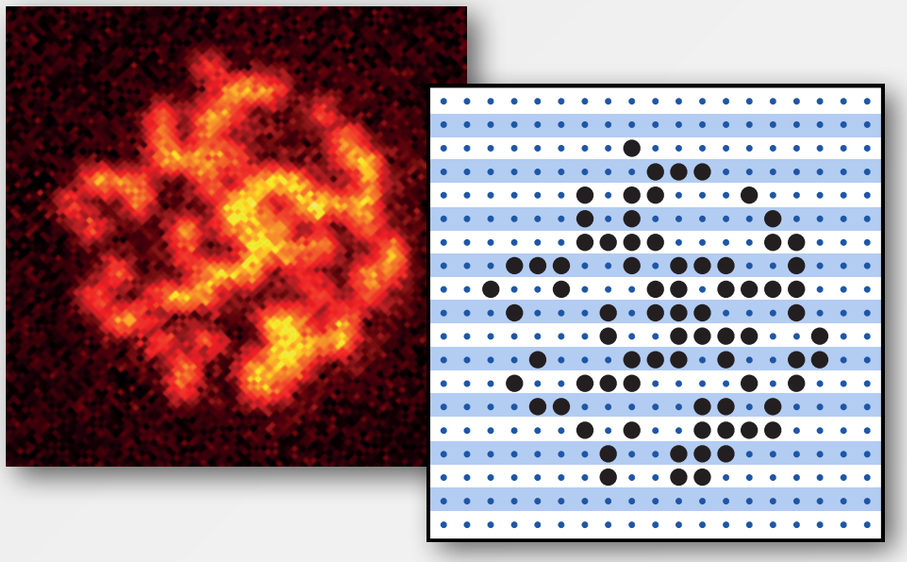

N-point parity correlations

$$\langle \hat{s}_{k_1} \hat{s}_{k_2} \dots \hat{s}_{k_N} \rangle \qquad \hat{s}_k = e^{i\pi\hat{n}_k}$$

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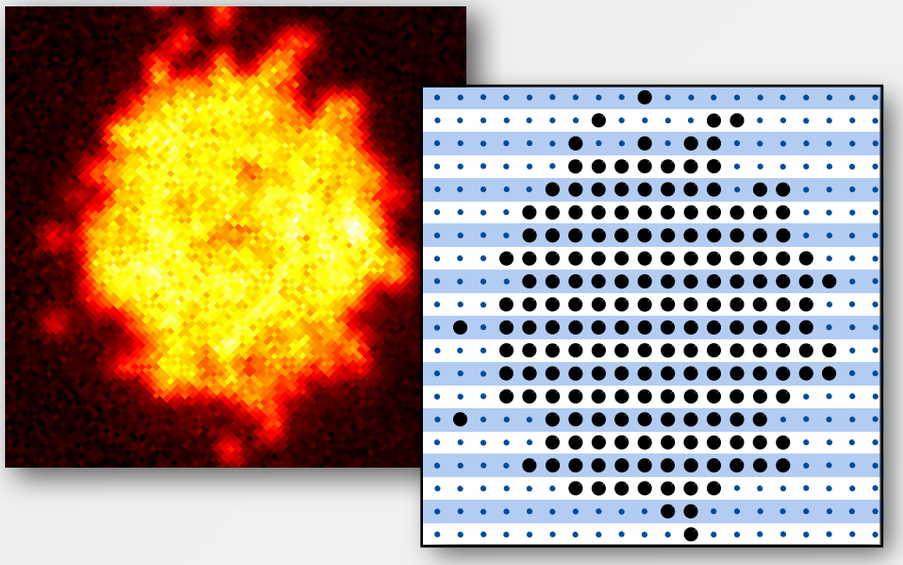

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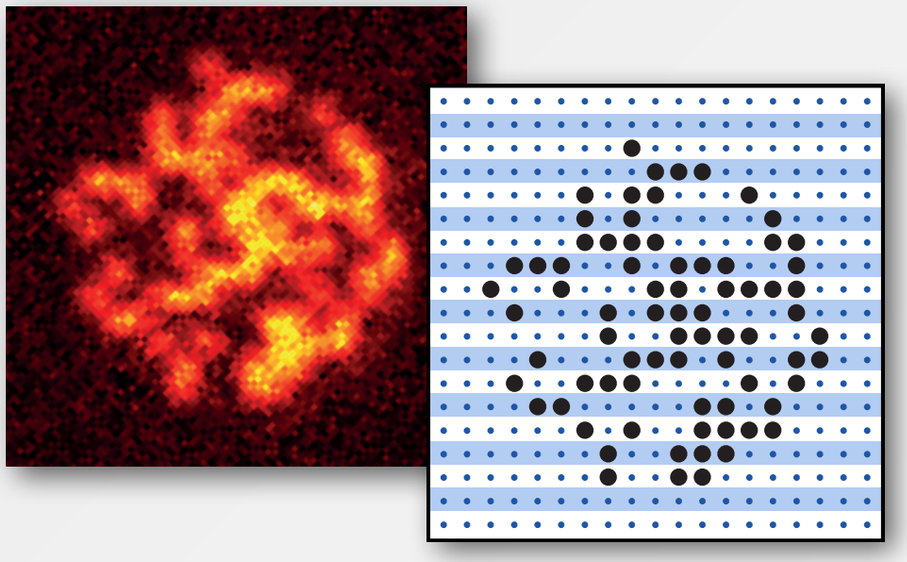

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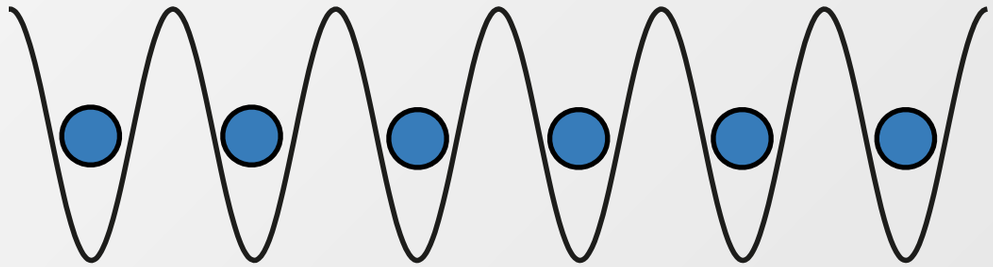
1 dimension
or
2 dimensions

How to observe the propagation of correlations?

Initial state:
deep in the MI phase

1 dimension



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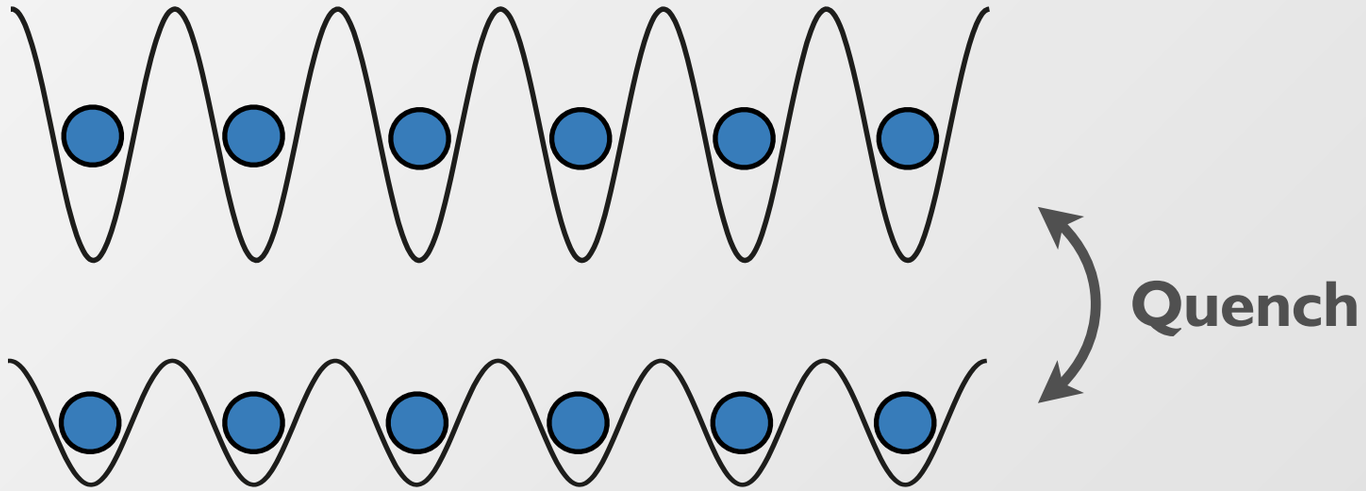

