Two dimensional attractive Fermi gas: balanced and highly polarized regimes at zero temperature.

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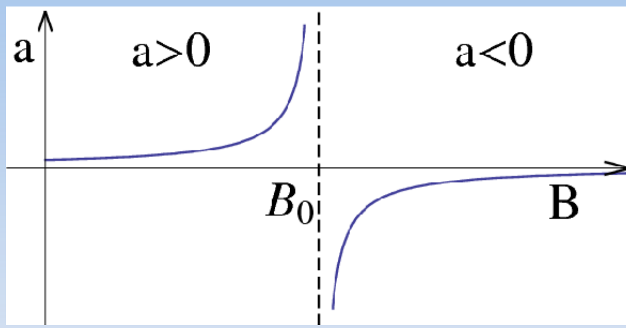
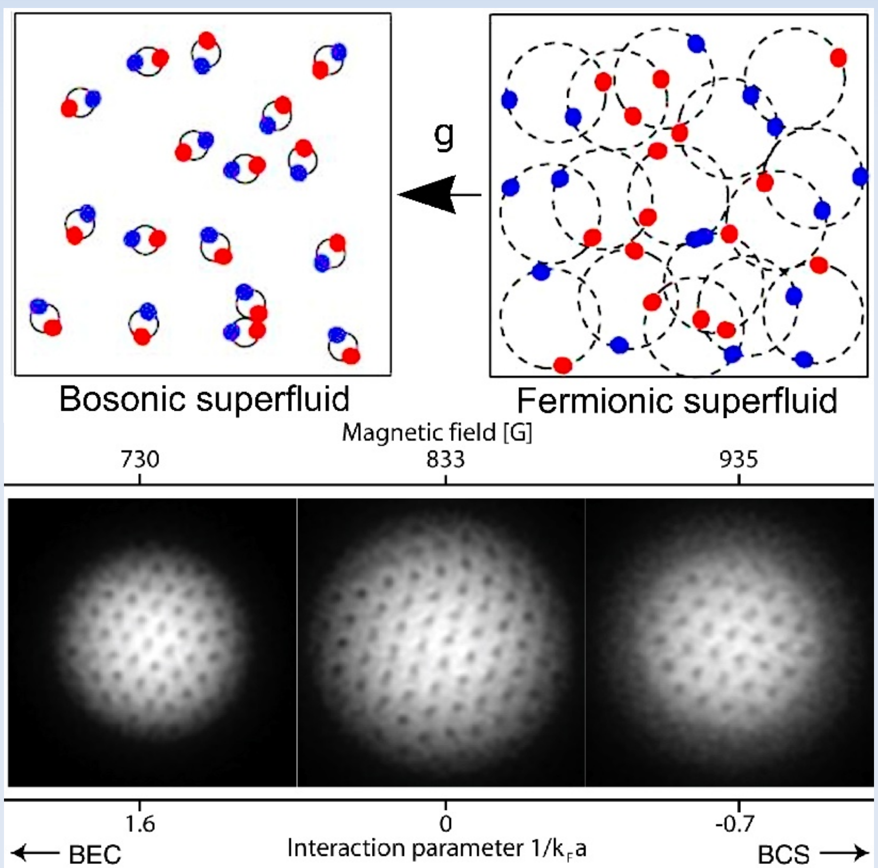
Ultracold Fermi gases

- Short range interaction R
- Scattering length a
- Diluteness $R \ll l \quad (k_F R \ll 1)$
- Low temperature $R \ll \lambda_T$
- Quantum degeneracy $n \lambda_T^d > 1 \quad (T < T_F)$
- Universality (T=0) $$\frac{E}{N} = E_{FG} f(k_F a)$$
- Relation to nuclear matter, neutron stars, high T_c superconductors

BCS-BEC crossover





2 species of fermions with attractive interaction at T=0 (Feshbach resonance)

- Weak coupling:
  BCS instability $\quad f \sim 1 + f_1(k_F a)$
- Strong coupling:
  BEC of dimers $\quad f \sim \dfrac{-c}{2(k_F a)^2}$
- Crossover:
  no phase transition and no small parameter for perturbation theory
  → QMC

Ketterle group, MIT (3D system)

Quantum Monte Carlo methods

- Expectation values of operators (Energy, correlation functions) in quantum mechanics at T=0
- N-body problem: calculation of integrals of DOF=d N variables

$$I=\int g(x)\,dx^{DOF}=\int p(x)\,g'(x)\,dx^{DOF}$$ provided we can sample x from p(x)

- Non perturbative (useful for strong coupling regimes)
- More efficient than fixed grid methods $DOF^{A} \leftrightarrow B^{DOF}$
- Assessment of the statistical error (from variance) $\sigma \propto 1/\sqrt{W}$

DRAWBACKS:

- Probability distributions must be positive, so difficulties arise with fermions and excited states
- Need for finite size analysis
- With DMC: not all correlation functions can be exactly sampled