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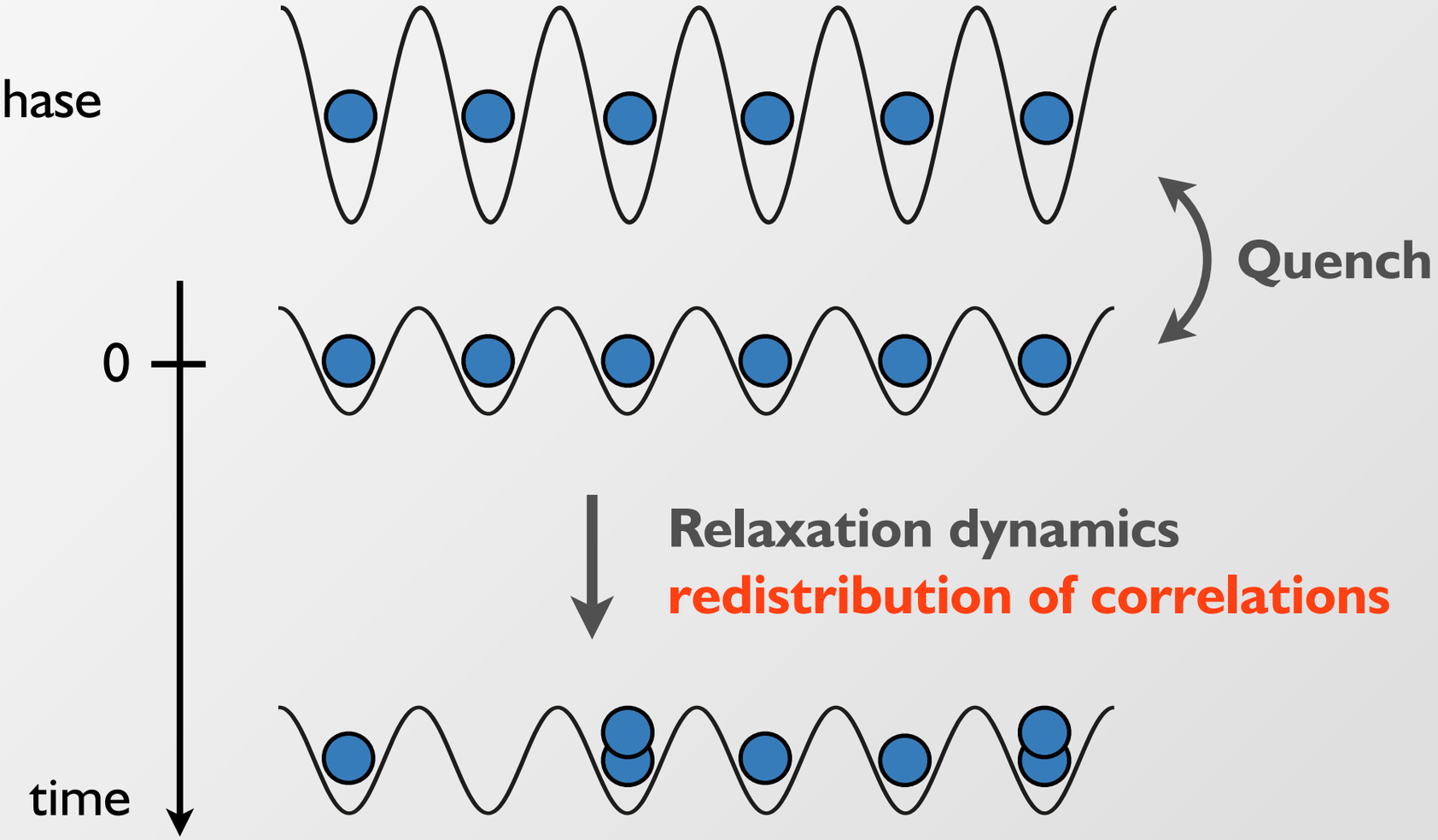

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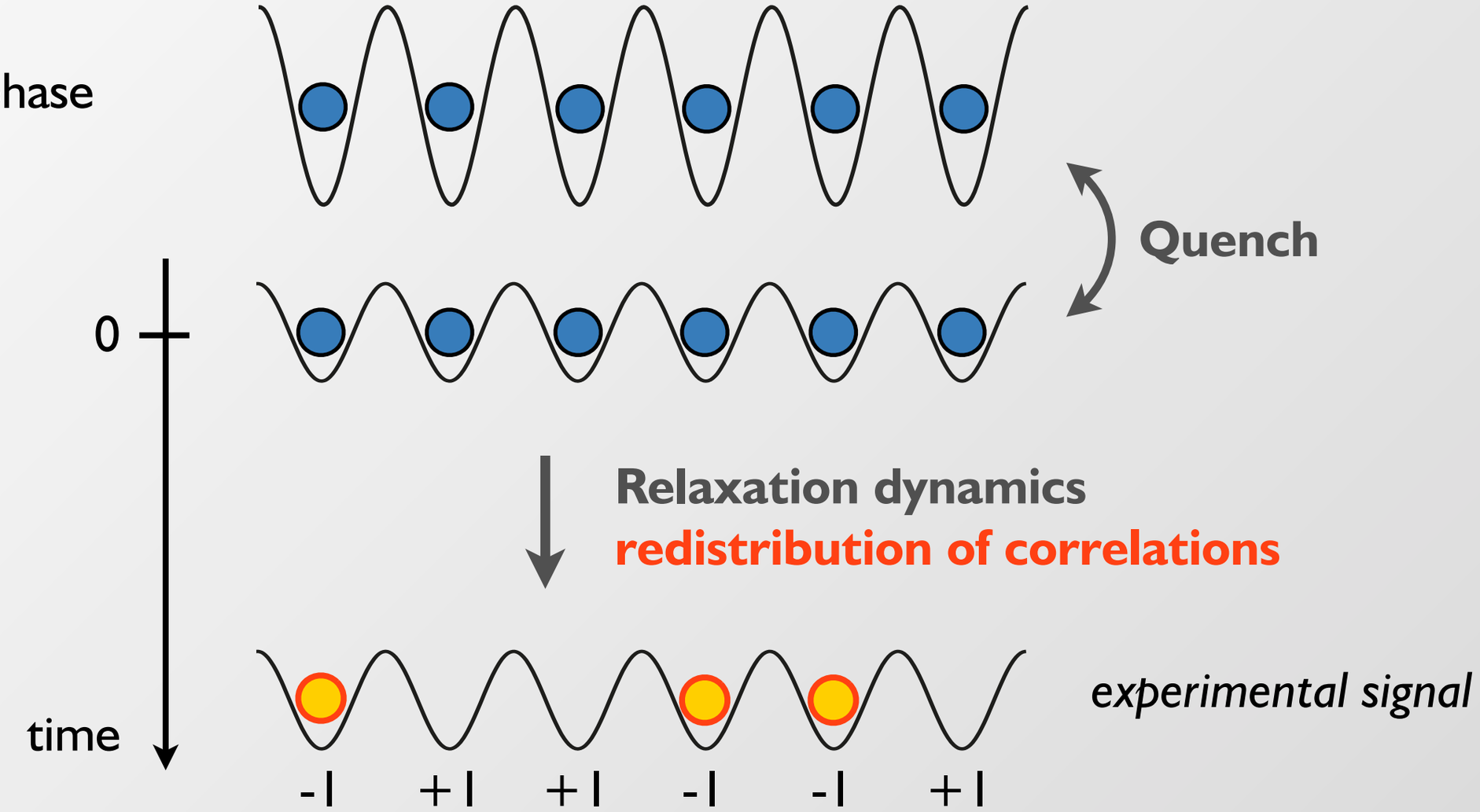

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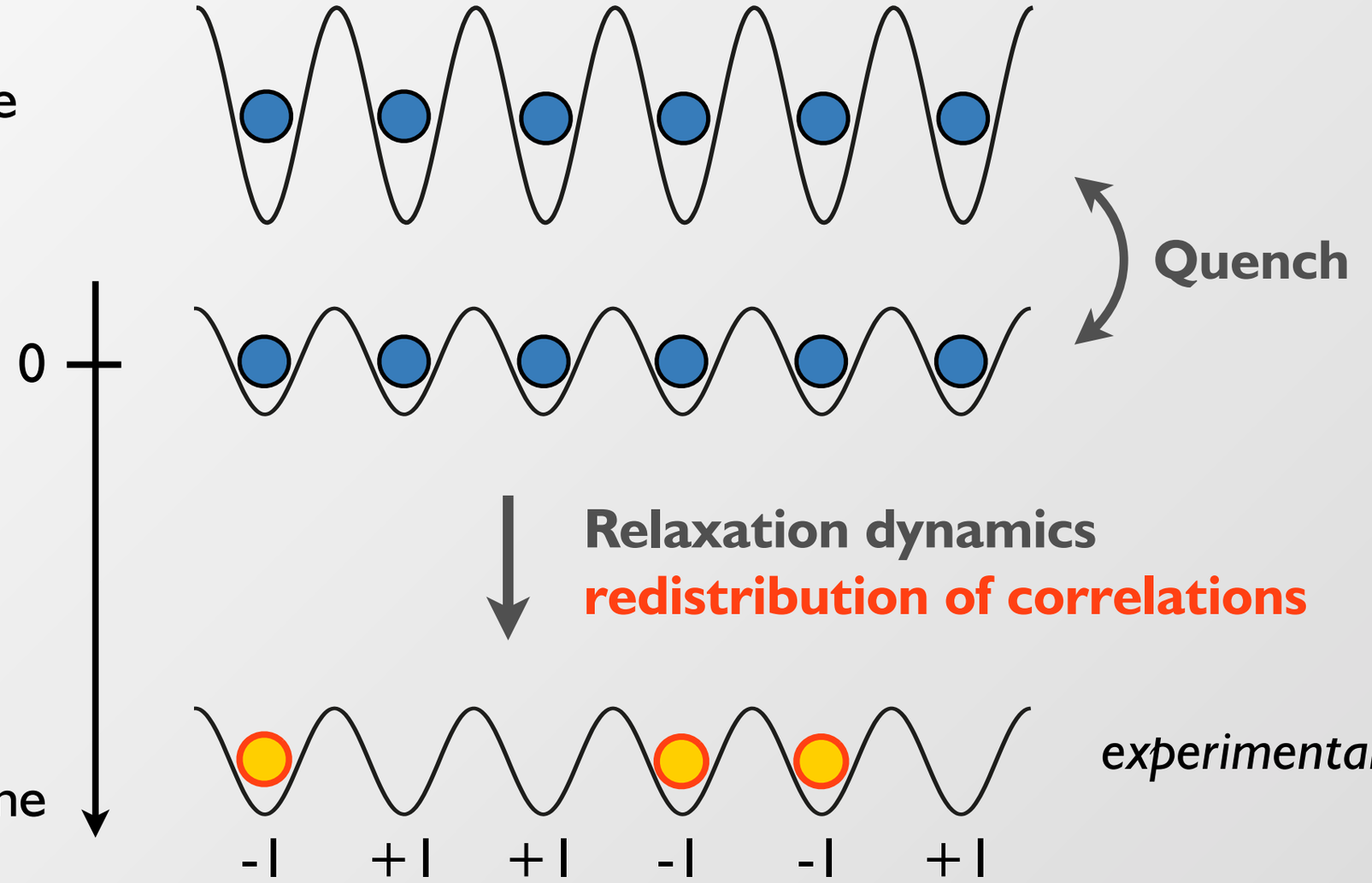

connected two-point parity correlation function

$$C(d,t) = \langle \hat{s}_k \hat{s}_{k+d} \rangle - \langle \hat{s}_k \rangle \langle \hat{s}_{k+d} \rangle$$

Experimental results

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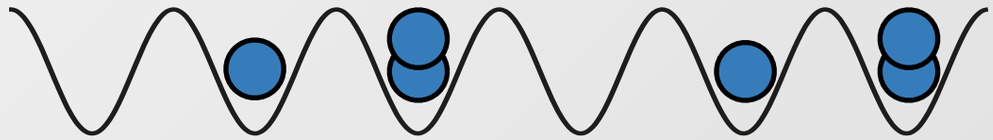

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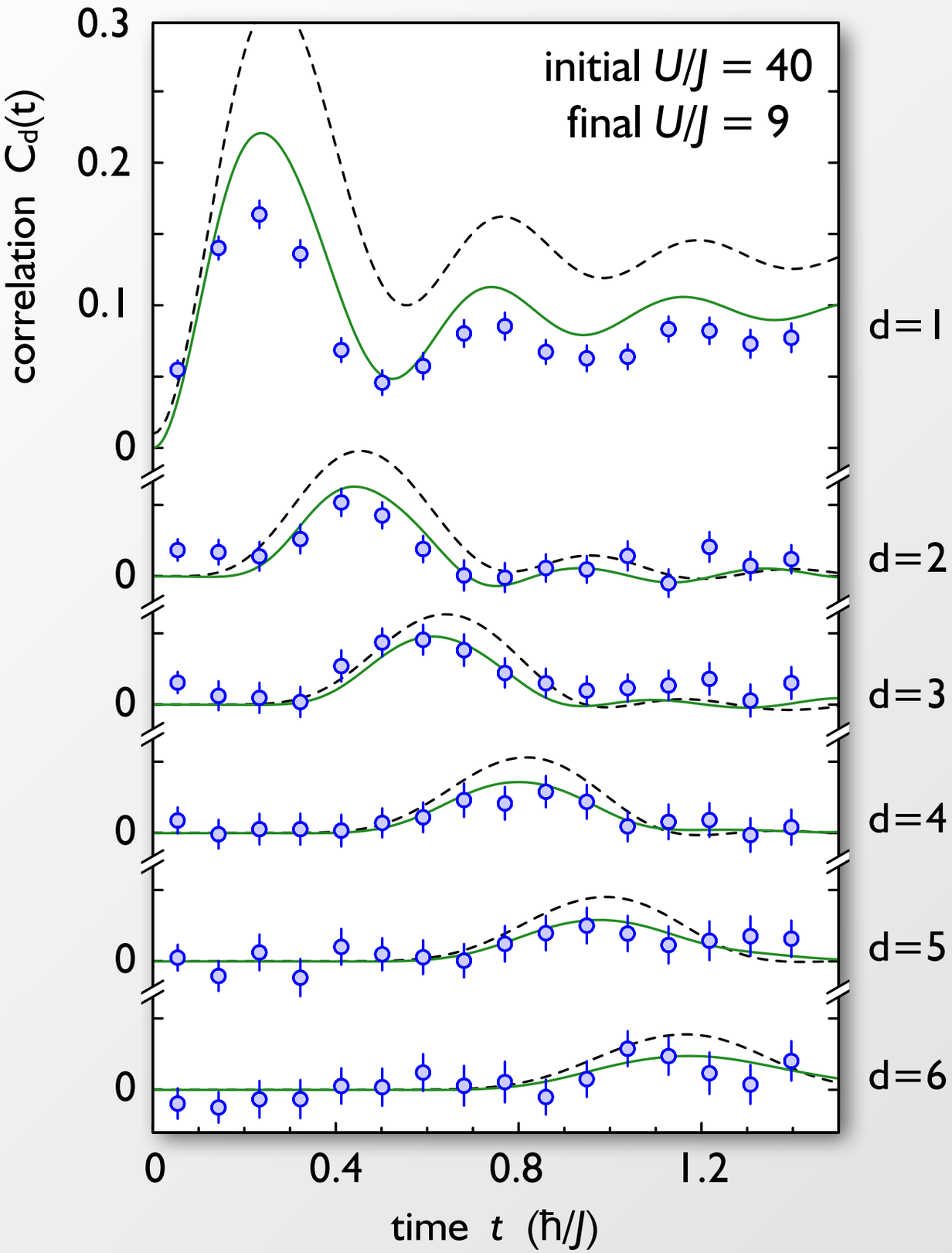

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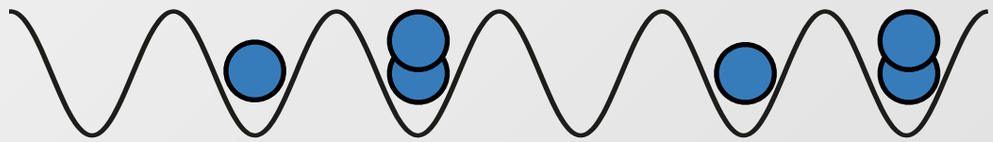
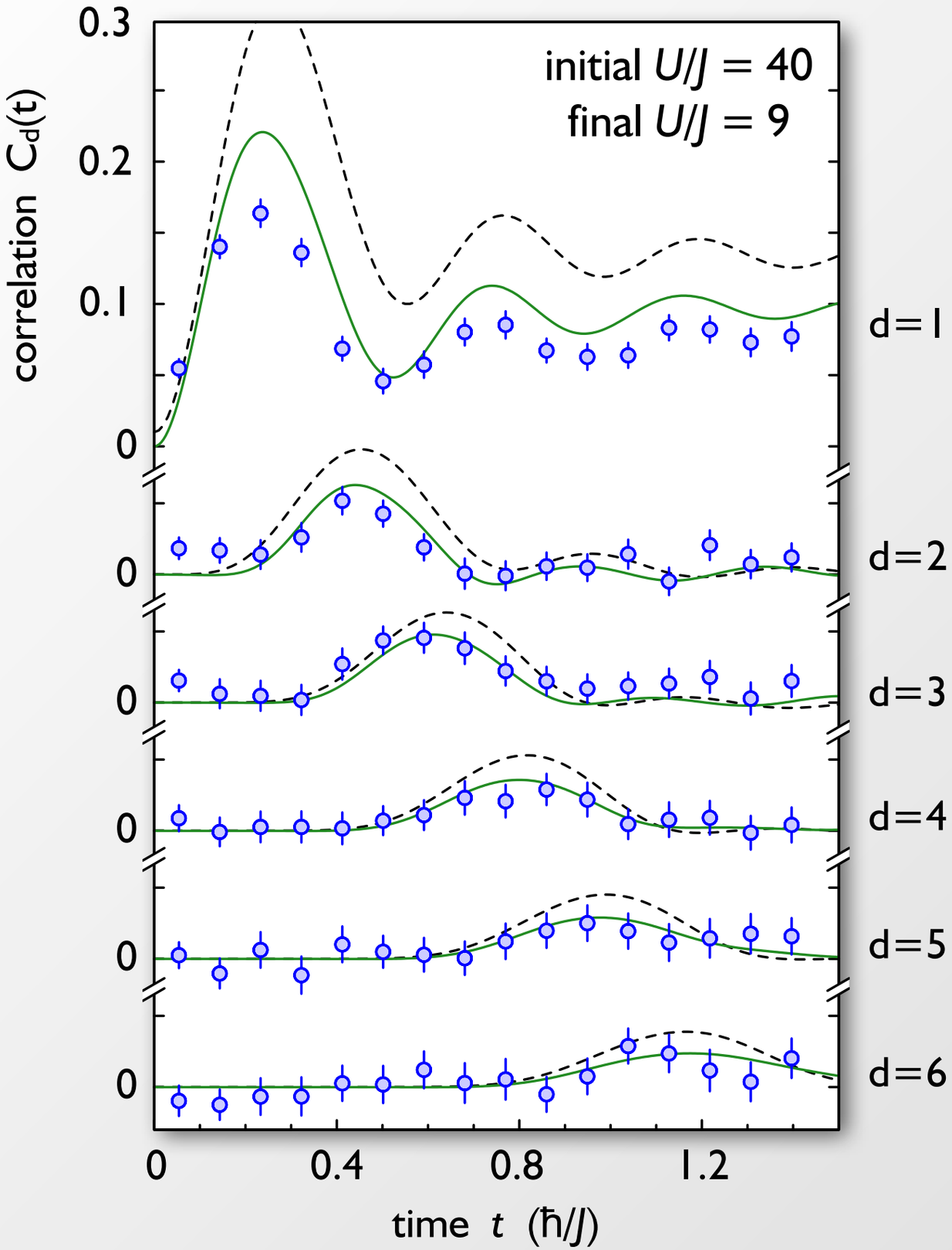


Theory: (solid and dashed line) DMRG & QM
zero temperature, homogeneous system

Experimental results



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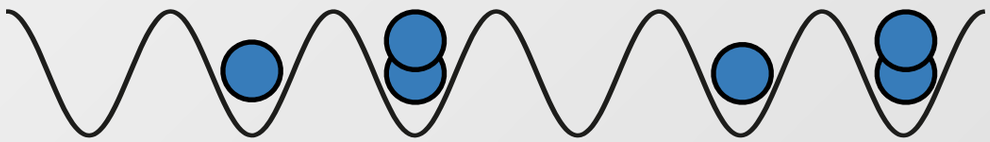