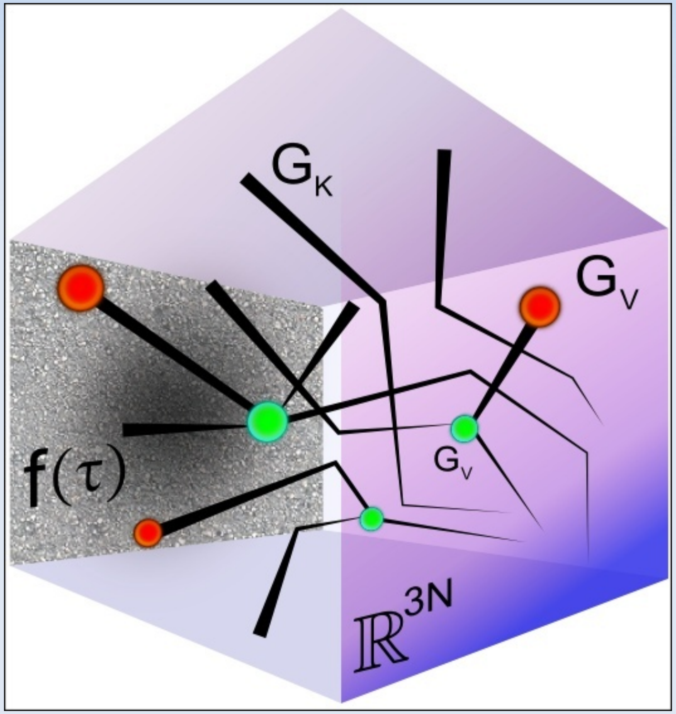
Diffusion Monte Carlo (DMC)

Schroedinger equation in imaginary time (N particles) $-\frac{\partial}{\partial \tau}\Psi = \hat{H}\Psi$

Expand $\Psi(x,\tau) = e^{-\hat{H}\tau}\Psi_T(x)$ in energy eigenfunctions $\psi_n(x,\tau) = e^{-E_n\tau}\psi_n(x)$

$\psi_0(x,\tau)$ is the main contribution for $\tau \to \infty$

$\Psi_T : \langle \Psi_T | \Psi_0 \rangle \neq 0$ Trial (guide) wavefunction



Basic Algorithm
(small time-step expansion of evolution operator)
Random walk + Drift + Branching

Bosons : *exact (ground state)*
Fermions : *Fixed Node approximation*
(*variational principle*)

$p(x) = \Psi(x)\Psi_T(x) \geqslant 0$

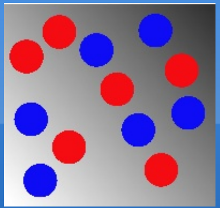
GS Energy

$\hat{H} = \hat{K} + \hat{V}$

$$\langle \hat{H} \rangle = \frac{\langle \psi_0 | \hat{H} | \Psi_T \rangle}{\langle \psi_0 | \Psi_T \rangle} = \frac{\int \psi_0(x,\tau)\Psi_T(x)\frac{\langle x | \hat{H} | \Psi_T \rangle}{\Psi_T(x)}dx}{\int \psi_0(x,\tau)\Psi_T(x)\,dx} = \int p(x,\tau)E_L(x)\,dx$$

BCS-BEC crossover in 2D

G. Bertaina and S. Giorgini, Phys. Rev. Lett. 106, 110403 (2011).

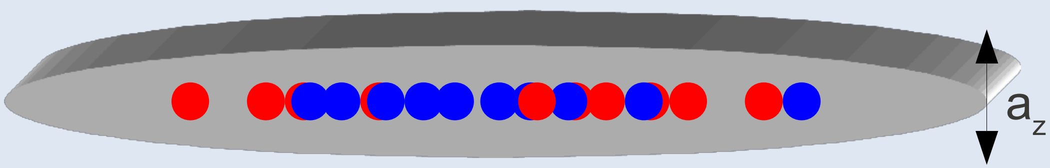


Low dimensions: important role of quantum fluctuations, failure (?) of BCS mean-field

BKT (finite temperature) physics
Ferromagnetism
Kondo effect
FFLO states (finite momentum condensate)

Optically trapped gas: Quasi-2D

$$\varepsilon_F \ll \hbar\omega_z$$
$$\varepsilon_b \ll \hbar\omega_z$$



Experiments:
Martiyanov et al. PRL (2010)
Fröhlich et al. PRL (2011)
Feld et al. Nature (2011)
Orel et al. NJP (2011)
Sommer et al. PRL (2012)
Koschorreck et al. Arxiv (2012)

Mapping: Quasi-2D → Pure 2D

$$a_{2D} \propto a_z \exp\left(-\sqrt{\frac{\pi}{2}}\frac{a_z}{a_{3D}}\right)$$

Bound state $\varepsilon_b = -\frac{\hbar^2}{m a_{2D}^2}\frac{4}{e^{2\gamma}}$

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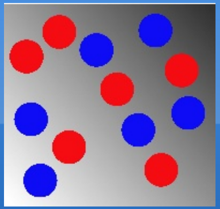

Model interaction: Square well (in universal regime $nR^2 << 1$)

$$\varepsilon_b = -\frac{\hbar^2}{m a_{2D}^2} \frac{4}{e^{2\gamma}}$$

- Trial Wavefunctions (used to fix the nodal surface in DMC):
- Weak coupling: Jastrow-Slater

$$\Psi_{JS} = J_{\uparrow\downarrow} D_{\uparrow}(N_{\uparrow}) D_{\downarrow}(N_{\downarrow})$$

Jastrow factor: $J_{\uparrow\downarrow} = \prod_{ii'} f_{\uparrow\downarrow}(x_{ii'}) > 0$ f: two-body problem

- Strong coupling: BCS

$$\Psi_{BCS} = A[\phi(x_{11'}) \dots \phi(x_{N_{\uparrow} N_{\downarrow}})]$$

ϕ : bound state

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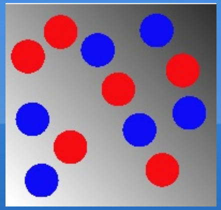
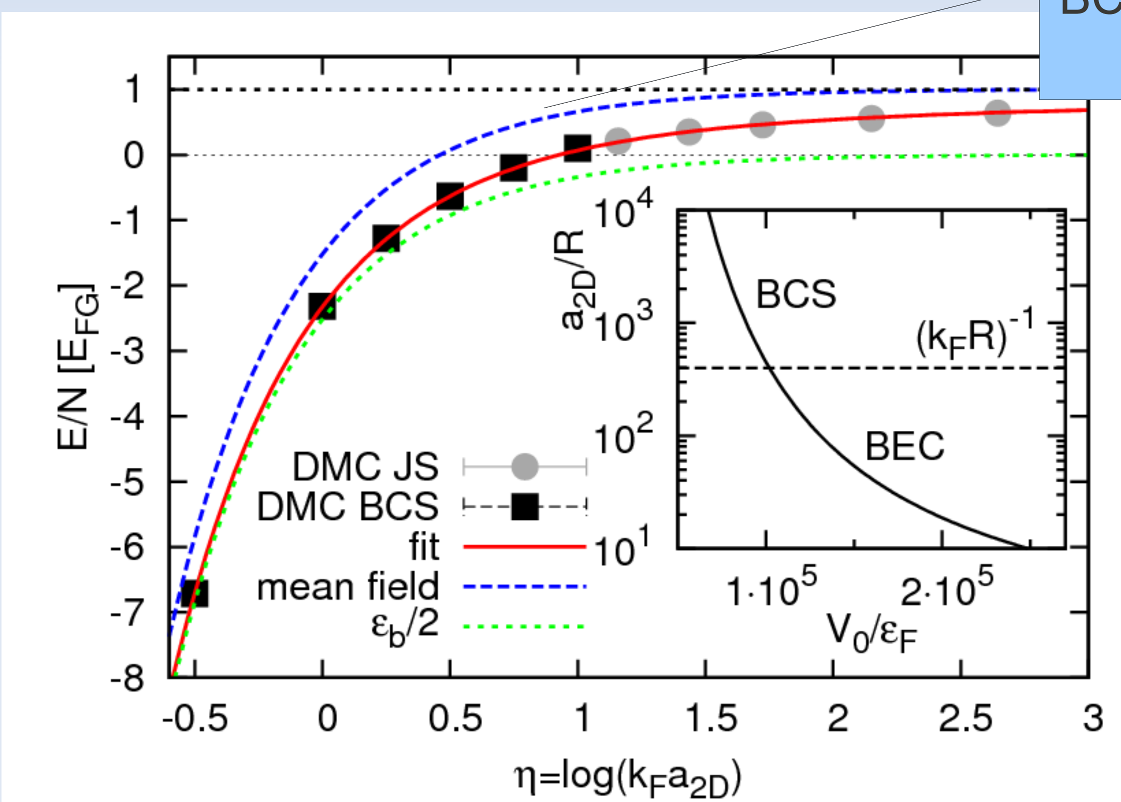


Energy per particle

Crossover parameter $$\eta = \log(k_F a_{2D}) \simeq \frac{1}{2}\log\left(\frac{\varepsilon_F}{|\varepsilon_b|}\right)$$



BCS selfconsistent theory (Randeria et al.)
Analytical solution

$$\frac{E}{N} = E_{FG} - \frac{|\varepsilon_b|}{2}$$

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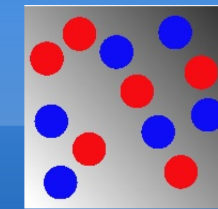


Energy per particle minus binding energy
(observation of beyond leading order contributions)