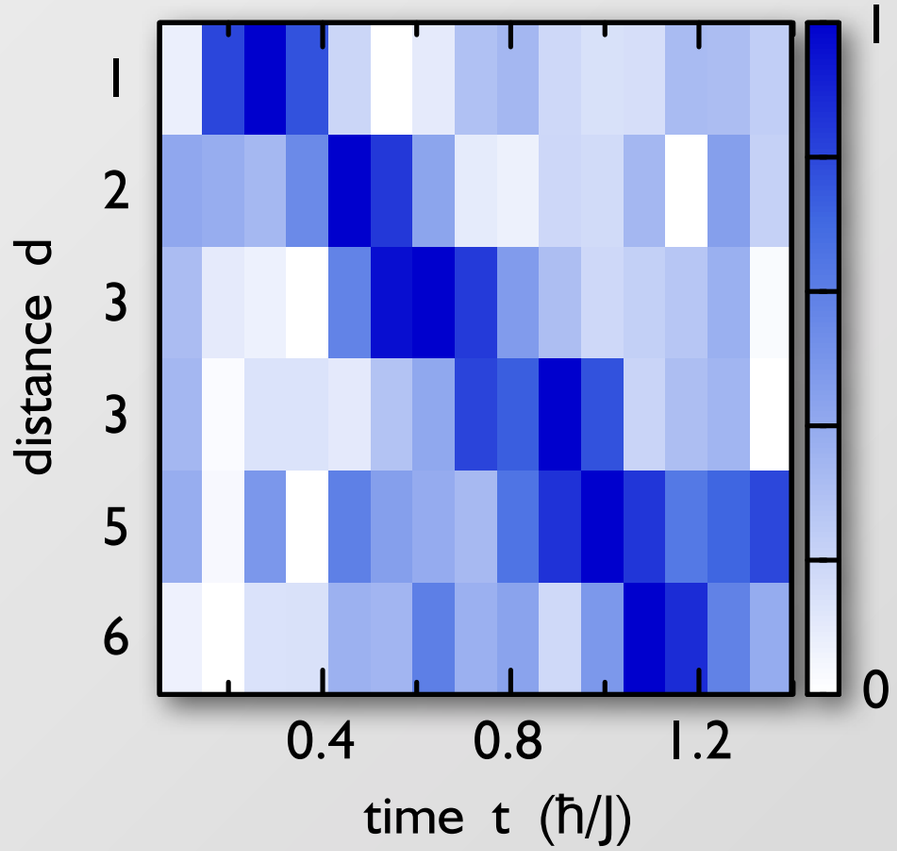

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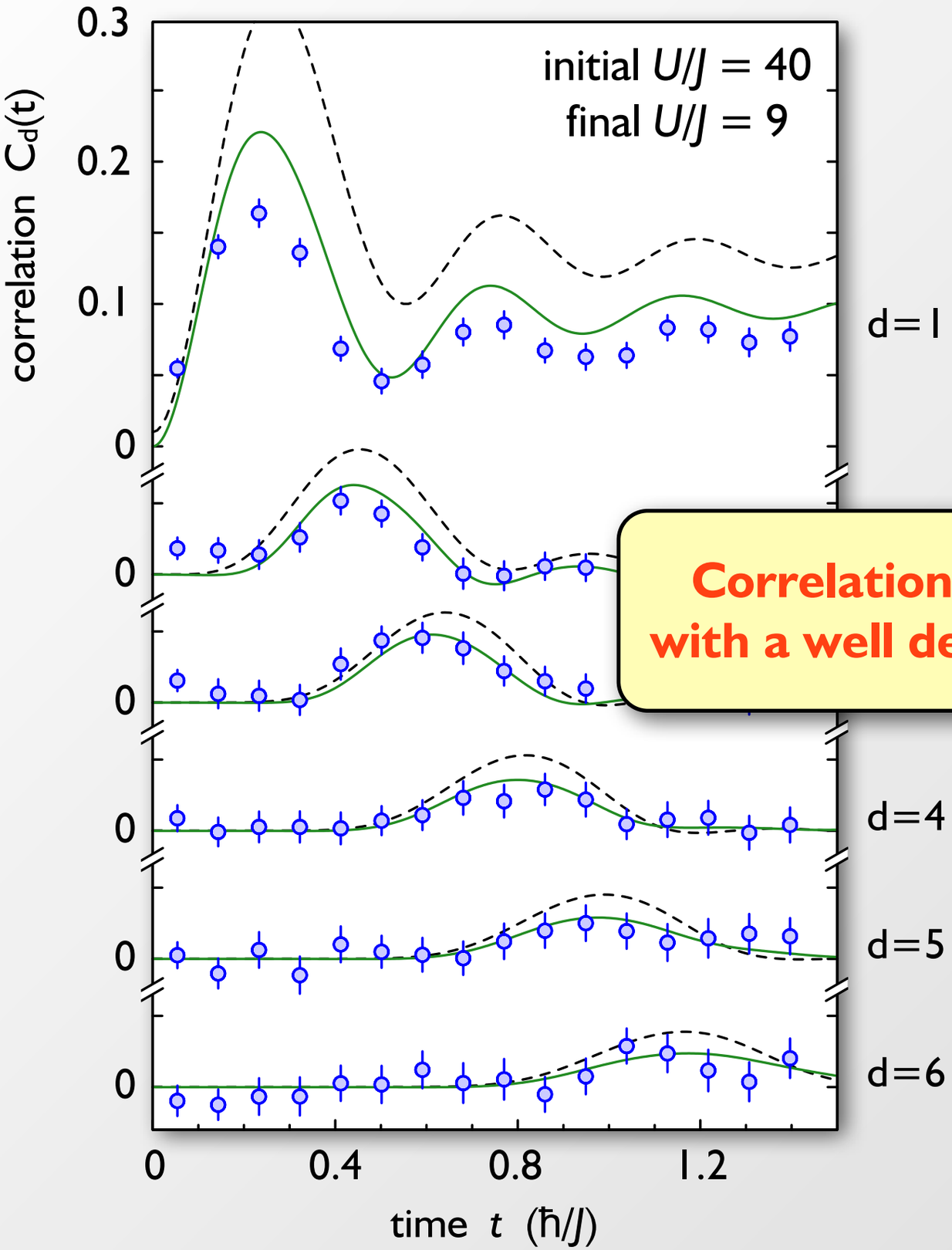

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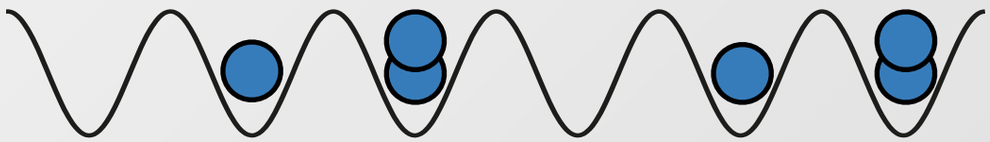
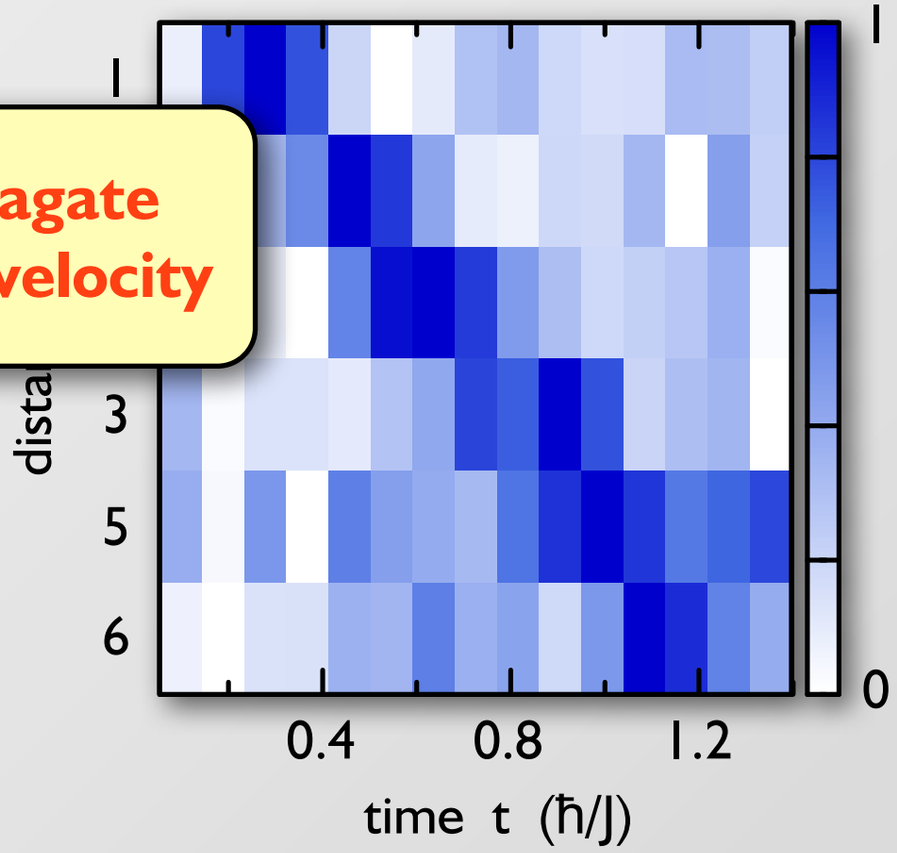
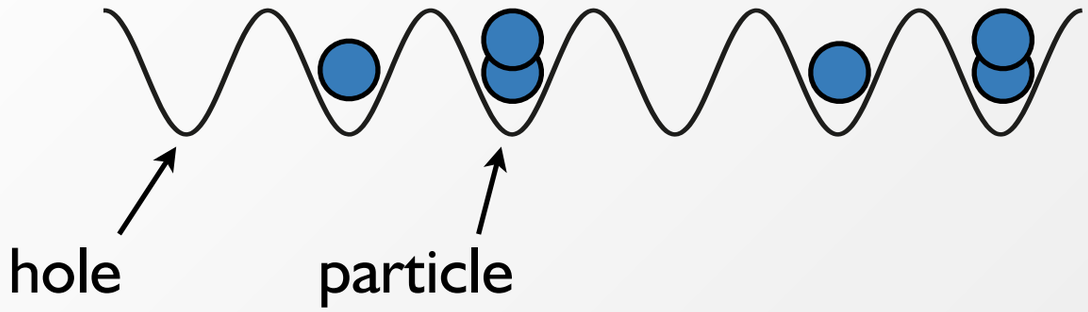