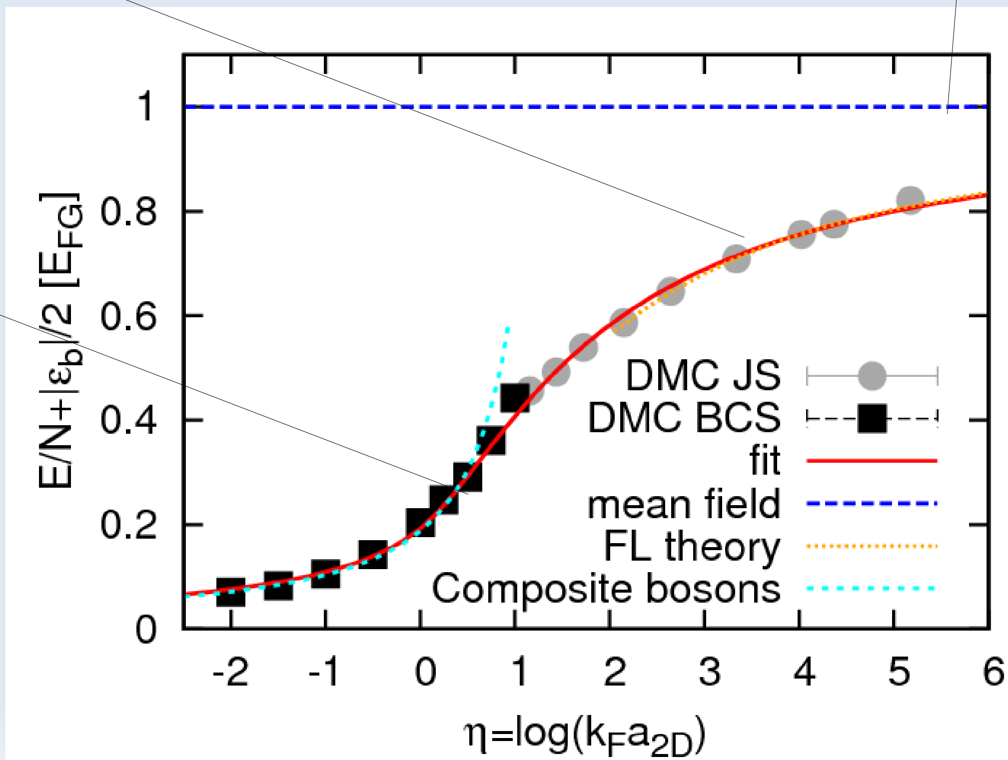
BCS selfconsistent theory
misses interaction among bosons

Fermi liquid energy functional (small gap)

$$\frac{E}{N}=E_{FG}\left(1-\frac{1}{\eta}+\frac{A}{\eta^2}\right)$$

Composite bosons: a_{dd} ~ 0.55(4) a_{2D}
(Petrov et al.)

$$\frac{E}{N}=-\frac{\varepsilon_b}{2}+\frac{\pi\hbar^2 n_d}{m_d}\frac{1}{\log\frac{1}{n_d a_{dd}^2}}\left[1+2^{nd}\,order\right]$$



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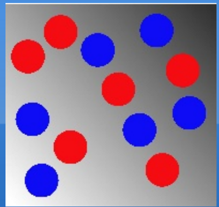
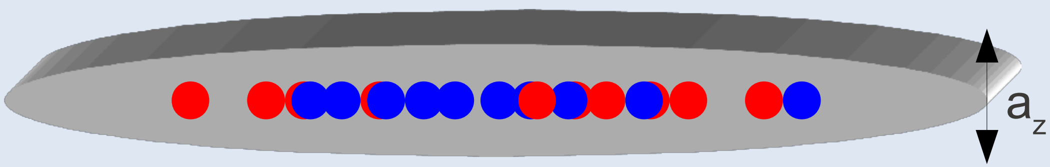
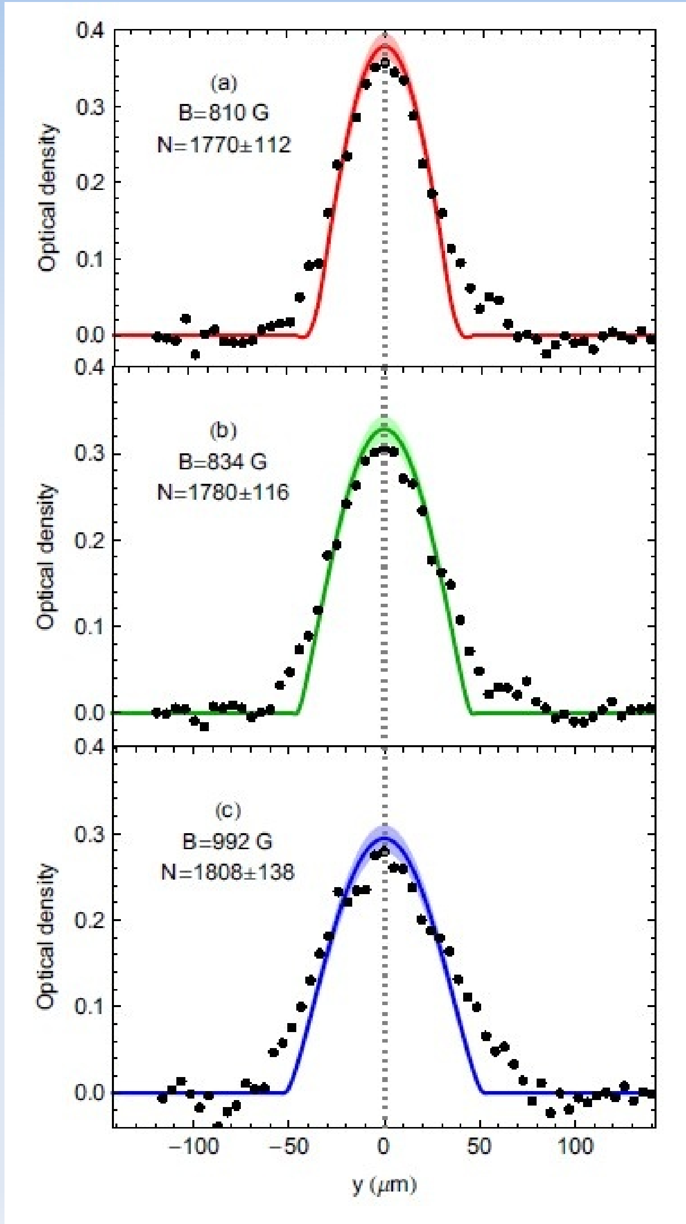