Correlations propagate with a well defined velocity

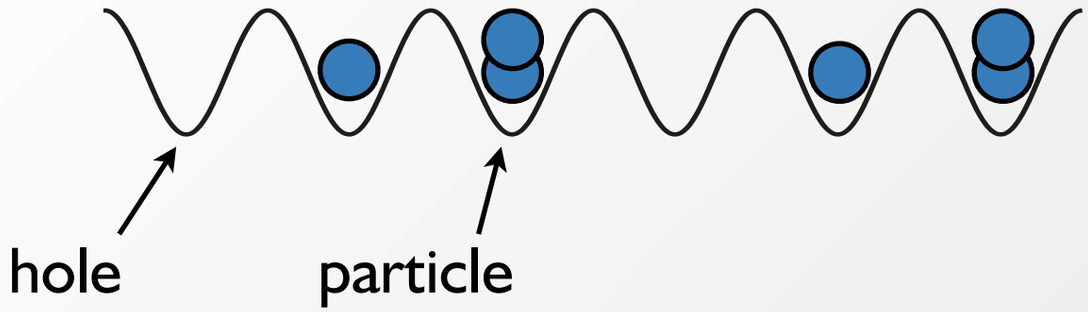
Theory: (solid and dashed line) DMRG & QM zero temperature, homogeneous system

Quasiparticle model



Quasiparticle pairs are emitted and **propagate correlations** across the system

Quasiparticle model

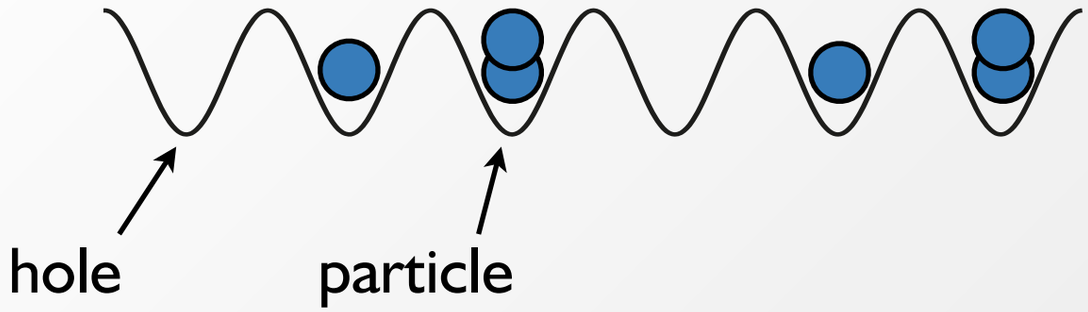


Quasiparticle pairs are emitted and **propagate correlations** across the system

Quasiparticle model:

- introduce two types of slave **bosons**: **holes** and **particles**

Quasiparticle model



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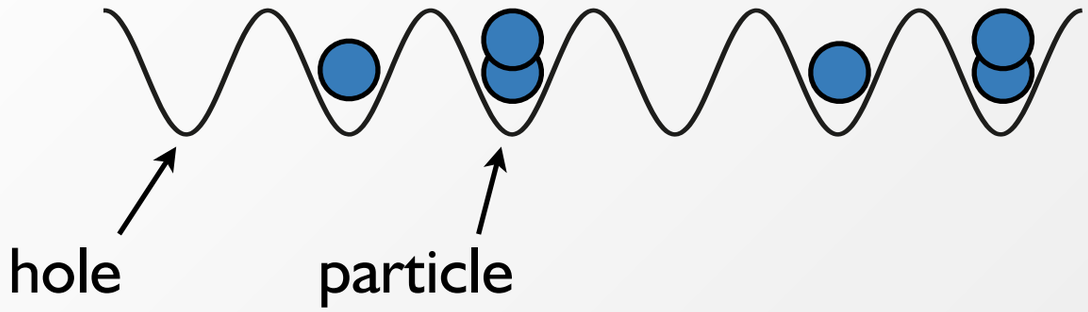
Quasiparticle model:

- introduce two types of slave **bosons**: **holes** and **particles**
- the **MI ground state** is the **vacuum** for the slave particles

Quasiparticle model



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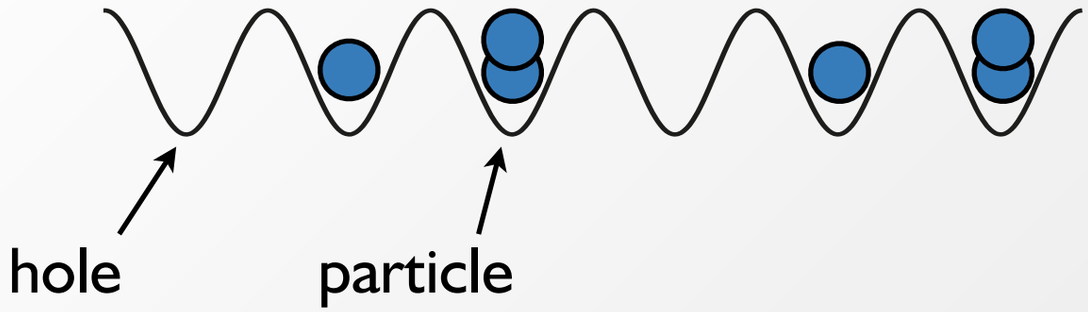

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low density of excitations

$U/J > 9$

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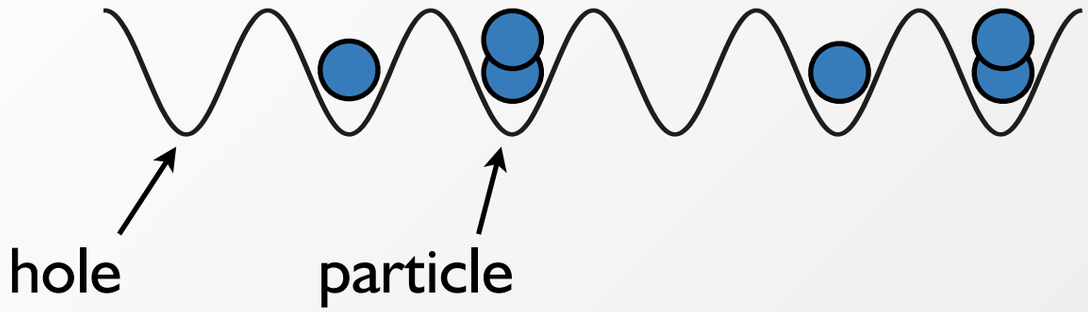

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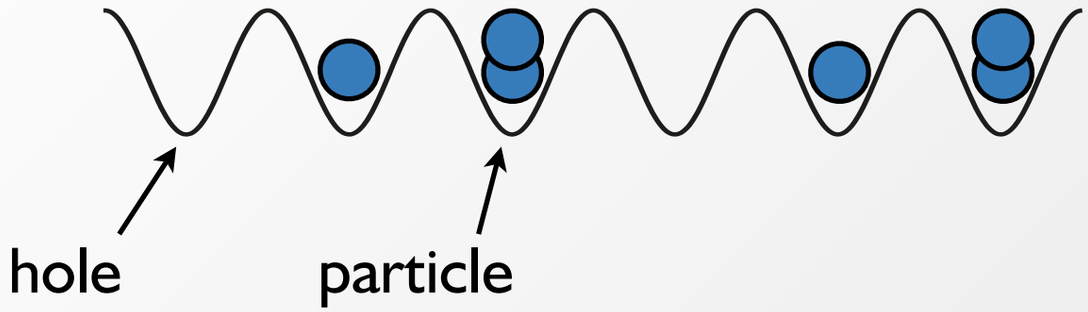

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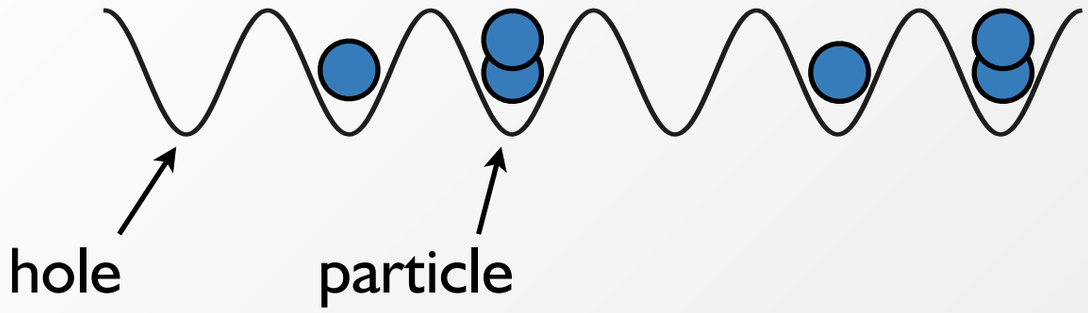

Quasiparticle pairs are emitted and **propagate correlations** across the system

Solve the many-body dynamics analytically

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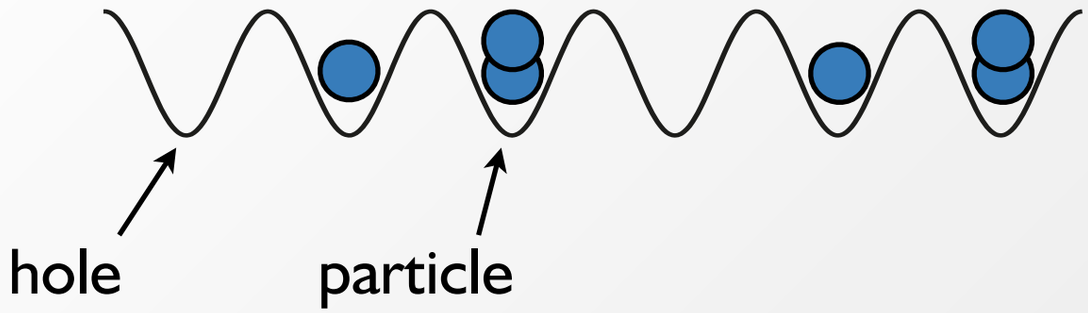