LDA + EOS → Description of non uniform system

It works in case of small T and large density





Orel et al. , New J. Phys. 113032 (2011).

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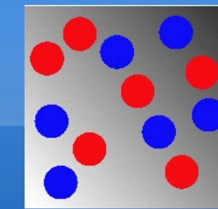
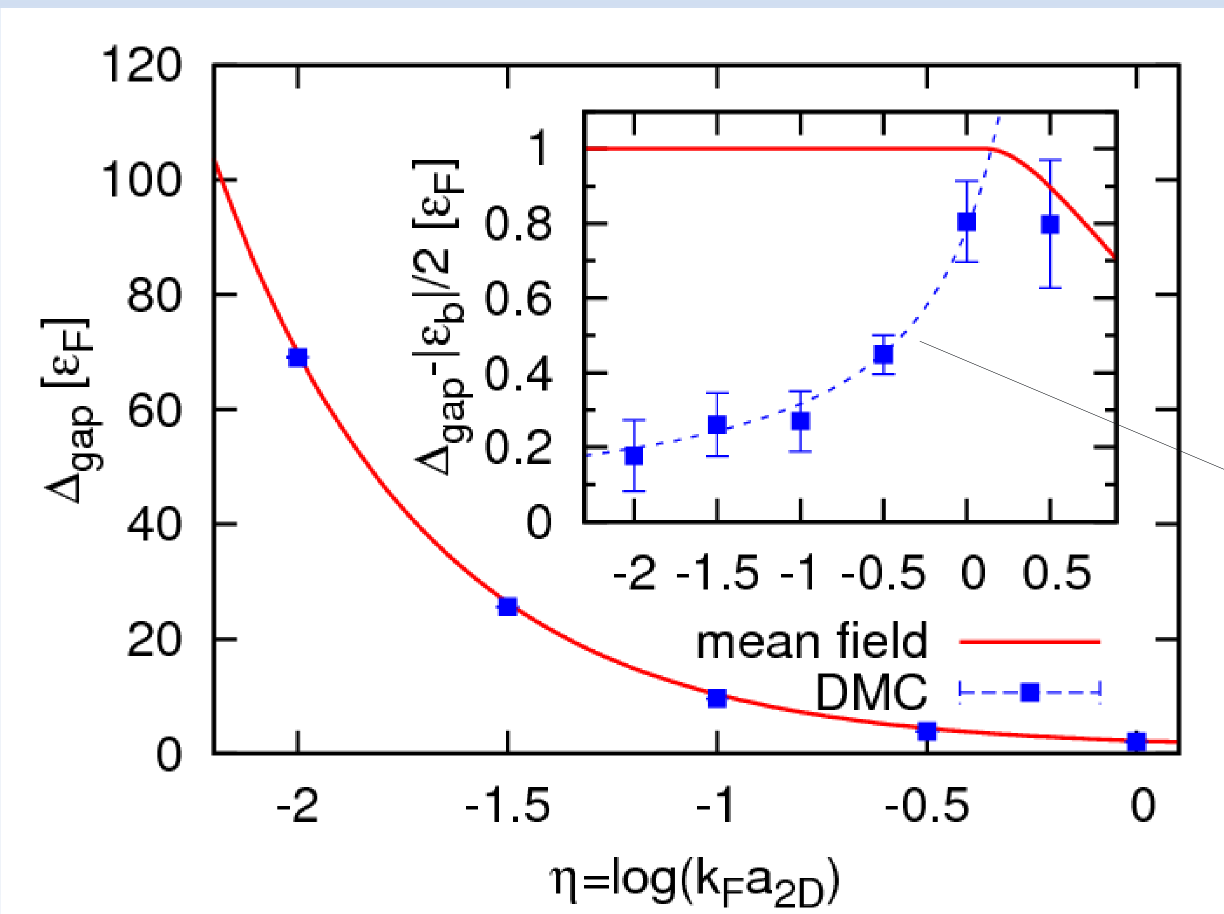


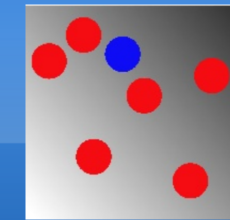
Gap in the spectrum

$$E(N_\uparrow+1, N_\downarrow) = E(N_\uparrow, N_\downarrow) + \mu_\uparrow + \Delta_{gap}$$