Quasiparticle pairs are emitted and **propagate correlations** across the system

Time-evolution of the many-body state (first order in J/U)

$$|\Psi(t)\rangle \simeq |\Psi_0\rangle + \frac{J}{U}\sum_k f(k)\left[1 - e^{-i[\epsilon_d(k)+\epsilon_h(k)]t/\hbar}\right]\left[d_k^\dagger h_{-k}^\dagger - d_{-k}^\dagger h_k^\dagger\right]|\Psi_0\rangle$$

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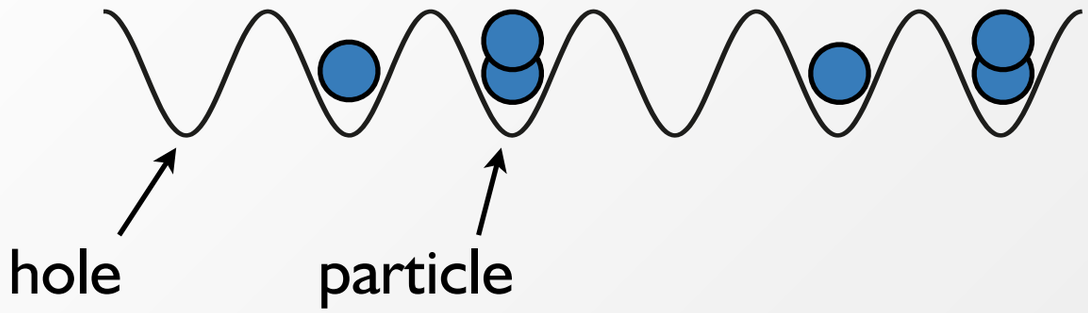

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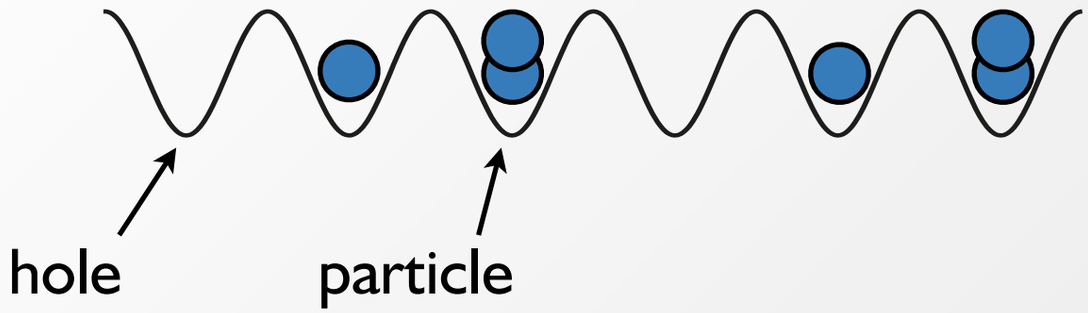
initial state

Quasiparticle model



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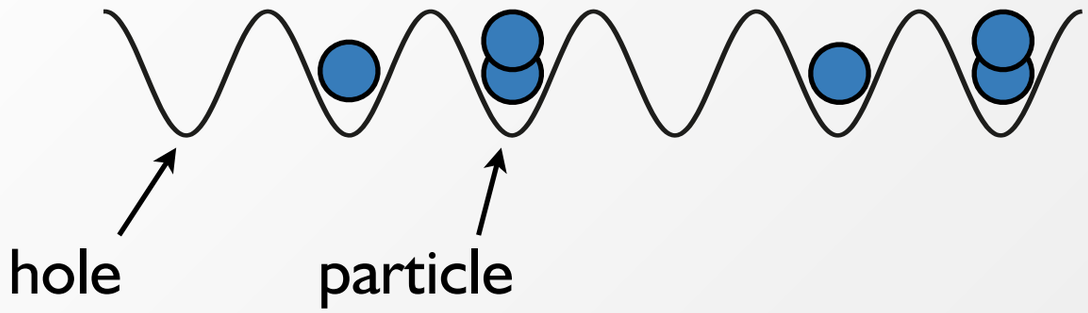
initial state

entangled pair
particle + hole

Quasiparticle model



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initial state

wave packet

propagating pair

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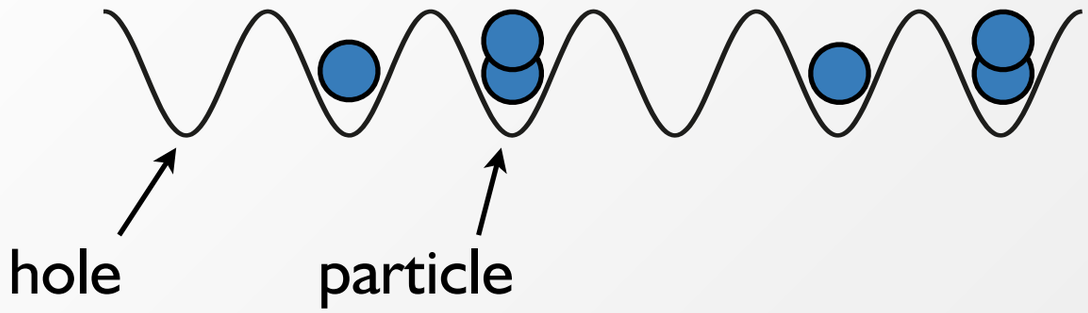

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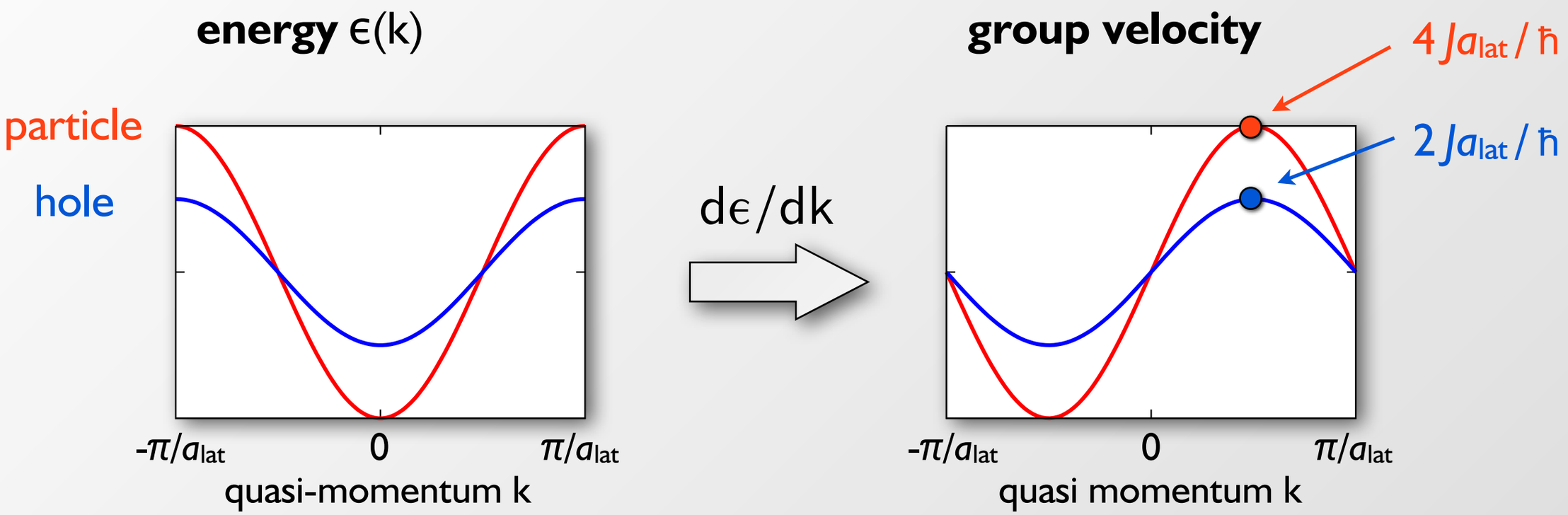
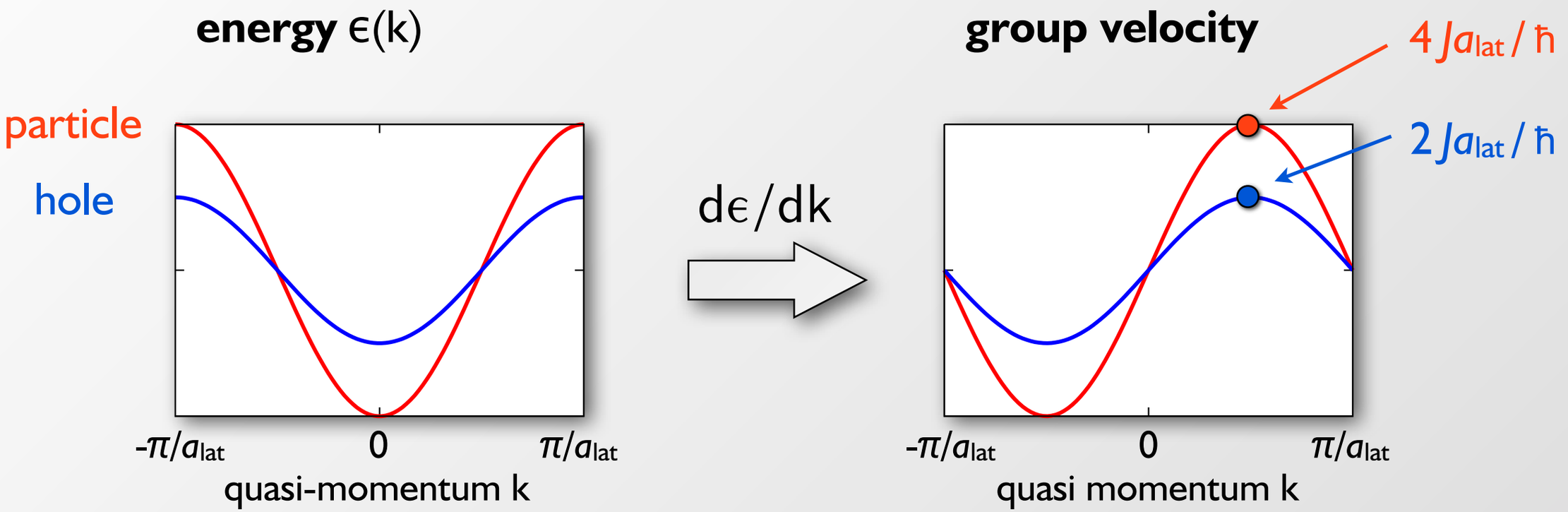
Group velocity