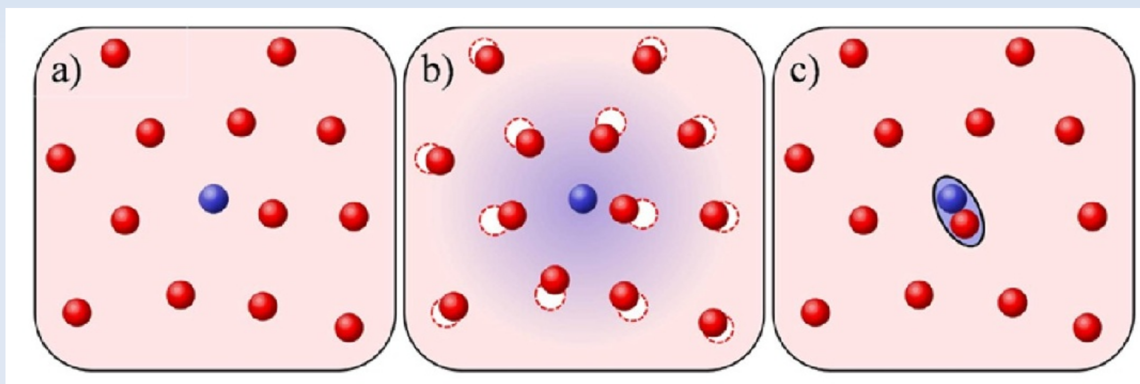


Interpretation:
One fermion in a BEC
$a_{ad} \sim 1.7(1)\, a_{2D}$

Polarons and molecules in 2D



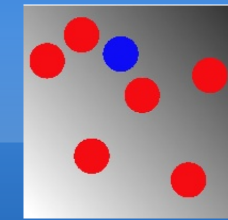
Same system, extremely polarized:
one ↓ in a ↑ Fermi sea
Many-body localized perturbation (Polaron)
or Two-body molecular state?



Picture from Schirotzek et al. PRL 102 (2009)

Available variational (particle-hole / T-matrix) calculations and experiments

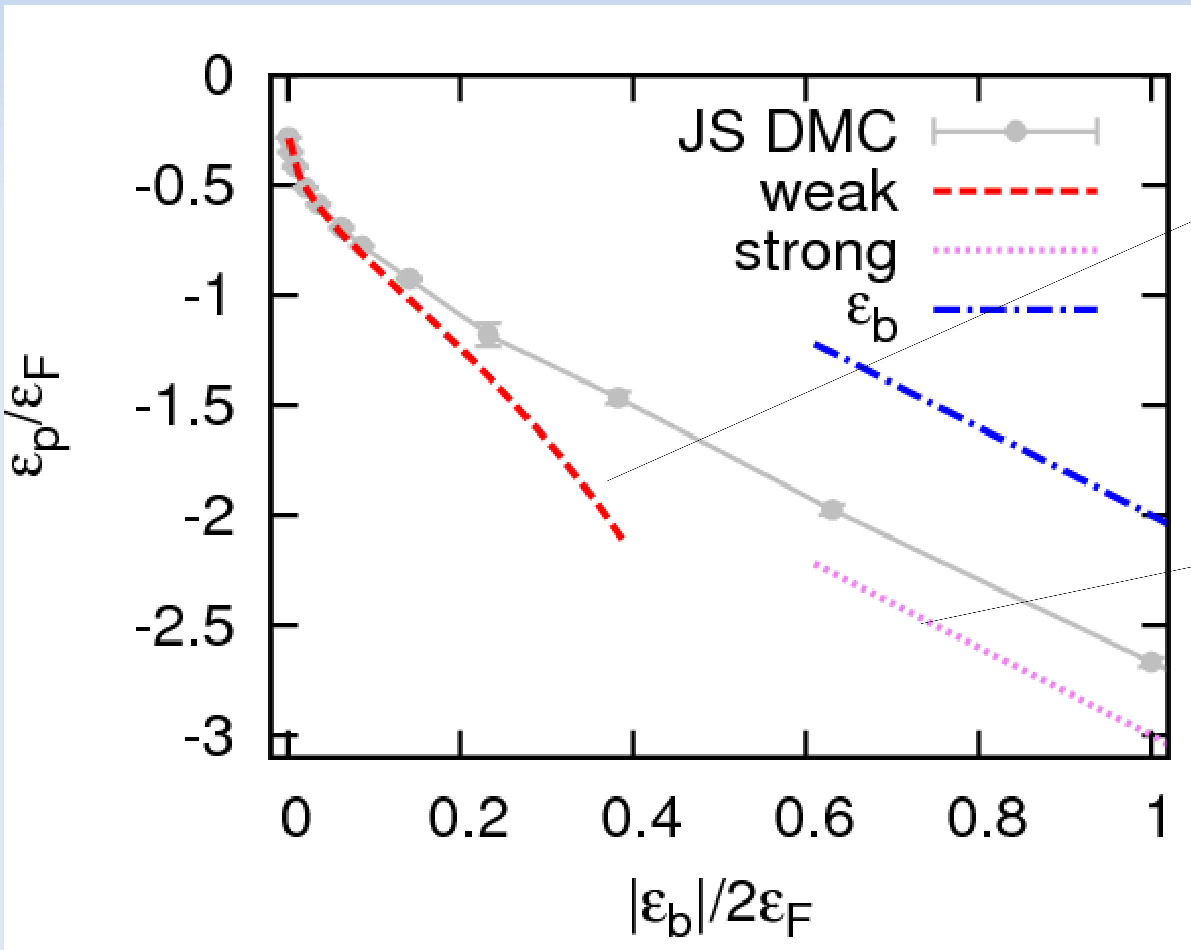
Polarons and molecules in 2D



Preliminary results

Nodal surface:
polaronic picture

$$\Psi_{JS} = J_{\uparrow\downarrow} D_{\uparrow}(N_{\uparrow})$$



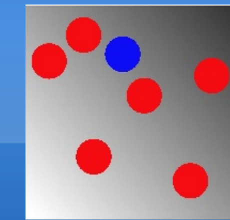
Fermi liquid
weak coupling expansion

Strong coupling,
massive limit $\varepsilon_P = \varepsilon_b - \varepsilon_F$

Finite size scaling:
$N_\uparrow$ = 13, 25, 37, 49

Polarons and molecules in 2D

Preliminary results



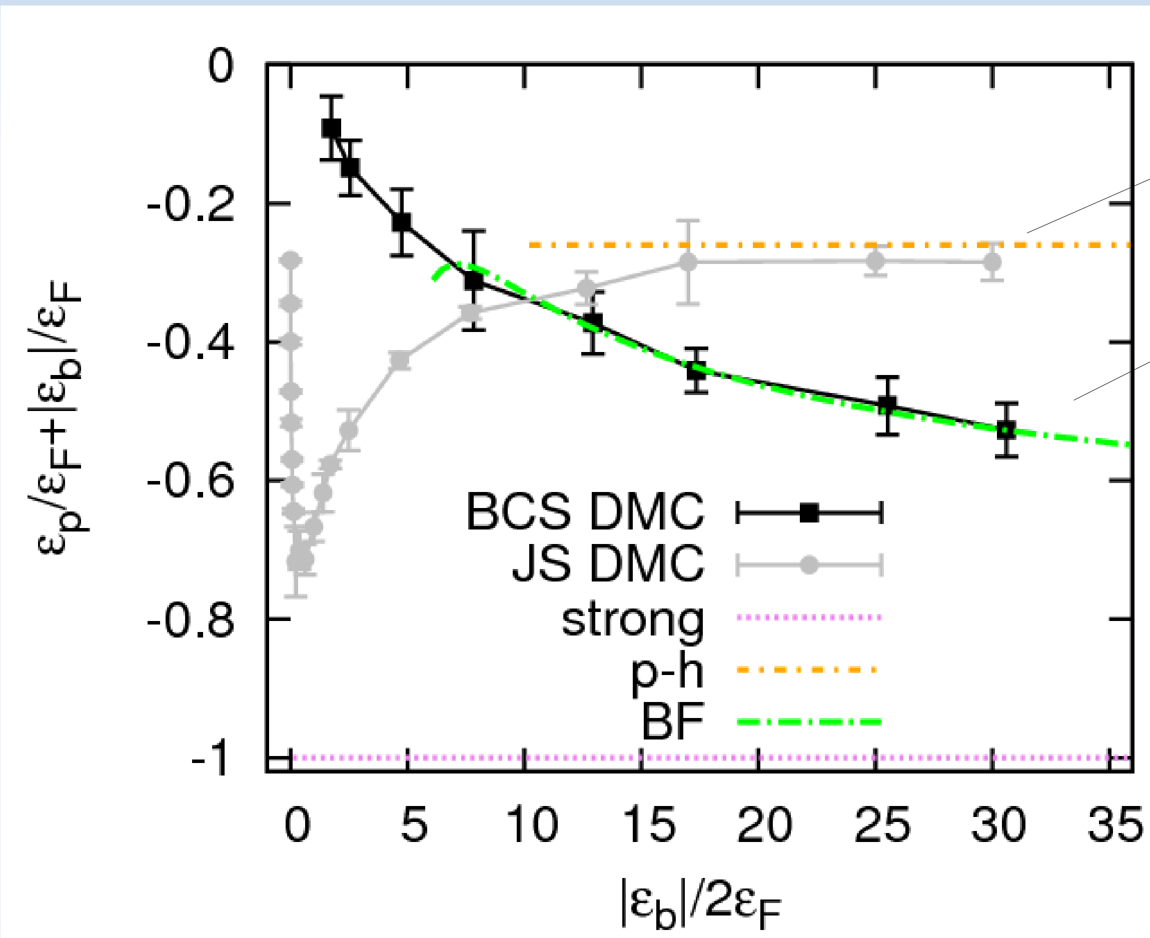
Nodal surfaces:

polaronic picture

molecular picture

$$\Psi_{JS} = J_{\uparrow\downarrow} D_{\uparrow}(N_{\uparrow})$$

$$\Psi_{BCS} = A[\phi(x_{11'})\psi_{k_1}(x_2)\ldots\psi_{k_{N_\uparrow}}(x_{N_\uparrow})]$$



Zollner et al. PRA 83 (2011)
$\varepsilon_P = \varepsilon_b - 0.26\,\varepsilon_F$