Due to the band structure, there exists a **maximum group velocity**

Group velocity



Due to the band structure, there exists a **maximum group velocity**

Notes:

- We artificially limit the local Hilbert space to 2 states
- The maximum group velocity increases with the lattice filling (Bose enhancement of the tunnel coupling)

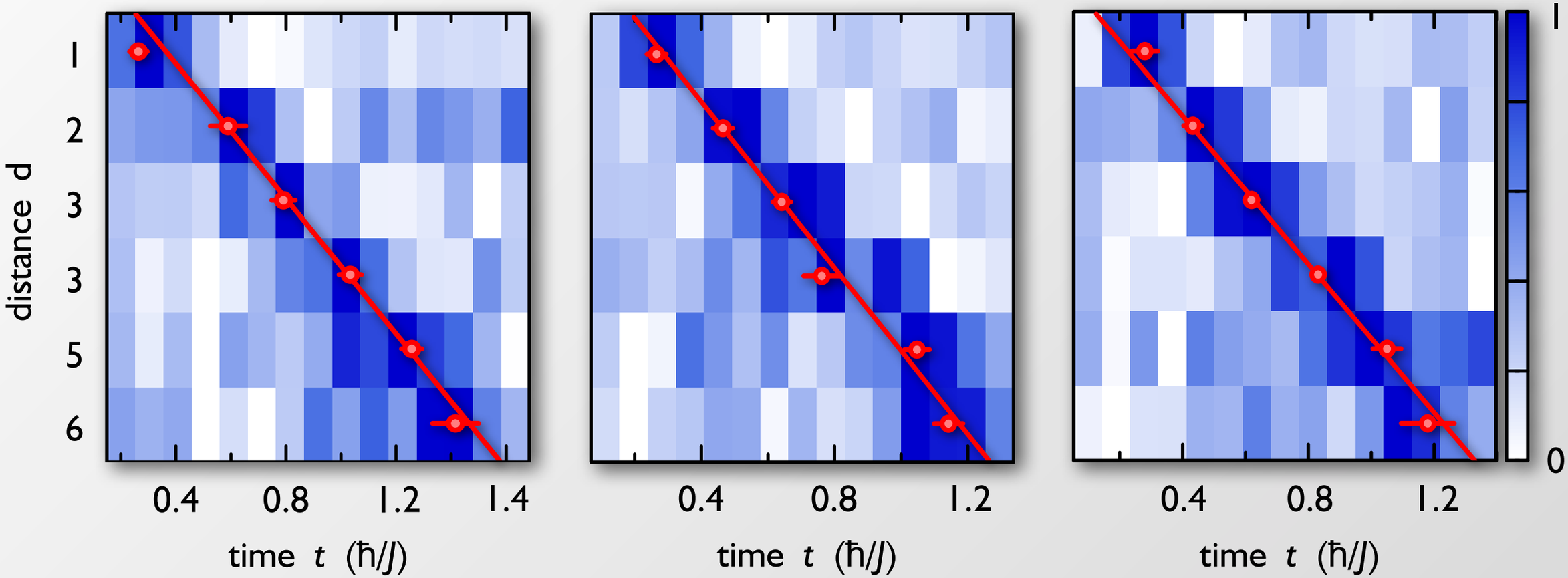
Beyond the quasiparticle model

Quasiparticle picture breaks down

final $U/J = 5$

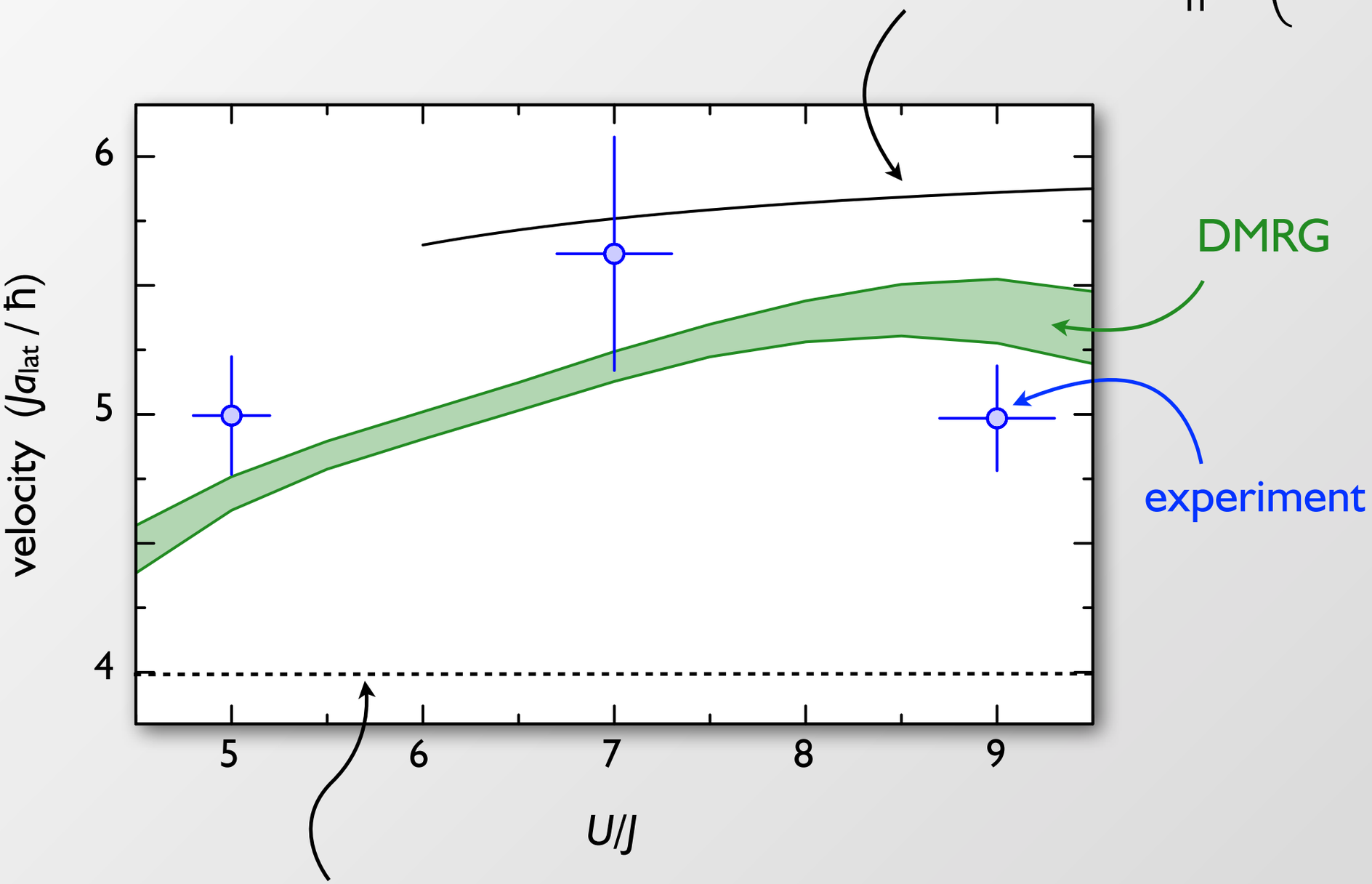
final $U/J = 7$

final $U/J = 9$



Spreading velocity

maximum quasiparticle velocity $\simeq \frac{6Ja_{lat}}{\hbar}\left(1-\frac{16J^2}{9U^2}\right)$



Conclusion

- Relaxation dynamics in the Mott regime driven by the propagation of entangled particle-hole pairs
  We can see them!!

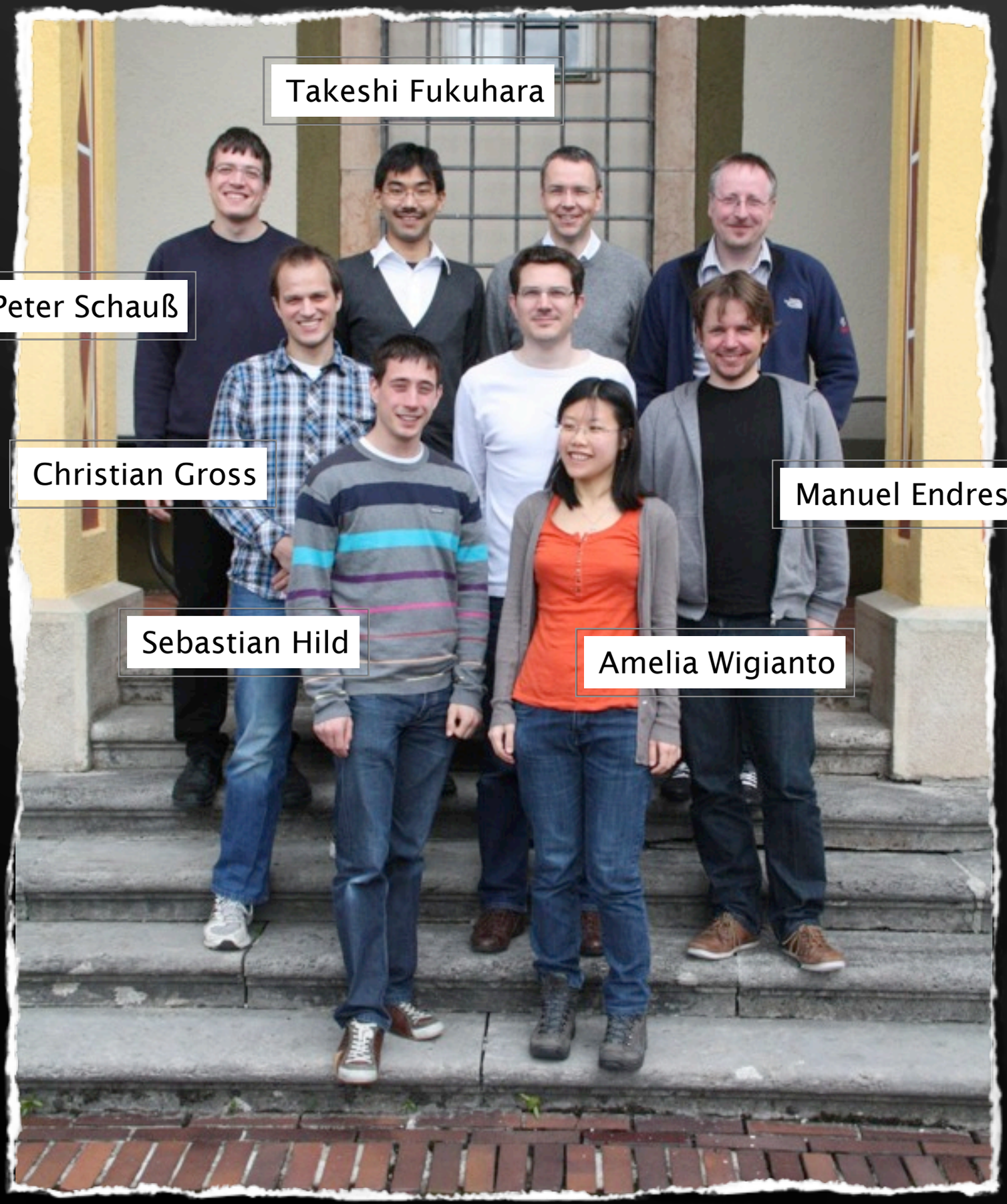
- At finite energy and particle number, there exists a maximum group velocity for the propagation of the quasiparticles

- Can we speak of a Lieb–Robinson bound?

- The light-cone dynamics is related to the linear increase in time of the entanglement entropy

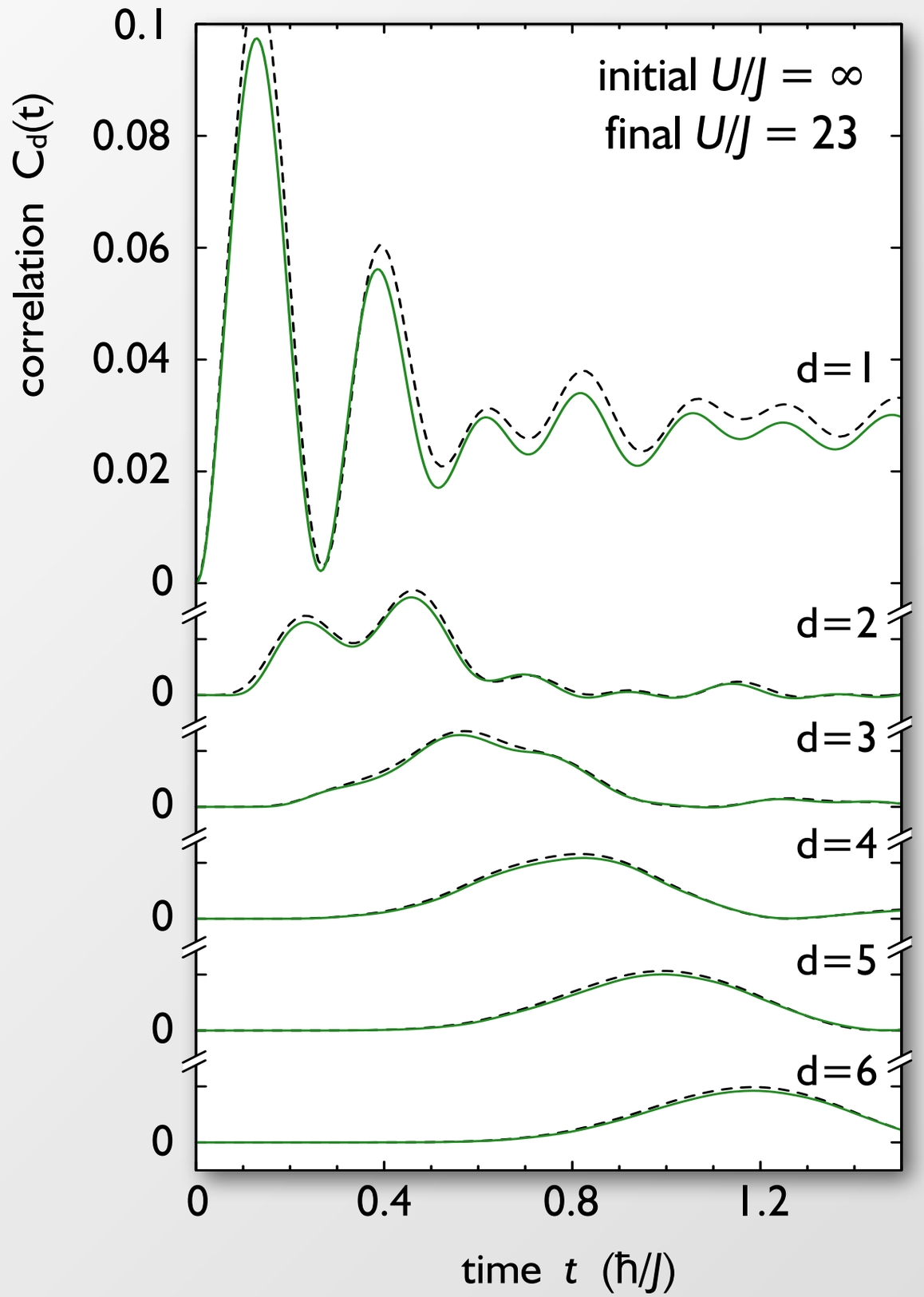
- What happens in 2D? (*Talk by Ludwig Mathey*)

- Is this a generic feature of quantum systems with finite-range interactions???

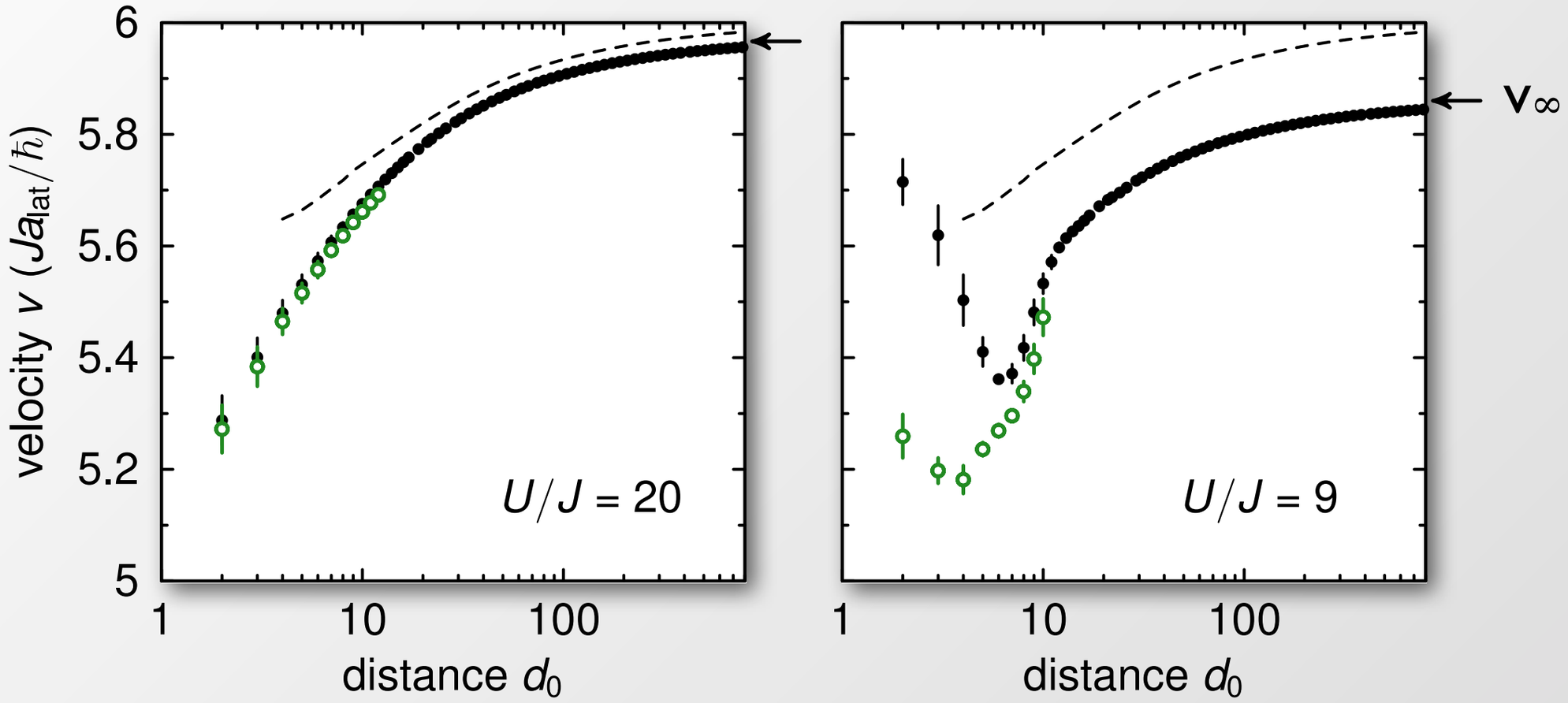

Takeshi Fukuhara
Peter Schauß
Christian Gross
Manuel Endres
Sebastian Hild
Amelia Wigianto

Thank you!

Quasiparticle model



Finite-time effects



Universal scaling: $$v(d) = v_\infty \left(1 - \xi d^{-2/3}\right)$$