A boson interacting with a Fermi sea:
$a_{ad} \sim 1.50(5)\, a_{2D}$
fit from 2nd order expansion

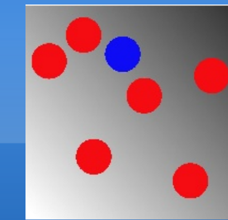
Transition: $\varepsilon_b/\varepsilon_F = 20(5)$

Parish PRA 83 (2011): $\varepsilon_b/\varepsilon_F = 10$

Experiment $\varepsilon_b/\varepsilon_F = 7(1)$

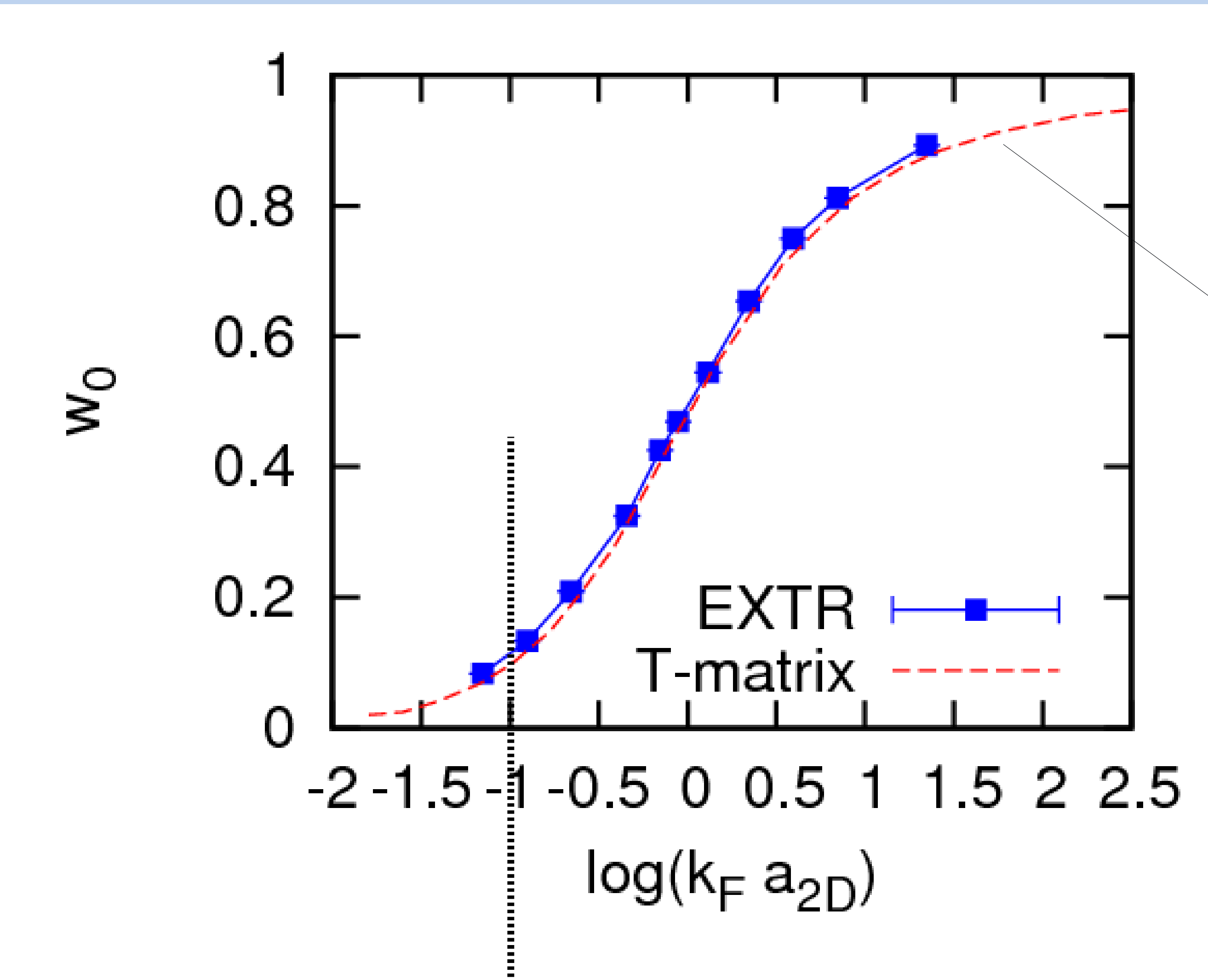
quasi-2D effects Levinsen et al.

Polarons and molecules in 2D



Preliminary results

Polaronic weight from n(k=0)



Measurable with RF spectroscopy

T-matrix from Schmidt et al. PRA 85 (2012)

Equivalence of p-h variational wf,
T-matrix,
FN approximation?

In 3D: Combescot et al.
PRL 98 (2007)
PRL 101 (2008)

Conclusions

- ✓ Relevance of beyond mean-field effects in 2D
- ✓ Need for next-to-leading order calculation of energy functionals

Outlook

- ✗ Phase diagram in 2D as a function of interaction and polarization
- ✗ Finite size effects in the polarized BCS nodal surface